

Experimental Verification of the Damage Locating Vector Method

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ABSTRACT

In recent years, numerous approaches have been proposed for detecting damage in structures, in which the flexibility-based Damage Locating Vector (DLV) method is one of the promising techniques. By computing a set of load vectors from the change of the flexibility matrix before and after damage and then applying them as static forces to analytical model for static computation, the DLV method is able to locate damage in structures. The main purpose of this paper is to experimentally verify this method. Following a brief overview of the DLV method and construction of the flexibility matrix from experimental data, the experimental setup is described. The test structure is a 15 ft long three-dimensional truss structure. To simulate damage in the structure, the original truss member is replaced by one with reduced stiffness. Results show that the DLV method can successfully detect the damage using limited number of sensors and truncated modes.

INTRODUCTION

The condition of civil infrastructure is in a the state of decline due to daily use, corrosion, natural hazards, etc., and thus monitoring its condition becomes a very important issue. By continuously monitoring the condition of the structure, necessary measures can be carried out at an early stage, which could substantially

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reduce the maintenance costs and reduce the likelihood of future collapse of the structure.

The first implementations of structural health monitoring (SHM) appear to have taken place in offshore structures and bridges. One class of damage identification methods employed for SHM measures the change in frequencies to determine structural damage. Vandiver [1] examined the change in resonant frequencies due to the damage in structural elements. Cha and Tuck-lee [2] examined the change in frequency response data and used this information for structural parameter updating. Changes in a structure's mode shapes have also been utilized. West [3] was perhaps the first to systematically use mode shape information for localization of structural damage without employing a prior finite element model. Another such technique takes advantage of the change in the flexibility matrix. Pandey and Biswas [4][5] presented a damage detection and localization method based on the flexibility changes in the structure. Recently, Bernal [6] proposed a promising new flexibility-based method, the Damage Locating Vector (DLV) method, for damage localization.

In this paper, following a brief overview of the DLV method, an approach to construct the flexibility matrix with inputs measured is presented. The DLV method is then verified through test of a 15 ft long, three-dimensional truss structure. Experimental results for two damage cases are presented.

PROBLEM FORMULATION AND SOLUTION

Health monitoring methods based on the flexibility matrix have recently been shown to be quite promising. Because an inverse relationship exists between the flexibility matrix and the square of the modal frequencies, the flexibility matrix is not sensitive to higher frequency modes. This unique characteristic allows the use of a small number of truncated modes to construct a reasonably accurate representation of the flexibility matrix.

The DLV Method

Bernal [6] proposed a flexibility-based damage localization method, the DLV method. This technique is based upon the determination of a special set of vectors, the so-called damage locating vectors (DLVs). The DLVs have the property that when they are applied to the structure as static forces at the sensor locations, no stress will be induced in the damaged elements. This unique characteristic can be employed to localize structural damage.

First, let's look at how to obtain the DLVs. For a linear structure, the flexibility matrices at sensor locations can be constructed from measured data before and after damage and denoted as \mathbf{F}_u and \mathbf{F}_d , respectively. Assume that we have a set of linear-independent load vectors \mathbf{L} , which satisfy the following relationship

$$\mathbf{F}_d \mathbf{L} = \mathbf{F}_u \mathbf{L} \quad \text{or} \quad \mathbf{F}_\Delta \mathbf{L} = (\mathbf{F}_d - \mathbf{F}_u) \mathbf{L} = 0 \quad (1)$$

This equation implies that the load vectors \mathbf{L} produce the same displacements at the

sensor locations before and after damage. From the definition, the DLVs are seen to also satisfy Eq. (1); that is, because the DLVs induce no stress in the damaged elements, the damage of those elements does not affect the displacements at the sensor locations. Therefore, the DLVs are indeed the vectors in \mathbf{L} .

To calculate \mathbf{L} , the singular value decomposition (SVD) can be used. The SVD of the flexibility difference matrix \mathbf{F}_Δ leads to

$$\mathbf{F}_\Delta = \mathbf{U}\mathbf{S}\mathbf{V}^T = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_0 \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_0 \end{bmatrix}^T \quad (2)$$

or, equivalently

$$\begin{bmatrix} \mathbf{F}_\Delta \mathbf{V}_1 & \mathbf{F}_\Delta \mathbf{V}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{U}_1 \mathbf{S}_1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{F}_\Delta \mathbf{V}_0 = 0 \quad (3)$$

Eqs. (1) and (3) indicate that $\mathbf{L} = \mathbf{V}_0$, *i.e.*, DLVs can be obtained from the SVD of the difference matrix \mathbf{F}_Δ .

Each of the DLVs is then applied to the undamaged analytical model of the structure, and the stress in each structural element is calculated. If an element has a zero normalized accumulative stress $\bar{\sigma}_j$, then this element is a possible candidate of damage. The normalized accumulative stress for the j th element is defined as

$$\bar{\sigma}_j = \frac{\sigma_j}{\max_k(\sigma_k)} \quad \text{in which} \quad \sigma_j = \sum_{i=1}^n \frac{\sigma_j^i}{\max_k(\sigma_k^i)} \quad (4)$$

in which σ_j^i = stress in the j th element induced by the i th DLV; σ_j = cumulative stress in the j th element. In practice, $\bar{\sigma}_j$ induced by DLVs in the damaged elements may not be exactly zero due to noise and uncertainties. Therefore, a small value of $\bar{\sigma}_j$ indicates a possible damage location.

Constructing Flexibility Matrix Using Limited Sensor Information

As shown in the previous section, the flexibility matrix needs to be constructed from the measurement data to implement the DLV method. When the input is measured, and there is at least one co-located sensor and actuator pair, the experimental data can be used to construct the flexibility matrix and no additional information is required [7].

First, the state space representation of the structure can be obtained using various realization algorithms, such as Eigensystem Realization Algorithm (ERA) [8]. The state space representation is given by

$$\begin{aligned} \dot{x} &= \mathbf{A}x + \mathbf{B}u \\ y &= \mathbf{C}x + \mathbf{D}u \end{aligned} \quad (5)$$

where x = state vector; u = input vector; and y = output vector. Taking the Fourier Transform of Eq. (5) yields the following relationship between the input and output

$$y(\omega) = [\mathbf{C}[\mathbf{I} \cdot i\omega - \mathbf{A}]^{-1}\mathbf{B} + \mathbf{D}]u(\omega) \quad (6)$$

Based on sensors used in the experiment, the displacement vector $y_D(\omega)$ can then be expressed as

$$y_D(\omega) = \frac{1}{(i\omega)^p}[\mathbf{C}[\mathbf{I} \cdot i\omega - \mathbf{A}]^{-1}\mathbf{B} + \mathbf{D}]u(\omega) \quad (7)$$

where $p = 0, 1,$ and 2 when outputs are displacement, velocity, and acceleration, respectively. If we note that the flexibility matrix relates the inputs to the outputs at $\omega = 0$, the flexibility matrix can be obtained as

$$\mathbf{F}_f = \lim_{\omega \rightarrow 0} \left[\frac{1}{(i\omega)^p} [\mathbf{C}[\mathbf{I} \cdot i\omega - \mathbf{A}]^{-1}\mathbf{B} + \mathbf{D}] \right] \quad (8)$$

$$\mathbf{F}_f = -\mathbf{C}\mathbf{A}^{-(p+1)}\mathbf{B} \quad (9)$$

Based on this derivation, note that each column of \mathbf{F}_f is associated with each input and each row is associated with a sensor location. We denote the flexibility matrix at sensor locations as \mathbf{F}_s ; columns of \mathbf{F}_s then correspond to inputs applied at sensor locations. So, if there are any inputs located at the sensor locations, the associated columns in matrices \mathbf{F}_f and \mathbf{F}_s will be the same. Define two Boolean matrices \mathbf{q}_f and \mathbf{q}_s which pick out these columns from \mathbf{F}_f and \mathbf{F}_s , respectively. We have

$$\mathbf{F}_s\mathbf{q}_s = \mathbf{F}_f\mathbf{q}_f \quad \text{and} \quad \mathbf{F}_s\mathbf{q}_s = -\mathbf{C}\mathbf{A}^{-(p+1)}\mathbf{B}\mathbf{q}_f \quad (10)$$

Expressing the flexibility \mathbf{F}_s and the system matrix \mathbf{A} in terms of their eigenvalues and eigenvectors gives

$$\frac{\varphi_m}{\mathbf{v}}\omega^{-2}\frac{\varphi_m^T}{\mathbf{v}}\mathbf{q}_s = -\mathbf{C}\boldsymbol{\psi}\boldsymbol{\lambda}^{-(p+1)}\boldsymbol{\psi}^{-1}\mathbf{B}\mathbf{q}_f \quad (11)$$

where φ_m = mode shapes at the measured degree of freedoms (DOFs); \mathbf{v} = matrix of mass normalized index; $\boldsymbol{\lambda}$ and $\boldsymbol{\psi}$ = eigenvalue and eigenvector matrices of the system matrix \mathbf{A} , respectively. It is important to note from Eq. (11) that, for classical damping, a mode-by-mode equality can be established. Eq. (11) can then be written as

$$[\varphi_m]_j(\omega_j\nu_j)^{-2}[\varphi_m^T]_j\mathbf{q}_s = -2 \cdot \text{real}(\mathbf{C}\boldsymbol{\psi}_j\lambda_j^{-(p+1)}\boldsymbol{\psi}_j^{-1}\mathbf{B}\mathbf{q}_f) \quad (12)$$

In Eq. (12), only v_j , the diagonal term of \mathbf{v} , is unknown. After v_j is calculated – i.e., the diagonal matrix \mathbf{v} is obtained – the flexibility matrix at the sensor locations can be constructed from the relationship

$$\mathbf{F}_m = (\boldsymbol{\phi}_m \mathbf{v}^{-1}) \boldsymbol{\omega}^{-2} (\boldsymbol{\phi}_m \mathbf{v}^{-1})^T \quad (13)$$

EXPERIMENTAL VERIFICATION

The DLV method is experimentally verified by using a 15 ft long, three-dimensional truss structure, which is shown as Fig. 1. In this section, the experimental setup is first described and the experimental results are then presented.

Experimental Setup

The three-dimensional truss structure was tested at the Smart Structures Technology Laboratory (SSTL) of the University of Illinois at Urbana-Champaign (<http://cee.uiuc.edu/sstl/>). The length of each bay of the truss is 1.3 ft on each side. The truss sits on two rigid supports. One end of the truss is pinned to the support, and the other is roller-supported. The pinned end can rotate freely with all three translations restricted. The roller end can move in the longitudinal direction and rotation about the longitudinal axis is not allowed.

The truss members are steel tubes with inner diameter of 0.428 in and outer diameter of 0.612 in. The joints of the elements are specially designed so that the truss member can be easily removed and replaced to simulate damage without disassembling the whole structure. A detailed picture of the joint is shown in Fig. 1. As can be seen, the truss member can be removed by unscrewing the collar towards the joint. On the other hand, if the collar is screwed away from the joint, this member can be easily installed.

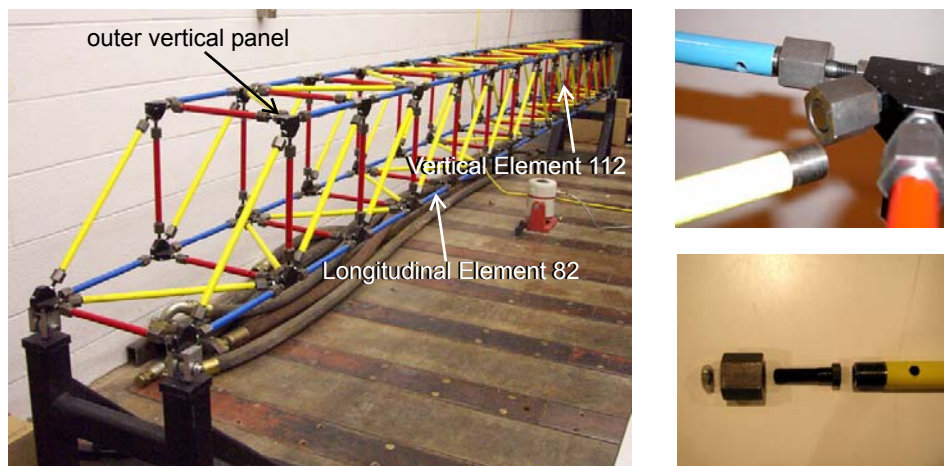


Figure 1. Fifteen-foot truss structure

The truss is excited vertically by a permanent magnetic shaker that can generate a maximum force of 20 lbs with a dynamic performance ranging from 1 Hz to 8000 Hz. The shaker is connected to the bottom of the outer panel using a stinger. A load cell is installed between the stinger and the bottom of the joint to monitor and measure the input to the structure.

Accelerometers are attached to the truss joints with magnetic bases. Siglab is used to drive the shaker and measure the accelerations and the excitation. Two four-channel 20-42 Siglab box are synchronized to measure eight channel of data simultaneously.

Using limited sensors to monitor all members in a complex structure might be difficult. In this experimental verification, the 53 elements in the outer panel (shown in Fig. 2), which are elements 14 through 25 and 79 through 119, are monitored using 13 accelerometers, which are installed vertically at the joints of the lower chord. Due to the fact that only 8 channels are available, the 13 accelerations are measured in two sequential experiments. In this way, mode shapes at these 13 DOFs can still be established. The shaker is connected to one of the joints at the bottom, so there is one co-located sensor and actuator pair.

Experimental Results

There are two damage cases studied in this experimental verification. One is a 40% stiffness reduction in a longitudinal element and the other is a 40% stiffness reduction in a vertical element.

CASE 1 STUDY

In this case, longitudinal element 82 in the lower chord is replaced by a tube with 40% stiffness reduction.

The transfer functions are measured first. Typical experimentally-measured transfer functions are shown in Fig. 3. The modal parameters are then obtained from these transfer functions before and after damage using the ERA. Here, the first 6 dominant natural frequencies, which are numbered in Fig. 3, are identified. The

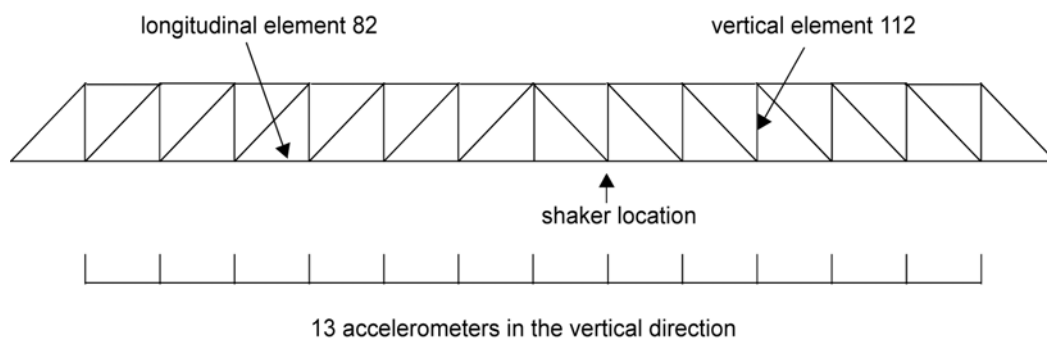


Figure 2. Sketch of the outer vertical panel (elements 14 through 25 and 79 through 119)

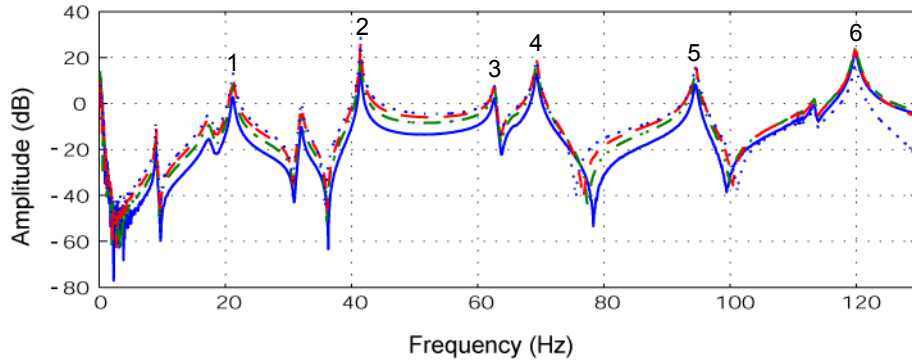


Figure 3. Experimentally measured transfer functions

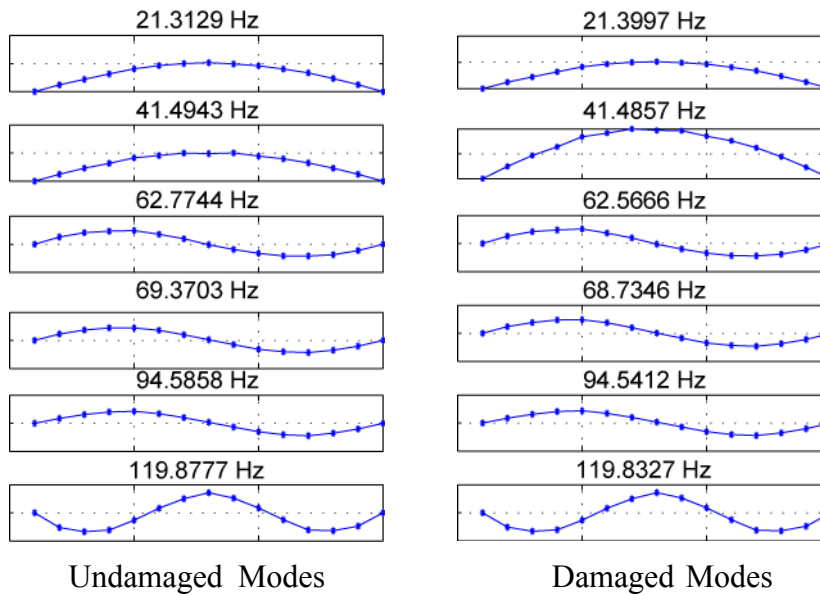


Figure 4. Experimentally identified mode shapes

corresponding mode shapes are also extracted as shown in Fig. 4. As noted, there is very little change in the frequencies and mode shapes. It is obvious that direct comparison between the frequencies and mode shapes to detect damage is difficult if not possible in this case. Herein, the DLV method is employed to detect the damage.

Once the modal parameters before and after damaged are obtained, the flexibility matrix at the sensor locations can be constructed following the procedure described above. The DLVs are then computed from the difference matrix F_{Δ} using Eqs. (2) and (3), and applied as vertical loads at the sensor locations to the undamaged analytical model for static computation. The normalized accumulative stress can then be obtained and employed to locate the damage in the structure.

The results of the normalized accumulative stress is shown in Fig 5. As can be seen, the normalized accumulative stress for element 82 is considerably smaller than others elements. However, element 17, which is not a damaged element, also has a

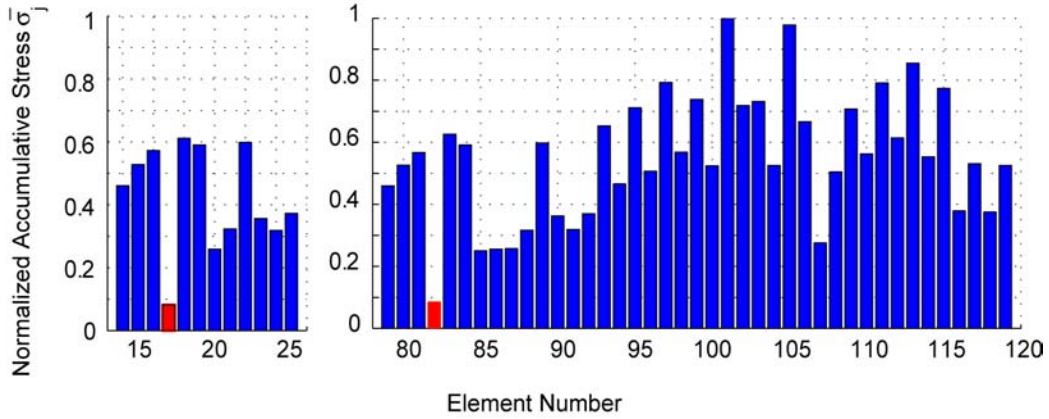


Figure 5. Normalized accumulative stress for the case when element 82 is damaged

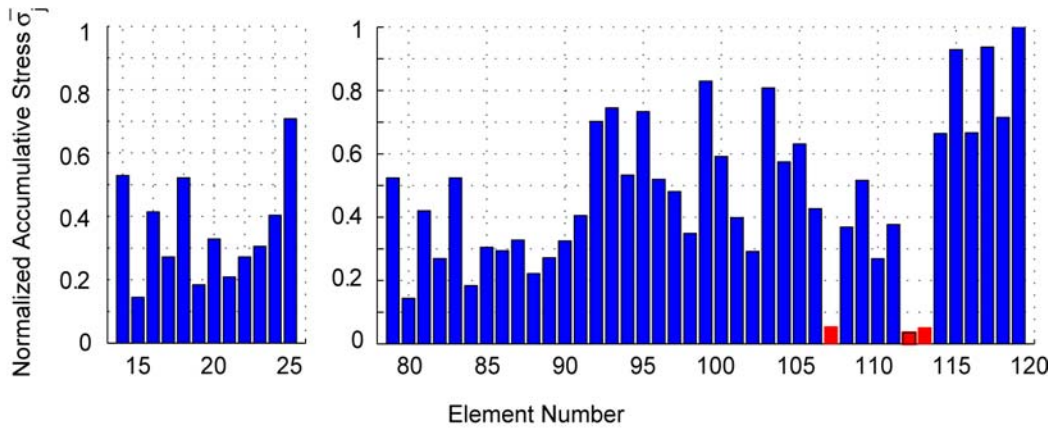


Figure 6. Normalized accumulative stress for the case when element 112 is damaged

small value of normalized accumulative stress. Therefore, element 82 can only be identified as a potentially damaged element.

CASE 2 STUDY

In this case, the vertical element, element 112, is replaced by a tube with 40% stiffness reduction instead of a longitudinal element as presented in CASE 1. Here again, the DLV method successfully detected this element as a possible damage element. The results are shown in Fig. 6. As illustrated, the damaged element, element 112, is correctly identified among the candidates of damage locations.

CONCLUSIONS

The DLV method has been successfully verified using experimental data. The experimental results show that the change of modal properties subjected to a 40% stiffness reduction of single member is very small. Direct comparison of the modal properties to detect damage is very difficult if it is not impossible for this truss. Techniques which are more sensitive to structural damage is highly desirable. By using a flexibility-based DLV method, damage in this truss can be correctly located using only limited number of sensors and truncated modes.

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