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Time evolution of modal parameters identified using WSN data collected by seismic structural monitoring of a monumental church

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ABSTRACT
The paper analyzes the occurrence of dynamic interactions between the nave walls and the tendons inserted as safety measures in the monumental historical structure, the Basilica S. Maria di Collemaggio, which was heavily damaged in the 2009 L'Aquila earthquake. Several information on the dynamic behaviour are obtained using acceleration data collected during long term structural monitoring which has been purposely designed to detect earthquake-induced vibrations through a wireless sensors network. The Wavelet Transform (WT) and the Short-Time Fourier Transform (STFT) are utilized to evidence the variation on time of modal parameters putting into evidence localization of energy in a certain frequency range. Such phenomena are related to the local vibrations of tendons elements used to protect inner and external walls by out-of-plane failure. Acceleration acquired during release tests and seismic events are used to interpret the dynamic behaviour together with the efficacy of the adopted protecting solution. The large data set is used to identify in the time domain parametric model by SSI procedure. The findings contribute to the general development of procedures to provide automated real-time structural assessment for use in seismic zones.

KEYWORDS: *Seismic Structural Monitoring, Wireless Sensor Network, Modal Identification, Monumental Structures*

1. INTRODUCTION

Dynamic response measurements are nowadays becoming easily obtainable by permanently installing inexpensive wireless communication sensor networks that furnish numerous information for structural monitoring. The present paper introduces and discuss some new insights that emerged from an ongoing multidisciplinary research effort conducted to develop easily-manageable and economically-affordable tools and instruments for the permanent structural health monitoring (SHM) of civil and historical structures, with specific applications in seismic areas. The main research findings regard the determination of the time-variability of the modal parameters using the seismic structural response of a monumental church registered by a wireless sensor networks [1].
Traditional structural monitoring systems consist of grids of sensors deployed throughout the target structure and connected to a central processing unit by means of a wired communication infrastructure. Usually, each sensor communicates with a central data acquisition system through a coaxial cable. Wired systems are currently widely used in civil engineering, even though they present several practical disadvantages, mainly related to the logistic difficulties in the deployment of the communication wires along the structure (sometimes due to their size as in the case of the bulk force-balance accelerometers). Other shortcomings relate to the installation and wire expenses, often corresponding to the heaviest outcome of the monitoring system cost table. Nevertheless, this type of system is still widely used both for long-term or temporary monitoring setups. Two recent cases, in which a wired measurement equipment has been used in the testing of monumental buildings, are the two-years monitoring of the Anime Sante Church in L'Aquila, Italy [2] and the experimental test carried out to measure traffic-induced vibration in the Basilica of Maxentium in Rome, Italy [3].
In recent years, the rapid extraordinary developments in the electronic device integration, the wireless communication technology and the sensor miniaturization (such as MEMS), have favoured the use of wireless

sensors networks (WSNs), which may be composed of heterogeneous devices (sensor nodes, or motes). Each node is able to record physical data from its environment, process the acquired data, and communicate with its neighbours. Sensors nodes are usually battery powered and are required to work with extremely low power consumption in order to extend their activity life. Their employment for long-term structural health monitoring overcomes several key disadvantages of wired systems through, for instance, the simplification of the installation operations and the reduction of both costs and visual impact (the latter extremely important in the case of historical or artistic structures), the use of a large number of (low cost) sensors nodes. Moreover, WSNs permit also the on-board real-time implementation of output-only identification techniques or some rapid elementary functions. A case study of operating wireless monitored system is the Jindo Bridge [4], whereas an application in masonry buildings is the monitoring project of the Torre Aquila (Aquila Tower) in Trento, Italy [5].

2. LONG TERM SEISMIC MONITORING OF THE BASILICA S. MARIA DI COLLEMAGGIO

This paragraph describe the design, deployment, management and performance of a WSN used for the vibration-based seismic monitoring of a monumental structure, the Basilica S. Maria di Collemaggio in L'Aquila after the partial transept collapse caused by the catastrophic 2009 earthquake. The church plan has a central nave, which measures 61 m in length and 11.3 m in width, and two side aisles measuring 7.8 m and 8.0 m in width, respectively. The nave and the side aisles are separated by the inner longitudinal walls, sitting on seven columns with a height of 5.3 m and an average central section of about 1 m in diameter. The inner and outer longitudinal walls, with a masonry thickness varying from 0.95 m to 1.05 m are transversally connected by the church façade and the transept structure. The church has a wooden gable roof supported by trusses orthogonal to the longitudinal walls. The dynamic behavior of the undamaged Basilica was characterized in numerical and experimental studies carried out in the early 90's, when a light retrofitting intervention was completed. The 2009 L'Aquila earthquake caused a partial collapse of the structure in the transept area [6]. After the earthquake, a permanent wireless structural monitoring system was developed and installed inside the damaged church (Figure 1). The main goals of this project were, (i) to investigate the possible causes of the collapse; (ii) to monitor the performance of the scaffolding structures and other installed reinforcements (tendons between the walls and temporary composite tape wrapped around the columns for confinement), (iii) to avoid the progression of damage, (iv) to explore possible advantages arising from the use of innovative technologies, and (v) to make a long-term analysis of the structure dynamic response and its modification after final retrofitting and reconstruction.

2.1. Wireless Sensors Network features

Sixteen sensor nodes were installed in the church on June 2011. The majority of sensor nodes were placed inside the structure: ten along the main nave, one at the base of a column, and one in the transept area. Two sensor nodes were placed outside at the top corners of the church's rear facade. Figure 2.1 illustrates the sensor locations. The main monitoring platform is based on a wireless communication platform, MEMSIC Imote2 mote, which includes a sensor board, MEMSIC SHM-A. The latter features an advanced 16-bit data acquisition system (QuickFilter QF4A512 model) and a MEMS tri-axial accelerometer (ST microelectronic LIS344ALH). Moreover the board includes also a temperature and humidity sensor (SENSIRION SHT11) and a luminosity sensor (TAOS 2561). The single node is programmable making use of an ISHMP Toolsuite, a software developed in the contest of Illinois Structural Health Monitoring Project (ISHMP), from which each node of the network actively perform the synchronization and the multi-hop. A node gateway collects the measurements recorded that are uploaded to a remote server using a 3G modem/router. The latter provides also an internet access useful when are performed local test. Electrical lines power the whole system. This choice is motivated by the need of measuring the dynamic response over a time interval as long as possible, without data losses nor maintenance interventions (such as node battery replacement). A scheduling algorithm able to alternate two groups of nodes, every 15 minutes, in the mentioned operation was developed within the project. In particular the nodes 8, 102, 105, 1, 87, 132e 37 constitute the Group I while the nodes 152, 137, 149, 35, 145, 40, 19 e 38 the Group II. In this way, a continuous coverage of the dynamic response of the building was obtained. A second monitoring network, including crackmeters and inclinometer sensors, was also installed in the Basilica. In this case the nodes are battery powered, this choice is appropriate in the case of crack width and inclination measurements due to the reduced number of measurements in time, which allows for an extensive use of duty cycle power saving techniques.

During the months following the installation, the monitoring system was continuously enhanced to the point of complete and automated operation in sensing seismically-induced vibrations. To date, several events with

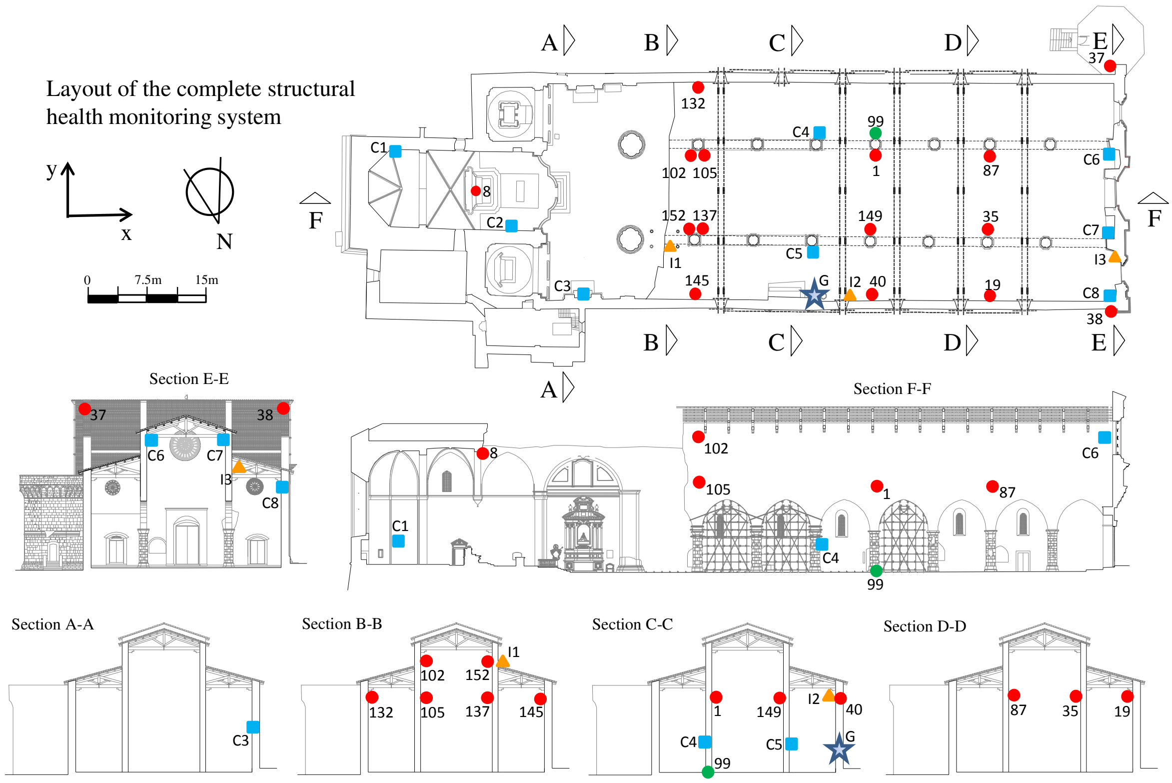

Figure 2.1 Location of the multifunction wireless sensors: 16 accelerometers (15 in elevation, red circle, and 1 on the ground, green circle), 8 crackmeters (blue squares), 3 wall inclinations (orange triangles), and 1 node gateway (star).

relevant dynamic effects have been observed and measured, among them structural accelerations induced by far- and near-field earthquakes. The information from the cases most significant for identification purposes has been reported in Table 1. The first column lists the location of the earthquake epicenter and the second column provides an estimation of the approximate distance from the epicenter. Date, time and earthquake magnitude follow. The last two columns show two major significant values coming from the monitoring system: the peak response acceleration amplitude registered by the WSN, and the group of sensors in current operation during the shake. Recorded structural responses show prevailing out-of-plane oscillations of the nave walls.

Table 1.1 Main seismic events recorded by the WSN at the Basilica S. Maria di Collemaggio.

	Earthquake/epicenter	D	Date	Time (UTC)	M	PRA [mm/s^2]	Group
E1	Main Emilia/Finale Emilia	F	20/05/2012	2:03 AM	5.9	70.4	I
E2	After Emilia/Vigarano	F	20/05/2012	1:18 PM	5.1	17.9	II
E3	After Emila/Cervia-Ravenna	F	06/06/2012	6:08 AM	4.5	10.9	I
E4	L'Aquila/Scoppito	N	14/10/2012	4:32 PM	2.8	71.7	II
E5	L'Aquila/Pizzoli-Scoppito	N	30/10/2012	2:52 AM	3.6	72.7	II
E6	L'Aquila/Pizzoli	N	16/11/2012	3:37 AM	3.2	83.2	I
E7	L'Aquila/Val di Sangro	N	14/02/2014	8:51 PM	2.9	26.2 (60.4)	I
E8	L'Aquila/Valle dell'Aterno	N	04/09/2014	3:55 PM	2.1	18.8	I

D: distance of the epicenter (F=far; N=near); M: magnitude. () relative to the node 37 in global X-direction.

3. MODAL IDENTIFICATION

Signal process methods for seismic structural response aim to extract significant information on the modal properties, such as frequencies or modal shapes. The variations of these parameters can help to detect damage in a structure, especially when a seismic event occurs. The modal identification has been classically performed with two different approach: time domain and frequency domain analysis. Among the others, the Enhanced Frequency Domain Decomposition (EFDD) and the Stochastic Subspace Identification (SSI) are certainly the most used for this specific purpose [7, 8, 9].
In recent years several time-frequency analysis techniques have demonstrated a good efficiency in the evaluation of these characteristics especially when they may vary over time, as for example in the case of a seismic response. In particular the Short Time Fourier Transform (STFT) and the Wavelets analysis are two techniques which have the advantages of being very easy to use and providing useful information. The STFT divides the entire signal into small time intervals and then performs the Fourier Transform for all short intervals. The STFT is clearly described by the following expression:

$$STFT\,(\tau,\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x(t)h(t-\tau)\,e^{-i\omega t}\,dt \tag{1}$$

where $h(t-\tau)$ is an appropriately chosen windows function that emphasizes the signal $x(t)$ around the time sample τ. This operation can give information of the instantaneous content in the harmonics.
In more or less four years of continuous monitoring, low magnitude events have been acquired furnishing a sufficient level of energy to overcome the noise threshold of the monitoring system and allowing the recording of acceleration data characterized by a sufficient structural signature. Consequently, spectral analysis has been used to extract preliminary information from the recorded data, clearly evidencing the change in the main natural frequencies with respect to the values measured in the pre-earthquake configuration.
In the Figure 3.1 are reported different PSDs for four different events. These correspond to far-field (Figure 3.1 a, c) and near-field (Figure 3.1 b, d) seismic events. The Figure 3.1 reports the Power Spectral Densities (PSDs) of the response measured on one sensor node located in one of the internal nave wall (node 105). Looking at the corresponding plots disposed row-wise, the presence of different amplitudes in the range between 2.5 and 4.5 Hz is evident. The reason of this difference may be attributed to the different nature of the seismic inputs. In particular, the left and the right column refer to a seismic event having a very long (E3 in table 2, more or less 300 Km) or very short (E6 in table 2, about 10 Km) epicentral distance from the Basilica, respectively. In far-field E3 event, the low energy of the earthquake, probably contained in a narrow range of frequencies due to a filtering process obtained for the distance of the epicenter, apparently applied a sort of small impulse, as evident in the PSDs where the response peaks at 1 Hz can be associated to a structural mode. On the contrary, in the near-field E6 event the high energy was spread in a wide range of frequencies, inducing the interaction between the structure and the safety system (involving probably and mainly the wall-cable interaction), but also amplifying the modal components belonging to higher modes.

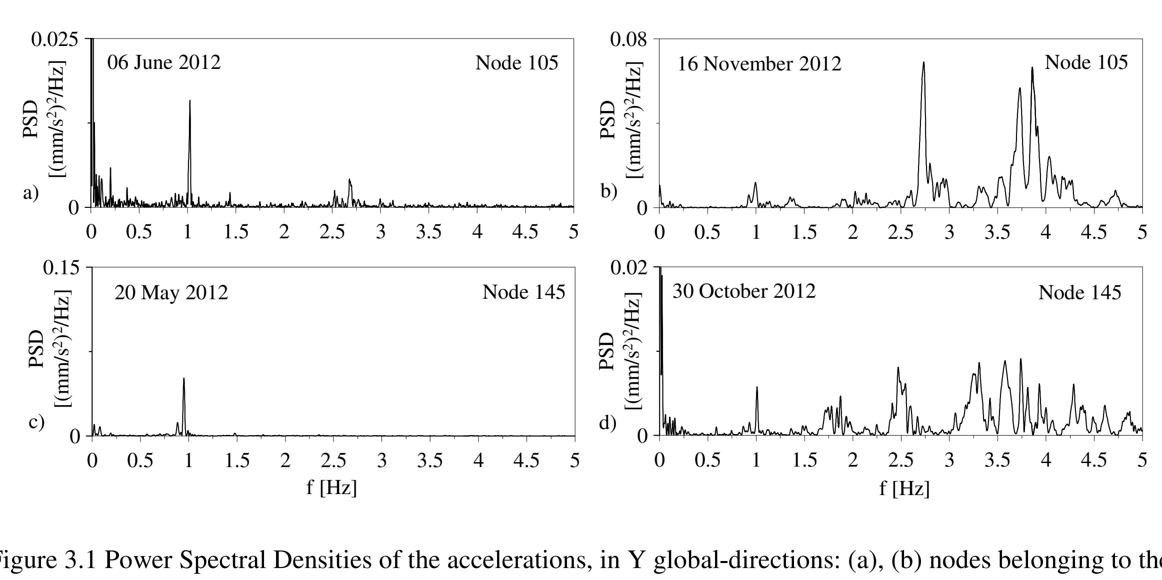

Figure 3.1 Power Spectral Densities of the accelerations, in Y global-directions: (a), (b) nodes belonging to the Group I (a), (b) and Group II (c), (d).

More information on the measured phenomena previously described looking at the PSDs, it can be found performing the STFT on the experimental. In particular in the Figure 3.2 are reported the STFT of the accelerations, in transversal Y-direction, measured during the event of the 16 November 2012. The results show, in every nodes, a great contribution in the first part of the seismic response in the range between 2 Hz and 5 Hz. As observed previously, this phenomena could be due to the interaction that occurs between the structure and the scaffolding system. Regarding the last part of the signal, the STFT highlights the presence only of the fundamental structural mode allocating in 1 Hz. This is well observable looking at the Figures 3.2 a and 3.2 c relative to the node 102 and 105 that have been placed in the damaged ends of the one inner longitudinal wall (see the Figure 2.1). This consideration can represent a justification about the collocation of the fundamental structural mode in 1 Hz.

SSI based-procedure has here been used to derive a reliable parametric models, from which the modal parameters have then been determined under a different hypothesis. The ability of SSI procedures to handle large amounts of noisy data has made these techniques appealing in the treatment of seismic monitoring data. The use of combined input-output (or output-only) SSI procedures has been recently discussed with regard to numerical simulations of the excitation of a tower's structural supports due to passage of trucks on a traffic plateau, the excitations being considered as a measured (or unmeasured) deterministic input [13].

In the present case, the identification of several parametric dynamic models has been performed considering the E6 event, in which the highest peak acceleration amplitudes have been registered, as a reference case. Both SSI-COV and SSI-DATA procedures have been used; in particular, in the second case the concepts of both reference-based and combined-subspace identification have been applied to extract valuable information regarding the robustness of the modal parameters extracted from noisy measurements. Figure 3.3 shows some of the results of the identification process in the form of stabilization diagrams. The bias and variance errors of this stabilization diagrams, based on the experience observed in [14], have been removed using in opportune way the so-called stabilization criteria (such as the expected proprieties of mode shapes or the expected damping ratio range).

The method has permitted identification of a series of parametric models with increasing system dimension (order of the identified system). The stability diagrams have been performed using both the measurements coming from the Group I (Figures 3.3 a and 3.3 b) and Group II (Figures 3.3 c and 3.3 d). Increasing the order of the identified system the diagrams seem to have a more robustness in the results. Moreover using the

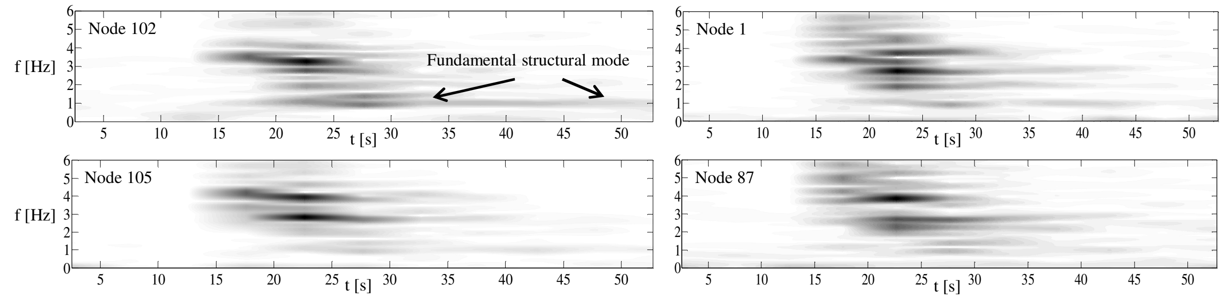

Figure 3.2 Short Time Fourier Transform of the accelerations acquired during the seismic event E6 (Group I).

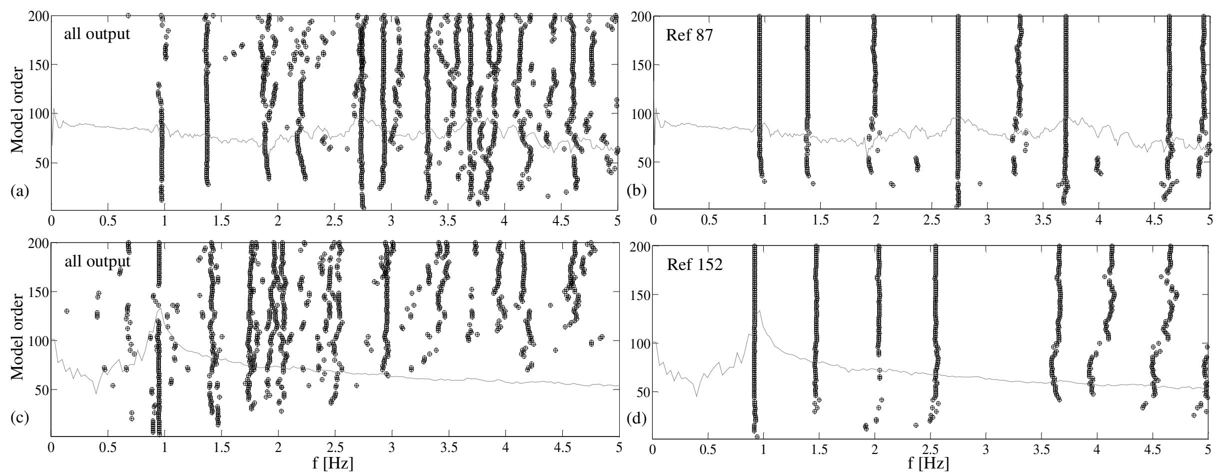

Figure 3.3 Stability diagrams obtained using the accelerations acquired during the seismic event E6 (a), (b) and E2 (c), (d). SSI-COV considering all output (a) and (c); SSI/Ref considering as Ref node 87 and 152.

measurement of the sensors 87 and 152 as reference-based sensors (in the sense described in [8]) the diagrams are more cleaned (see Figure 3.3 b and 3.3 d, respectively for the nodes belonging to the Group I and II). In order to evaluate the influence of the selected reference point in the eigenfrequency estimation, different sensors have been selected to serve as reference ones determining different analyzed cases. The results obtained for the first three frequencies are summarized in Table 3.1 in which the SSI-COV and SSI-DATA correspond to the Stochastic Subspace Identification covariance-driven and data-driven, respectively. Instead, the CSI is the Combined Subspace Identification in which is assumed known the input, i.e. the base acceleration.
The results reported in Table 3.1 are quite stable and in conclusion the identified frequencies for the first three modes have been collocated in 0.98 Hz, 1.40 Hz and 1.92 Hz. Looking at the modal shapes associated to the average values of the natural frequencies, obtained by averaging the set of all identified modes, a reasonable deflection of the central nave wall is recognized for the first three modes (Figure 3.4). The first modal shape evidences a sort of cantilever deformation of the nave wall, which is restrained at the end by the strong in-plane stiffness of the facade (Figure 3.4 a). Coherent with this result, the second and third modes have shapes in which the occurrence of positive and negative displacements is observed (Figure 3.4 b, 3.4 c).
Similar evaluations have been pursued using the measurements obtained by the sensors belonging to group II but are not reported for sake of brevity.

Table 3.1 First three identified frequencies during the E6 seismic event in the group I sensor nodes.

	SSI-COV			SSI-DATA			CSI		
	Frequency [Hz]			Frequency [Hz]			Frequency [Hz]		
	mode 1	mode 2	mode 3	mode 1	mode 2	mode 3	mode 1	mode 2	mode 3
all	0.991	1.389	1.936	0.983	1.375	1.917	0.983	1.385	1.915
Ref 87	0.977	1.379	1.907	0.995	-	-	0.971	1.406	1.921
Ref 1	0.971	1.368	1.891	0.920	1.374	-	0.976	1.403	1.891
Ref 105	0.985	1.359	2.014	0.920	1.350	-	0.980	1.395	1.894
Ref 102	0.976	1.386	1.845	0.999	1.363	-	0.981	1.409	1.931
Ref 87-1	1.003	1.374	1.974	0.973	1.384	1.893	0.975	1.406	1.916
Ref 1-105	1.002	1.374	1.978	0.985	1.368	1.958	0.978	1.388	1.907
Ref 105-102	-	-	1.971	0.985	1.390	1.920	0.977	1.387	1.901
μ	0.986	1.376	1.940	0.970	1.372	1.922	0.978	1.397	1.910
σ	0.012	0.010	0.052	0.030	0.012	0.023	0.004	0.009	0.013

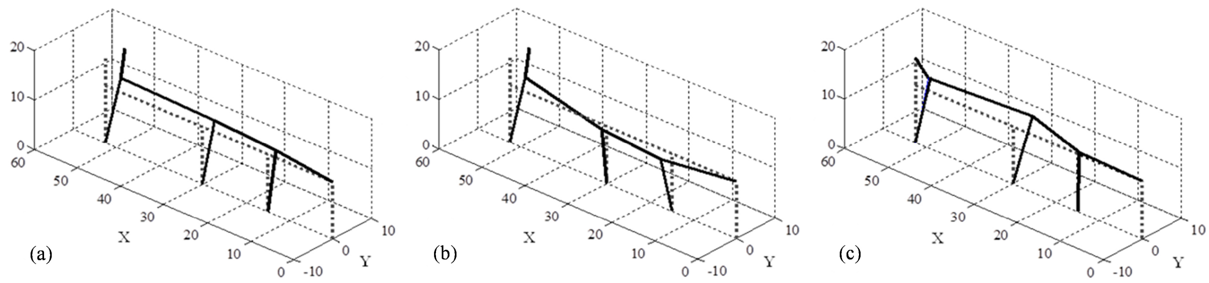

Figure 3.4 Modal shapes identified by the SSI procedure. 3D view: (a) first, (b) second, (c) third mode.

4. MANUAL MODEL UPDATING

Finite element models have been used to analyses the structural behaviour of the church, taking in account also the interaction due to the presence of the scaffolding system. The models were implemented in SAP2000 in order to respect all possible geometric characteristics of the church, such as the openings in the walls and the imperfect parallelism between the walls of the nave. Standards triangular and isoparametric two-dimensional plate were used to reproduce, as accurately as possible, the real geometry of the masonry walls of the Basilica. The columns of the central walls were modeled by beam elements. Truss elements were used to describe the wooden beams of the roof as well as the steel truss structural system built at roof level. To take into account the nonhomogeneous material, related to the historical work operated in the church, it was reasonable to assign values to the mechanical parameter (Young and Poisson moduli, mass density, etc…), valid on average for a large areas of masonry, to be calibrate at the global level with the use of dynamic test. In this phase the finite

element model, representative of the pre-earthquake configuration, have been updated by minimizing the errors between the modes and frequencies identified experimentally from the data obtained during the dynamic test campaign performed in the year 2000 and the numerical ones as reported in [11].
The first three modal shapes of the updated model are reported in Figure 4.1 while the relative MAC values are presented in Table 4.1. This evaluation has been performed considering only the four modal transversal components of the numerical nodes having the same coordinate of the point in which are applied the sensor nodes (nodes 102, 105, 1, 87, see the Figure 2.1) or through a model-based expansion, described in [35], used to reconstruct complete modal shapes from the measured modal shape components. The updating process has been conducted varying both the material characteristic and the internal connection at the roof level of the internal walls. Operating only in such manner it has been reached a good agreement between both identified and model frequencies and modal shapes, compared by the Modal Assurance Criteria (MAC).
The results show that enriching the experimental modal shapes knowledge through the finite element model permits to obtain an enhancement in the agreement between model and experimental data.

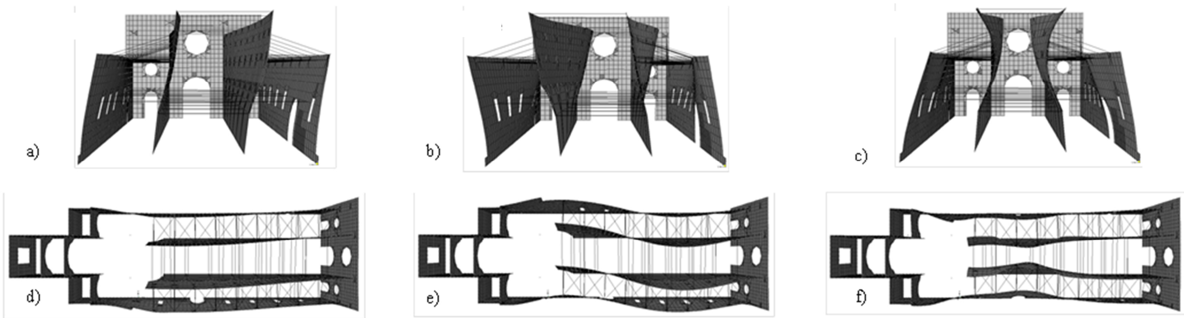

Figure 4.1 Modal shapes of the updated numerical model: above internal perspective view, below view from above. (a), (d) first, (b), (e) second, (c), (f) third mode.

Table 4.1 Finite model updating: Frequencies and MAC.

	Frequencies			MAC					
Modes	Identified	Numerical	D %	Reduced			Expanded		
1	0.9780	0.9500	2.95	0.9885	0.2081	0.5100	0.9929	0.0078	0.0010
2	1.3970	1.5600	-10.45	0.4118	0.9727	0.0555	0.0079	0.9706	0.0209
3	1.9100	2.0100	-4.98	0.1327	0.0940	0.7840	0.0343	0.0240	0.9447

4. CONCLUSIONS

The paper describes the design, deployment, management and performance of a WSN used for the vibration-based seismic monitoring of a monumental structure, the Basilica S. Maria di Collemaggio in L'Aquila, Italy, after the partial transept collapse caused by the catastrophic 2009 earthquake.
The project has permitted the practical consideration of various critical issues related to the development of a WSN composed of MEMS accelerometers, and designed to furnish reliable measurements, comply with the different technical needs of a continuous (24 hours per day) monitoring program, and capture the structural response to occasional seismic events.
The data have been treated using different techniques operanting in frequency, time-frequency and time domain. Especially regarding the last procedure has been tested the potentiality of the SSI and CSI algorithms to extract modal information from the seismic-induced vibration.
The robustness of the procedures has permitted both the identification of the main modal characteristics and has permitted to update a finite element model of the Basilica in the current state. The important role played by the protective systems installed after the partial transept collapse in the complex dynamics of the monument has been highlighted, evidencing some persistent sources of high seismic vulnerability of the church.

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