6th International Conference on Advances in Experimental Structural Engineering
11th International Workshop on Advanced Smart Materials and Smart Structures Technology
August 1-2, 2015, University of Illinois, Urbana-Champaign, United States

Optimal Design of Visco-Elastic Devices Coupling Two Simple Oscillator Under Seismic Excitation

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ABSTRACT
Optimal design of two simple oscillators coupled by visco-elastic connection under seismic excitation is discussed. A solution to determine the optimal values of the coupling stiffness and damping coefficient is presented. The connection is described by both Maxwell and Kelvin-Voigt damper model. The response of the system under white noise seismic excitation is used to evaluate selected performance indexes varying the two control parameters. The procedure is enriched by considering the effects of ground acceleration represented by a Kanai-Tajimi filtered non-stationary process.

KEYWORDS: *Coupled oscillators, Structural control, Optimal parameters, Seismic response*

1. INTRODUCTION

The research field related with the mitigation of the seismic demands belongs to the structural control topics. Actually all methodologies used for the structural control are divided in three section: active, semi-active and passive control [1, 2]. The first categories require external power source to control the systems (eventually inducing energy into the system in the active case and only changing the mechanical characteristic of the device in the semi-active one) and the use of sensors and real-time controllers and actuators. The passive control has the aim to increase the damping in the system enhancing its dissipative capacity. In the semi-active control is possible change the structural proprieties of the devices (stiffness and damping) on the base of the information obtained during the seismic response of the structures using sensors and real-time controllers appropriately disposed in the structure. In the last two decades great effort has been made in this fields obtaining considerable results [3]. The design solutions offered in the recent available literature differ in addressing either the damping system or the damped structure in a continuous attempt to balance the opposite needs of synthesis and representativeness. A pair of simple oscillators or a pair of equivalent one-dimensional beams, coupled with a variety of damping devices, have often been employed as a synthetic but a representative model to describe a wide class of structural realizations – for instance, adjacent tall buildings – or quasi-independent sub-systems composing a single complex structure.
Several studies have been carried out for the optimization and design of the structures. For example, different strategies have been proposed for the optimal placement of viscous-type coupling devices into seismic joints to dissipate energy and avoid hammering phenomena [4, 5]. A series of studies has also been devoted to dissipative interconnections realized through hysteretic dampers [6], friction dampers [7] or semi-active devices [8]. In spite of the inherent complexity related to the description of the peculiar properties of each specific device, the simplest elasto-viscous constitutive laws are reproduced by the Kelvin–Voigt (KV) or Maxwell (Ma) model, fully described by two parameters, a stiffness and a viscous damping coefficient, whose assessment may play an important role in the preliminary design stage. Since the earliest studies [9], in which the role of the stiffness was neglected, it has been evident that the viscous damping coefficient may attain an optimal value, not necessarily the highest possible, depending on the relative characteristics of the coupled oscillators. An optimization criterion has been introduced in [10], in which the minimization of the total energy has been used to find the optimal relations.
In the present work a comparison of the performances of a pair of simple oscillator subject to a different excitation (white noise and seismic ground motion) is pursued. The focus is on a device for which the structural behaviour is described by a Kelvin-Voigt model and designed using different criteria. The final scenario obtained will be successively compared using a Maxwell model for the constitutive laws of the device.

2. EQUATION OF MOTION

Consider two simple linear oscillators with mass M_j and stiffness K_j, (j=1,2), coupled by a passive damper (Figure 1.1). Denoting U_1 and U_2 the relative horizontal displacements and F the mutual force applied by the coupling damper, the dynamic response of the two-degrees-of-freedom (*dofs*) system to a synchronous horizontal ground displacement U_g, is governed by the equations:

$$\begin{aligned} M_1\ddot{U}_1 + K_1U_1 - F &= -M_1\ddot{U}_g \\ M_2\ddot{U}_2 + K_2U_2 + F &= -M_2\ddot{U}_g \end{aligned} \tag{2.1}$$

where dot indicates derivative with respect to time t. Denoting L a convenient reference length, and the following dimensionless variables and parameters can be introduced

$$u_j = \frac{U_j}{L}, \quad u_g = \frac{U_g}{L}, \quad \omega_j^2 = \frac{K_j}{M_j}, \quad \beta = \frac{\omega_2}{\omega_1}, \quad \rho = \frac{M_2}{M_1}, \quad u = \frac{F}{\omega_1^2 M_1 L}, \quad \tau = \omega_1 t \tag{2.2}$$

where the dimensionless force u is understood as the control variable, and the relevant parameters ρ and β stand for the mass and frequency ratio between the two uncoupled oscillators, respectively.
The equations of motion can be rewritten in the synthetic form

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{s}\,u(\mathbf{u},\dot{\mathbf{u}}) = -\mathbf{M}\mathbf{r}\,\ddot{u}_g \tag{2.3}$$

where $\mathbf{u}$ is the displacement vector, $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices, $\mathbf{s}$ and $\mathbf{r}$ are the position vectors of the control and external forces

$$\mathbf{u} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}, \quad \mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} 1 & 0 \\ 0 & \rho\beta^2 \end{bmatrix}, \quad \mathbf{s} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}, \quad \mathbf{r} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \tag{2.4}$$

Different rheological models of the coupling damper are introduced to define the constitutive law $u(\mathbf{u},\dot{\mathbf{u}})$, relating the control force to the displacement/velocity vector.
Adopting a state-space representation, with the use of the state vector $\mathbf{x} = \{\mathbf{u}^T, \dot{\mathbf{u}}^T\}^T$, Eq. (2.3) can be rewritten as follows:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}w + \mathbf{d}\ddot{u}_g \tag{2.5}$$

where the state matrix $\mathbf{A}$, the allocation control vector $\mathbf{b}$, the external input vector $\mathbf{d}$ are, respectively

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & \mathbf{0} \end{bmatrix}, \quad \mathbf{b} = \begin{Bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{s} \end{Bmatrix}, \quad \mathbf{d} = \begin{Bmatrix} \mathbf{0} \\ -\mathbf{r} \end{Bmatrix} \tag{2.6}$$

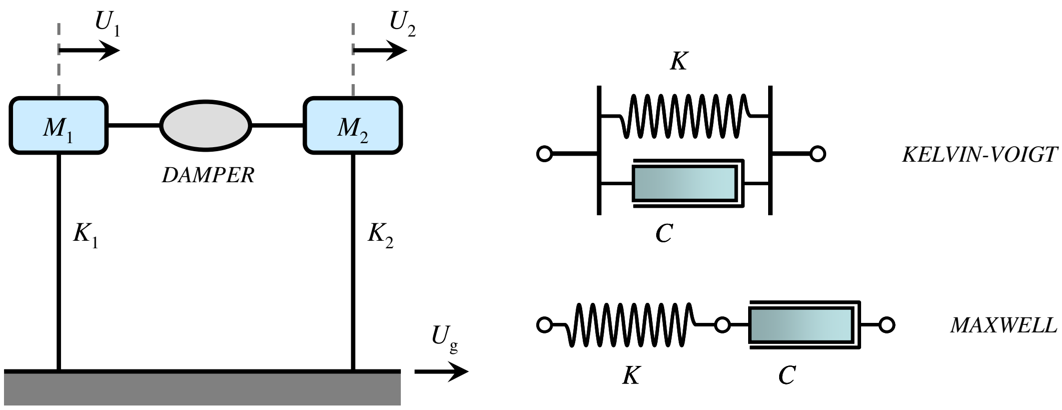

Figure 1.1 Two simple oscillators coupled by a visco-elastic damper.

Constitutive models describing with increasing complexity the damper behavior can be formulated joining, in different combination schemes, simple elements: a linear spring with elastic constant *K*, and a linear dashpot with viscous constant *C*. Introducing the dimensionless parameters

$$\eta = \frac{K}{\omega_1^2 M_1}, \qquad \gamma = \frac{C}{\omega_1 M_1} \tag{2.7}$$

the Kelvin-Voigt (*KV*) and the Maxwell (*Ma*) model correspond to the alternative parallel or series combination of the spring and the dashpot, respectively. Consequently, the constitutive law $u(\mathbf{u}, \dot{\mathbf{u}})$ reads

- KV model $\quad u = \eta(u_2 - u_1) + \gamma(\dot{u}_2 - \dot{u}_1)$ (2.8)
- Ma model $\quad u = 2\gamma(\dot{u}_2 - \dot{u}_1 - \dot{u}/\gamma)$ (2.9)

1.1. White noise excitation

Consider two simple oscillators forced by white noise, **w**(t), for which the following assumptions hold

$$E[\mathbf{x}] = 0 \text{ if } \mathbf{x}(0) = \mathbf{0} \text{ and } E[\mathbf{w}(t)\mathbf{w}(t+\tau)] = 2\pi S_0 \delta(\tau) \tag{2.10}$$

in which the first term is the mean of the state vector **x** while the second represents the autocorrelation function of the white noise vector.
The covariance matrix is defined as following

$$\mathbf{\Gamma}_\mathbf{x} = \begin{bmatrix} \sigma_{u1}^2 & \sigma_{u1u2}^2 & \sigma_{u1\dot{u}1}^2 & \sigma_{u1\dot{u}2}^2 \\ \sigma_{u2u1}^2 & \sigma_{u2}^2 & \sigma_{u2\dot{u}1}^2 & \sigma_{u2\dot{u}2}^2 \\ \sigma_{\dot{u}1u1}^2 & \sigma_{\dot{u}1u2}^2 & \sigma_{\dot{u}1}^2 & \sigma_{\dot{u}1\dot{u}2}^2 \\ \sigma_{\dot{u}2u1}^2 & \sigma_{\dot{u}2u2}^2 & \sigma_{\dot{u}2\dot{u}1}^2 & \sigma_{\dot{u}2}^2 \end{bmatrix} = \begin{bmatrix} E[u_1^2] & E[u_1 u_2] & E[u_1 \dot{u}_1] & E[u_1 \dot{u}_2] \\ E[u_2 u_1] & E[u_2^2] & E[u_2 \dot{u}_1] & E[u_2 \dot{u}_2] \\ E[\dot{u}_1 u_1] & E[\dot{u}_1 u_2] & E[\dot{u}_1^2] & E[\dot{u}_1 \dot{u}_2] \\ E[\dot{u}_2 u_1] & E[\dot{u}_2 u_2] & E[\dot{u}_2 \dot{u}_1] & E[\dot{u}_2^2] \end{bmatrix} \tag{2.11}$$

with the covariance matrix governed by the following differential equation

$$\dot{\mathbf{\Gamma}} = \mathbf{A}\mathbf{\Gamma} + \mathbf{\Gamma}\mathbf{A}^\mathrm{T} + \mathbf{b}(2\pi S_o)\mathbf{b}^\mathrm{T} \tag{2.12}$$

where **A** is the space-state matrix while **b** is the vector for the allocation of the control forces described in the previous Eq. 2.6. Of course, to find the stationary covariance the following Lyapunov equation has to be solved:

$$\mathbf{0} = \mathbf{A}\mathbf{\Gamma} + \mathbf{\Gamma}\mathbf{A}^\mathrm{T} + \mathbf{b}(2\pi S_o)\mathbf{b}^\mathrm{T} \tag{2.12}$$

2. PASSIVE CONTROL STRATEGIES

Among several methods proposed in the literature to define the design parameters characterizing fluid-viscous or viscous-elastic dampers for vibration mitigation [11, 12], in this paper a comparison between the performance of the design points determined by the underlying strategies is pursued. They are based on the choice of the design parameters, η and γ, such that the following objectives are reached:

1. Eigenvalue coincidence of the space-state matrix **A**.
2. Minimum of the standard deviation $\sigma_{u1.}$
3. Minimum of the standard deviation $\sigma_{u2.}$
4. Minimization of the max between the covariance σ_{u1} and σ_{u2}.

A depth analysis of the performance provided by all possible system designed applying the first strategy is deeply described in [13]. This methodology has been ideated looking at the eigenvalue parameter loci in the Argand plane. In Figure 2.1 (a) is illustrated, for example, the loci of the two complex eigenvalue obtained varying the γ-parameter for different values of η, for a pair of oscillators characterizing by the structural parameters ρ = 6.67 and β = 4.

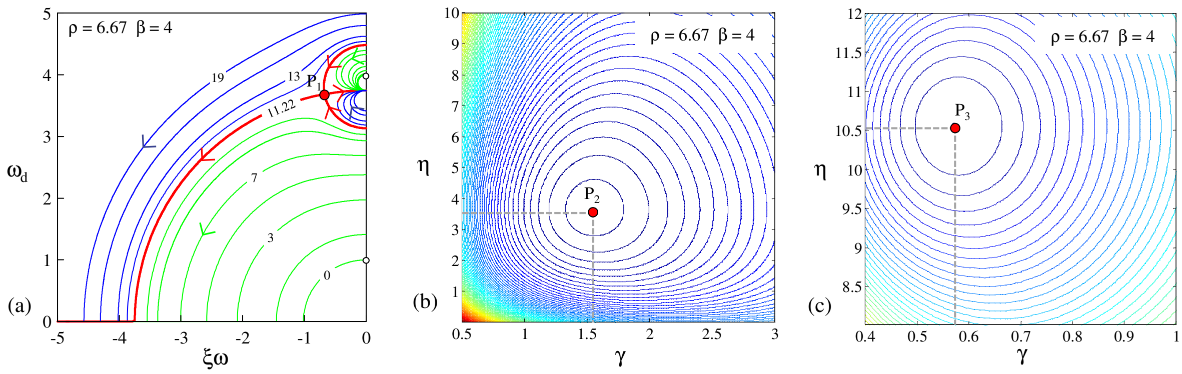

Figure 2.1 Two oscillator coupled by a Kelvin-Voigt damper: (a) iso-η eigenvalue loci varying γ parameter; contour plots of the variances σ_{u1} (b) and σ_{u2} (c) varying η and γ parameter.

Moreover it is right to remember that the imaginary parts correspond to the damped frequency while the real part is proportional to the modal damping ratio. It can be immediately notice that for a particular value of the pair η and γ the eigenvalues assume the same real and imaginary parts. This special occurrence (P_1 in the Figure 2.1 a) is considered as possible design point. Figure 2.1 regards a system coupled by a *KV* damper model but the same behaviour can be observed in also for a *Ma* damper model as well described in [13]. Moreover, in [13], for the *KV* model, are also reported the analytical formulas that determine the values of η and γ realizing the coincidence of eigenvalues for fixed the structural parameters ρ and β. In Figures 2.1 (b) and 2.1 (c) are illustrated the contour plots of the standard deviations σ_{u1} and σ_{u2} varying η and γ parameters for the same oscillators pair considered for the Figure 2.1 (a). Also in this case a system coupled by a *KV* model is taken into account. For this two situation, in a wide space of design parameters, η and γ, is evident the presence of a minimum for both σ_{u1} and σ_{u2}. The points that identify the minimum of the two contours are named P_2 and P_3 and they can be considered as other two point of design for the same structural system. Moreover it is right to observe that in a wide space of the design parameters, one of the two deviation standard is always greater than the other one and so, regarding the fourth strategies, the design point will be equally obtained in the second or third strategies. In the oscillators pair, taken for example in this paper, the values of σ_{u1} are superior to those of σ_{u2} as well shown in the 3D view of the Figure 2.2 a. Therefore, in this situation, the design point determined applying the second and forth strategy are equal. In the below table 2.1, the values of the real and imaginary part of the system realized by the three points shown in Figure 2.1 are reported. Looking at this values, it is possible make the following considerations. The second and third strategies, for their definition, aim to optimize the performance of one of the two oscillators while the first identify a point that realize a sort of balance between the two oscillators. Indeed in term of damping, the real part of the coincident eigenvalues (0.67 in P_1) is collocated, more or less, in the middle of the minimum and maximum eigenvalue shown by the system in P_2 (1.59 and 0.18). On the others hand, looking at the stiffness, taken in account by the value of the imaginary part, the value in P_1 (3.69), is inserted between the two imaginary part calculated in the system obtained in P_2 (4.32 and 1.49). The last three columns are relative to the results of the deviations standard for the systems P_1, P_2 and P_3. Of course the minimum for the displacements u_1 and u_2 is obtained applying the second and third, respectively. For these two strategies the value of the displacement not optimized is always greater than the value determined by the first strategy. The application of this last strategy confirm the balance provided by the first method even if looking at the mean square standard deviation (σ_m) the minimum value is obtained in the system P_2. In Figure 2.2 (b), (c), (d) and (e) are illustrated the sections of the two manifold developed in the Figure 2.2 (a). They are obtained for fixed values of η and γ, which are those calculated in the points P_1 (black line), P_2 (red line) and P_3 (blue line). The main findings are the following: the behaviour of the deviation standard in P_1 is always near to that of the component not optimize in the others strategies; in the sections of the Figure 2.2 (b), (c), and (d) the values of the deviations standard never intersect with each other and moreover the deviation in P_1 is always placed within the other two deviations; for the section reported in the Figure 2.2 (e), the previous consideration is true only in a range of the minimum while distant from it the deviations intersect with each other and moreover the system P_3, in which there was the minimum, assume the greatest value.

Table 2.1 Values of the design parameter, eigenvalue and deviations standard for the systems P_1, P_2 and P_3.

Design Point	η	γ	$\xi_1\omega_1$	$\xi_2\omega_2$	ω_{d1}	ω_{d2}	σ_{u1}	σ_{u2}	σ_m
P_1	11.22	1.17	0.67	0.67	3.69	3.69	1.18	0.64	0.67
P_2	3.65	1.54	1.59	0.18	1.56	3.88	0.93	0.81	0.62
P_3	10.55	0.57	0.30	0.35	3.14	4.32	1.49	0.57	0.80

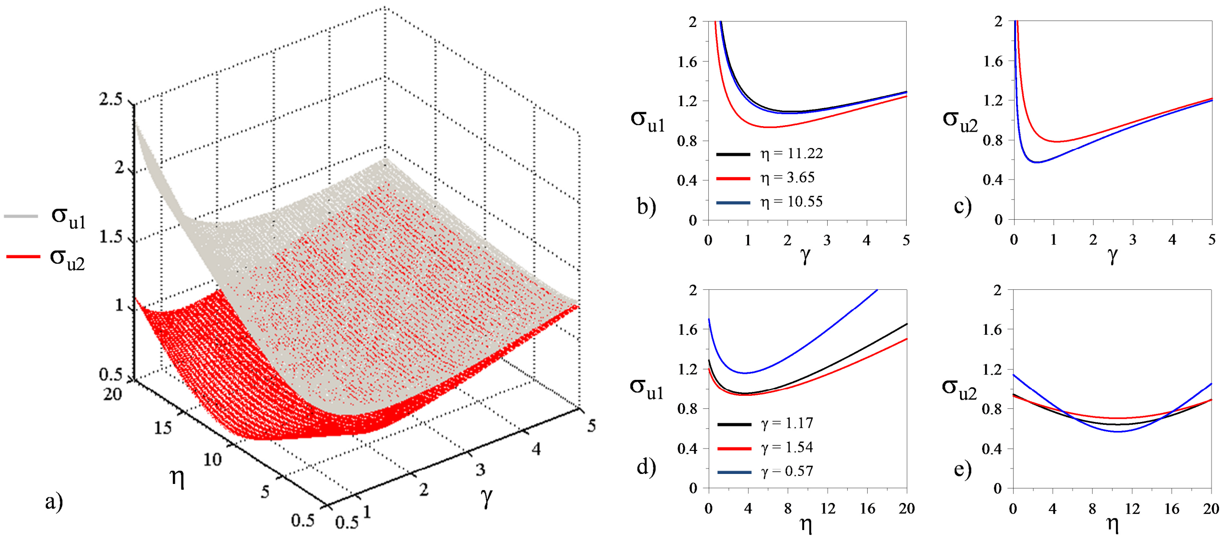

Figure 2.2 (a) Deviations standards manifold in the (η,γ)-plane for the system β=4 and ρ=6.67. Sections for given values of η for σ_{u1} (b) and σ_{u2} (c). Sections for given values of γ for σ_{u1} (b) and σ_{u2} (c).

3. NUMERICAL SIMULATION

The performance of the previously described design criteria are evaluated for different cases of white noise excitation and three different synchronous seismic ground motion. For the first situation have been calculated artificially 100 events using the numerical tool in Matlab "*wgn*" that generates automatically different time-histories of white Gaussian noise. The natural earthquakes ground motion El Centro 1940, Kobe 1995, L'Aquila 2009 have been used in the simulations. The effectiveness and the robustness of the design strategies have been evaluated according to the following performance indexes:

$$I_{ui} = \frac{\max|u_i(\tau)|}{\max|u_i(\tau)|_{P1}}, \qquad I_{\ddot{u}i} = \frac{\max|\ddot{u}_i(\tau)|}{\max|\ddot{u}_i(\tau)|_{P1}} \tag{3.1}$$

$$I_{\bar{u}\_Pi} = \frac{\sqrt{\max|u_1(\tau)|^2_{Pi} + \max|u_2(\tau)|^2_{Pi}}}{\sqrt{\max|u_1(\tau)|^2_{P1} + \max|u_2(\tau)|^2_{P1}}}, \qquad I_{\ddot{u}\_Pi} = \frac{\sqrt{\max|\ddot{u}_1(\tau)|^2_{Pi} + \max|\ddot{u}_2(\tau)|^2_{Pi}}}{\sqrt{\max|\ddot{u}_1(\tau)|^2_{P1} + \max|\ddot{u}_2(\tau)|^2_{P1}}} \tag{3.2}$$

The indexes defined in the Eq. 3.1 make a comparison between the maximum obtained by the strategy 2 and 3 (minimum of the standards deviation σ_{u1} and σ_{u2}, respectively) with those calculated with the first criterion (coincidence of the eigenvalues) in terms of displacements and accelerations. Indeed, in the indexes of Eq. 3.2, the same maximum have been used for calculate both the mean square displacement and acceleration. The results regarding the coupled system forced by white noise are shown in the Figures 3.1, 3.2, and 3.3. In particular in the Figure 3.1 (a) and 3.1 (b) are reported the performance evaluated through the index in Eq. 3.1 of the design parameter (η and γ) selected by the second criterion. The observations made in the previous sections have been found in the numerical results. Indeed, in this first case, the index concerning the displacement u_1, is on average less than one (0.7979 in the Figure 3.1 a) vice versa happens for the other displacement where the indexes' mean is 1.2055 (see Figure 3.1 b). In this particular case seems that the gain achieved in the first displacement is lost in the second. The same situation can be found when in the numerator of the first index in the Eq. 3.1 (see Figure 3.2 b) there are the maximum obtained by the second strategy. Of course, in the case the average value of the indexes result less than one regarding the displacement u_2 (0.8854) while is greater than one for the displacement u_1 (1.2247). For the second coupled system the best performance found for the displacement u_2 not compensate the opposite behaviour of the displacement u_1. About the results concerning the indexes that compare the accelerations, small variation of the performance are observed. Indeed, looking at the relative graphs, namely the Figures 3.1 c, 3.1 d, 3.2 c and 3.2 d regarding the coupled systems determined by the second and third criterion respectively, the average value of the index result very near to one. Nevertheless a small improvement can be observed in the index for the accelerations $\ddot{u}_1$ and $\ddot{u}_2$, in the Figure 3.1 (c) and 3.2 (d) respectively. All the variances are very small denoting the stability of the average evaluated indexes.

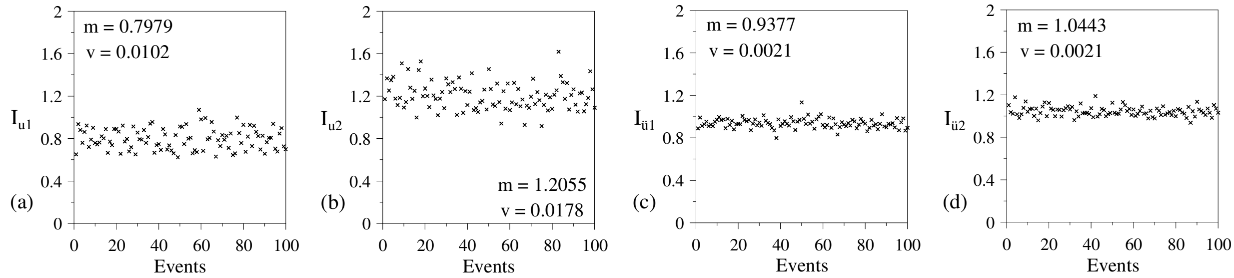

Figure 3.1 Performance of the design points P_1 vs P_2 under white noise. m=mean; v=variance.

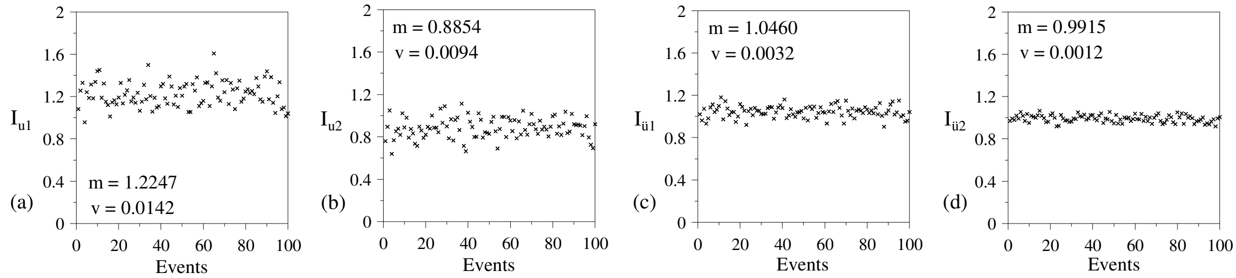

Figure 3.2 Performance of the design points P_1 vs P_3 under white noise. m=mean; v=variance.

Concerning the indexes for the mean square displacements and accelerations, reported in Figure 3.3, the average values show a best performance in the case of the coupled system realized by the second criterion (see Figure 3.3 a and 3.3 c where the index for the mean square displacement is 0.9113 and that for the mean square acceleration is 0.9872). An opposite behaviour is observed for the coupled system realized by the second criterion in which the mean indexes' values are both greater than one.

In Table 3.1 and 3.2 are reported the results for the performance indexes in terms of displacements and accelerations, respectively, varying the natural earthquakes ground motion. In the first Table, regarding the displacement u_1, the better seismic response is obtained applying the second strategy for which the index I_{u1} is equal to 0.69 in the case of El Centro earthquake. Instead the performances of the displacement u_2 are optimized by using the third strategy for which the minimum of the index I_{u2} is equal to 0.83 in the case of El Centro earthquake. The observations concerning the mean square displacement are the following: for the El Centro and Kobe earthquakes the improvement that is obtained using the second strategy, compared with the first strategy (about the -14% for El Centro and -9% for Kobe), is, practically, equal to the worsening resulting applying the third criterion (about the +18% for El Centro and +13 Kobe). For this index, the results achieved when the

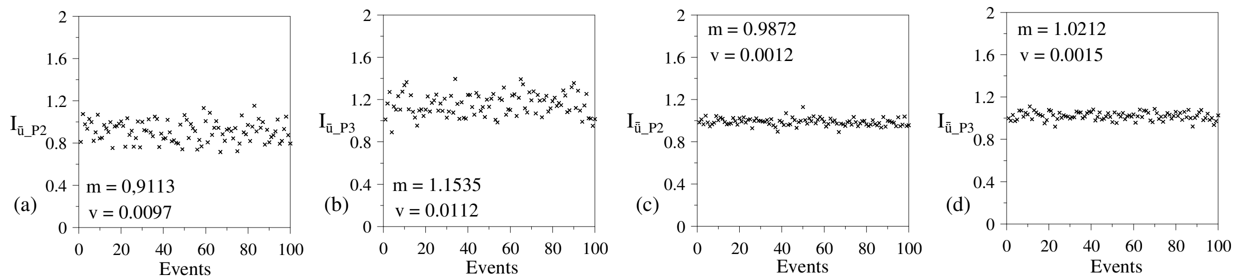

Figure 3.3 Performance of the design points P_1 vs P_2 (a) and (c), P_1 vs P_3 (b) and (d) under white noise in term f mean square displacement and mean square acceleration. m=mean; v=variance.

Table 3.1 Performance indexes, in terms of displacements, varying the natural earthquakes ground motion.

	El Centro 1940			Kobe 2005			L'Aquila 2009		
	I_{u1}	I_{u2}	$I_{\bar{u}}$	I_{u1}	I_{u2}	$I_{\bar{u}}$	I_{u1}	I_{u2}	$I_{\bar{u}}$
P_2 vs P_1	0.69	1.23	0.84	0.86	1.07	0.91	1.05	1.43	1.14
P_3 vs P_1	1.27	0.83	1.18	1.19	0.84	1.13	1.19	0.92	1.14

Table 3.2 Performance indexes, in terms of accelerations, varying the natural earthquakes ground motion.

	El Centro 1940			Kobe 2005			L'Aquila 2009		
	$I_{\ddot{u}1}$	$I_{\ddot{u}2}$	$I_{\bar{\ddot{u}}}$	$I_{\ddot{u}1}$	$I_{\ddot{u}2}$	$I_{\bar{\ddot{u}}}$	$I_{\ddot{u}1}$	$I_{\ddot{u}2}$	$I_{\bar{\ddot{u}}}$
P_2 vs P_1	0.71	1.14	0.88	0.72	1.05	0.84	0.77	1.21	0.93
P_3 vs P_1	1.17	0.85	1.07	1.07	0.91	1.02	1.28	1.09	1.23

coupled system is subjected to the L'Aquila earthquake, in the case of the index related to the mean square displacement, put ahead always the use of the first criterion because its value is always greater that one.
Regarding the accelerations more or less is observed the same situation. In this case the main consideration concern the indexes of the mean square acceleration and the acceleration $\ddot{u}_1$. Indeed, for these indexes, the second strategy shows always the better performance for all earthquakes and only the index $I_{\ddot{u}2}$ assume values greater than one. The optimization of the acceleration $\ddot{u}_2$ is obtained applying the third criterion for which the best performance is achieved in the case of El Centro earthquake.

4. CONCLUSIONS

In this paper the performances of four design strategies for a visco-elastic device coupling two simple oscillators has been compared. The system equation has been written in the space of the dimensionless parameters which describe both the mechanical characteristics of oscillators and device. In the studied case the device has been modelled by a linear KV rheological description. It has been designed using the four selected criteria based on the minimum of the standard deviations and the coincidence of the eigenvalues. Successively the different optimal systems have been submitted to different excitation: white noise and three different seismic motion. The performances have been evaluated by different indexes and the whole scenario indicates that optimal solutions are dependent on the selected response quantities as performance indicators (for example minimum of displacements or acceleration) therefore each selected strategy can be used for the related objective. Moreover the analyses will be completed with the Maxwell model and applying a Kanai-Tajimi filtered non-stationary process.

AKCNOWLEDGEMENT
The research leading to these results has received funding from the project DPC-ReLUIS 2014-2016.

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