6th International Conference on Advances in Experimental Structural Engineering
11th International Workshop on Advanced Smart Materials and Smart Structures Technology
August 1-2, 2015, University of Illinois, Urbana-Champaign, United States

Preliminary research on the longitudinal girder-pier collision issue of simply-supported girder bridges

T. Peng [1] , W. Tang[2]

1 *Associate Professor, College of Civil Engineering, Tongji University, China.*
E-mail: ptb@tongji.edu.cn
2 *Graduate student, College of Civil Engineering, Tongji University, China..*
E-mail: 13tangwy@tongji.edu.cn

ABSTRACT (10 pt)
The collision issue between girders in the earthquake has been a great concern of the researchers, but the research on the girder-pier collision issue was still blank. For simply-supported girder bridges, the theoretical method and finite element method were used to study the longitudinal girder-pier collision issue in this paper. A whole structural model and an SDOF model of the longitudinal collision issue were established to study influences of the pier stiffness and pier mass on longitudinal collision responses except for bridges rigid piers. It’s found that the pier mass has little effect on the collision response, but the pier stiffness has a great influence. The larger the pier stiffness is, the higher the peak collision force is. The SDOF model can estimate the collision force and girder displacement fairly accurately if the pier stiffness is minor.

KEYWORDS: *simply supported girder bridge, collision, one dimensional wave theory, SDOF harmonic theory, block*

1. INTRODUCTION

The collision issue of bridges under earthquakes has been a research focus studied by a lot of scholars in recent years. Some kinds of classical methods were proposed for the issue. For example, stereo mechanical approach [1, 2], which is a theoretical approach based on momentum conservation theorem of two particles, was used by several researchers. In stereo mechanical approach, the coefficient of restitution (COR) was used to represent the ratio of speeds after and before an impact. Pairs of particles with COR = 1 collide elastically, while particles with COR < 1 collide inelastically. For a COR = 0, the particles effectively "stop" at the collision. The concept of the approach is simple and complicated calculations are avoided. However, a main shortcoming is that the collision time must be very short to neglect effects of other forces in the collision process.

Compared with the stereo mechanical approach, several kinds of contact element methods were more often used for its convenience for commercial finite element software [3, 4]. In these methods, the impact of two collision objects is simulated by a contact element inserted between the two objects. These methods have been improved a lot in order to simulate different types of collisions. Contact element methods include several kinds of models, such as linear spring model, Kelvin model, Hertz and Hertz damping model. Contact elements are built by springs, dampers and gaps by series-parallel connections. The concept of contact element method is clear and different types of collisions can be simulated using a variety of models. Collision force can be calculated using finite element software easily.

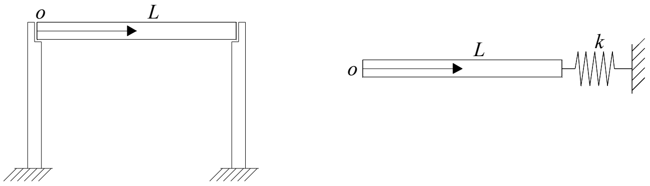
Goldsmith suggested studying the collision issue using one dimensional wave method theoretically [5]. Based on the method, Watanabe and Kawashima [6] proposed a contact element method to simulate the coaxial collision issue between adjacent girders. Parametric analyses of mesh density and stiffness of the contact element were conducted, and it was found the results obtained by FEM and by the one dimensional wave method would be very close if the stiffness of the contact element was close to the axial stiffness between two adjacent particles after meshing of the girder. And it was also found the finer the meshing of the girder was, the closer the results obtained by FEM was to the results obtained by the one dimensional wave method. Contact element methods have been often used to simulate collision issues between adjacent girders. Kim S and Shinozuka M et al. [7] studied the coaxial collision issue between adjacent girders in the longitudinal direction, the vibration properties of the bridge and collision effects under longitudinal earthquake were explored.

However, there is little research on the girder-pier collision issue, but this kind of phenomenon was far from rare. Girder-pier collisions would appear between higher piers and lower girders, or between the girder and unseating prevention blocks in the longitudinal and transverse directions in earthquakes. The collision effects were destructive and should receive adequate attention.

Because of the flexibility of piers, the girder-pier collision issue is different from the collision issue between girders. And because the mass of the girder is far larger than that of piers, the pier mass could be neglected for simplicity. Taking a simply supported girder bridge as an example, the girder-pier collision issue in the longitudinal direction is studied using theoretical methods at first. According to the findings, a simplified single degree of freedom (SDOF) model is established and verified by test method. At last, influences of the pier stiffness and mass on collision responses of a simply supported girder bridge were studied by finite element method with two models.

2. THEORETICAL STUDIES OF THE GIRDER-PIER COLLISION ISSUE

Two theoretical models were used in this section to simulate the collision issue between a girder with an initial velocity and the top of a stationary pier as shown in Figure 2.1(a). The first model is based on the one dimensional wave theory [6]. The influence of the pier mass on the collision responses was neglected. In the one dimensional wave theory, the girder mass and pier flexibility were considered. The model is shown in Figure 2.1(b). Suppose that piers are not damaged during the collision, the pier stiffness is a constant.



(a) girder-pier collision issue (b) one dimensional wave theoretical model
Figure 2.1 model of the one-dimensional wave theory for the girder-pier collision issue

Physical meanings of parameters in the theoretical analysis are as follows:
E, A, L, ρ are elastic modulus, cross sectional area, length and density of the girder respectively.
V is the initial velocity of the girder before collision.
$P(t)$ is the time history of collision force.
β is the ratio of the horizontal stiffness of bridge pier to the axial stiffness of the girder.
$k = \beta EA/L$ is the horizontal stiffness of bridge pier.
$u(x,t)$ is the displacement of the particle which is at a distance of x from the non-collision end of the girder at time t. According to the one dimensional wave theory, $u(x,t) = f(x+ct) + g(x-ct)$. f and g are displacement waves propagating in the positive and negative directions, respectively. f' and g' are the first order derivatives of f and g. $c = \sqrt{E/\rho}$ is the propagation velocity of the displacement wave.
The model shown in Figure 2.1(b) is analysed theoretically using the one dimensional wave theory. According to initial conditions that the initial velocity is V and initial strain is zero when $t = 0$, we have:

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = c(f' - g') = V \\ \dfrac{\partial u(x,t)}{\partial x} = f' + g' = 0 \end{cases} \Rightarrow f' = -g' = \frac{V}{2c} \tag{2.1}$$

According to the boundary condition that the strain is zero at $x = 0$, we have:

$$f' = -g' \tag{2.2}$$

According to the boundary condition that the strain is $-\frac{P(t)}{EA}$ at $x = L$, we have:

$$f' = -\frac{P(t)}{EA} - g' \tag{2.3}$$

According to the one dimensional wave theory, the first order derivatives of displacement wave equations can be shown in Figure 2.2. The wave equations of the free end and collision end are determined based on Eq. 2.2 and 2.3.

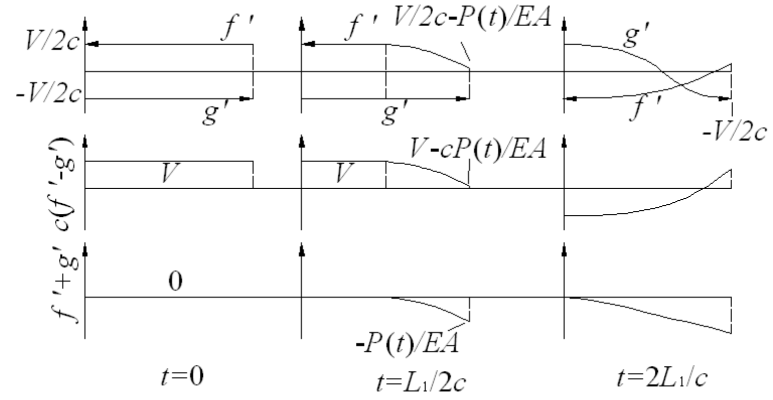


Figure 2.2 The waveforms of the first order derivatives of *f* and *g*.

Then, the velocity time history at $x = L$ for $t \in \left[0, \frac{2L}{c}\right]$ is as follows:

$$\frac{\partial u(L,t)}{\partial t} = -cf(L,t) + cg(L,t) = V - \frac{c}{EA}P(t) = V - \frac{\beta c}{L}u(L,t) \tag{2.4}$$

Solve the differential equations, we have:

$$u(L,t) = \frac{VL}{\beta c}\left[1 - \exp(\frac{-\beta ct}{L})\right] \tag{2.5}$$

And the time history of collision force is:

$$P(t) = \frac{EAV}{c}\left[1 - \exp(\frac{-\beta ct}{L})\right] \tag{2.6}$$

Using a similar method, the velocity time history at $x = L$ for $t \in \left[\frac{2L}{c}, \frac{4L}{c}\right]$ is as follows:

$$\frac{\partial u(L,t)}{\partial t} = V - \frac{c}{EA}P(t) - \frac{2c}{EA}P(t - \frac{2L}{c}) = V - \frac{\beta c}{L}u(L,t) - 2V\left[1 - \exp(\frac{-\beta ct}{L} + 2\beta)\right] \tag{2.7}$$

Solve the differential equations according to a continuous condition $u(L, \frac{2L}{c}) = \frac{VL}{\beta c}\left[1 - \exp(-2\beta)\right]$, we have:

$$u(L,t) = \frac{VL}{\beta c}\left[2\exp(-\frac{\beta ct}{L} + 2\beta) - \exp(-\frac{\beta ct}{L}) - 1\right] - \frac{4VL}{c}\exp(-\frac{\beta ct}{L} + 2\beta) + 2Vt\exp(-\frac{\beta ct}{L} + 2\beta) \tag{2.8}$$

And the time history of collision force is:

$$P(t)=\frac{EAV}{c}\left[2\exp(-\frac{\beta ct}{L}+2\beta)-\exp(-\frac{\beta ct}{L})-1\right]-\frac{4\beta EAV}{c}\exp(-\frac{\beta ct}{L}+2\beta)+\frac{2\beta EAV}{L}t\exp(-\frac{\beta ct}{L}+2\beta) \quad (2.9)$$

Introducing dimensionless collision force $R(t)=P(t)\frac{c}{EAV}$ and dimensionless time coordinate $\tau=\frac{ct}{L}$, the time history of dimensionless collision force using the one dimensional wave theory is as follows:

$$R(\tau)=\begin{cases}1-\exp(-\beta\tau) & 0\le\tau\le 2\\ 2\exp(-\beta\tau+2\beta)-4\beta\exp(-\beta\tau+2\beta)+2\beta\tau\exp(-\beta\tau+2\beta)-\exp(-\beta\tau)-1 & 2<\tau\le 4\end{cases} \quad (2.10)$$

According to the Eq. (10), time histories of the girder-pier collision force for different β can be drawn, as shown in Figure 2.3.

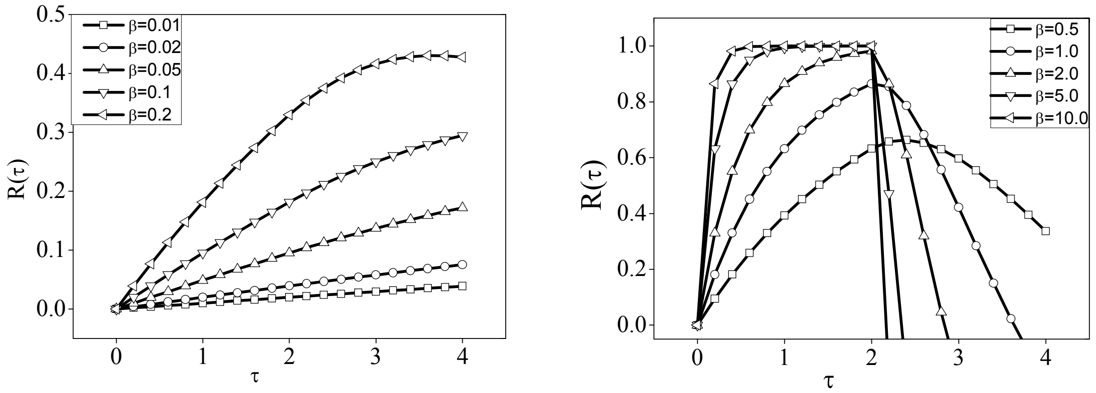


Figure 2.3 Time histories of the girder-pier collision force

As shown, for a minor β, the girder-pier collision force curve is close to a sinusoidal curve. And for a major β, the girder-pier collision force curve is close to a rectangular curve.

The second model is based on the SDOF harmonic theory [8]. According to the theory, the axial stiffness of the girder is assumed to be far larger than the pier stiffness, so the girder is be regarded as a rigid body and simulated by a particle with the girder mass. The model is shown in Figure 2.4. *m* is the girder mass in the figure.

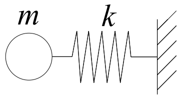


Figure 2.4 The model of the SDOF harmonic theory for the girder-pier collision issue

For the SDOF harmonic theory, the circular frequency of the time history of the girder-pier collision force is:

$$\omega=\sqrt{\frac{\beta EA/L}{\rho AL}}=\frac{c}{L}\sqrt{\beta} \quad (2.11)$$

Then the time histories of the collision force and dimensionless collision force are as follows:

$$P(t)=\sqrt{\beta}\frac{EAV}{c}\sin(\frac{ct}{L}\sqrt{\beta}) \quad (2.12)$$

$$R(\tau)=\sqrt{\beta}\sin(\sqrt{\beta}\tau) \quad (2.13)$$

Time histories of the collision force obtained using the two theoretical methods are compared in Figure 2.5. Solid lines indicate results of the one dimensional wave theory, and broken lines indicate those of the SDOF harmonic theory. As shown, for a minor β, results of the SDOF harmonic theory agree well with those of the one dimensional wave theory. However, for a major β, such as $\beta \geq 0.5$, the difference between the results is remarkable. Therefore the girder should not be regarded as a rigid body when the pier stiffness is close to the axial stiffness of the girder. As shown, as the axial flexibility of the girder is considered in the one dimensional wave theory, the collision duration will be extended and the peak collision force will be reduced.

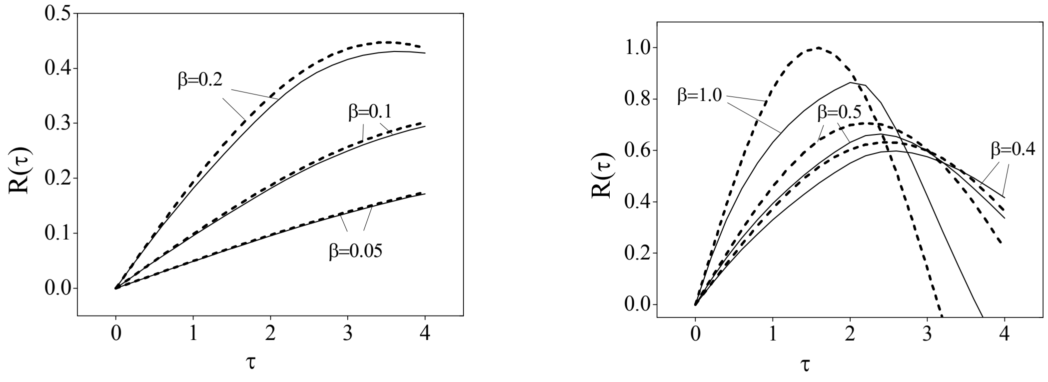


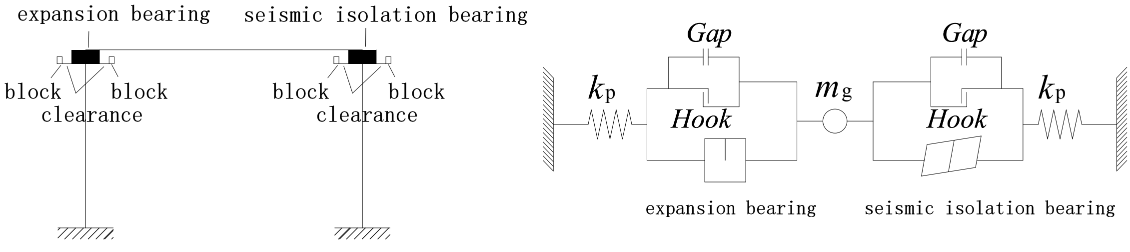
Figure 2.5 Comparisons of the collision force time histories between two models

3 FEM ANALYSIS OF THE GIRDER-PIER COLLISION ISSUE

In order to study collision responses under longitudinal earthquakes and the applicability of the SDOF harmonic theory, a simply supported girder bridge was designed. The span is 16m, and piers are both 5m high. The left end of the girder was supported on two same expansion bearings, and the right end of the girder was supported on two same seismic isolation bearings. Blocks for unseating prevention were installed on both sides of each bearing. Clearances are all set to be 0.1m for all the blocks. The girder-pier collisions would happen between the girder and the blocks under longitudinal earthquakes.

3.1 analysis models and seismic input

Two models of the simply supported girder bridge were established with the FEM software SAP2000. One is a whole structural model and the other one is an SDOF model. For the whole structural model, all the components are simulated as shown in Figure 3.1(a). The girder and piers are simulated with Frame elements. Bearings are simulated with Plastic Wen Link elements. Blocks are simulated with Gap and Hook Link elements. All the elements and their parameters are listed in Table 3.1. For the SDOF model, pier masses are neglected and the girder mass is concentrated on a particle as shown in Figure 3.1(b). k_p represents the horizontal stiffness of piers, and the girder mass m_g is 216ton .



(a) The whole structural model (b) The SDOF model

Figure 3.1 Models of a simply supported girder bridge

Table 3.1 Parameters of a simply supported girder bridge model

components	Parameters
girder	elasticity modulus is 3.45×10^7kPa, density is 2.5ton/m^3, cross sectional area is 5.4m^2

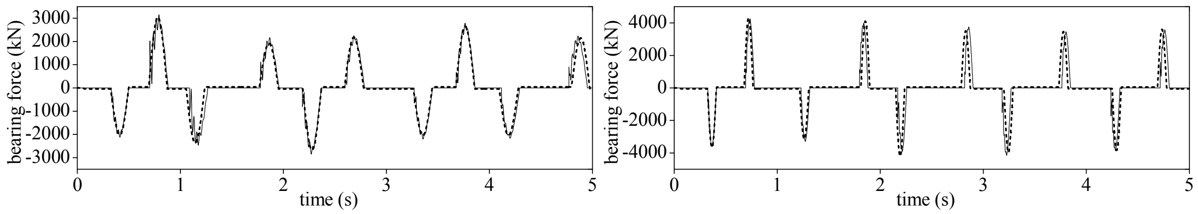
pier	elasticity modulus is 3.45×10^7kPa, density is 2.5ton/m^3, cross sectional area is 2.0m^2
expansion bearing	horizontal stiffness before yielding is 4.32×10^4kN/m, yield force is 43.2kN, horizontal stiffness after yielding is 0
seismic isolation bearing	horizontal stiffness before yielding is 4.32×10^4kN/m, yield force is 43.2kN, horizontal stiffness after yielding is 216kN/m
block	Stiffnesses of gap and hook link elements are 1.0×10^{10}kN/m, clearances are 0.1m

Sinusoidal acceleration wave was input as the seismic excitation for simplicity. Peak acceleration is 4m/s^2, frequency is 1Hz, duration is 5s, and integration step is 1ms.

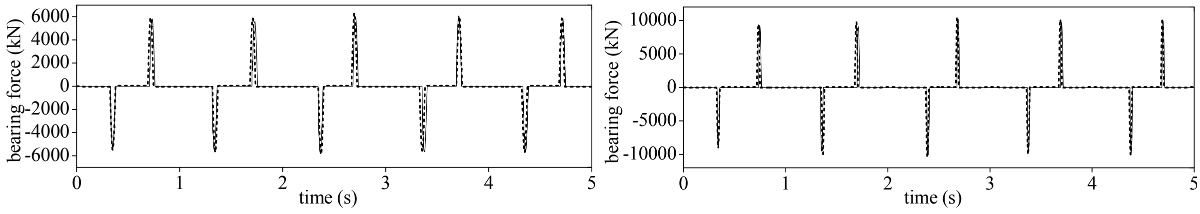
3.2 Applicability of the SDOF model

To evaluate the applicability of the SDOF model, results of the model for different pier stiffnesses are compared with those of the whole structural model. The influence of pier stiffness on the collision responses was studied by altering the elastic modulus of piers. The horizontal stiffnesses are designed to be 4.0×10^4kN/m, 1.6×10^5kN/m, 4.0×10^5kN/m, 1.2×10^6kN/m, 4.0×10^6kN/m, 1.16×10^7kN/m respectively. The axial stiffness of the girder is 1.16×10^7kN/m. So the stiffness ratios are 0.0034, 0.0138, 0.034, 0.103, 0.345 and 1.0, respectively.

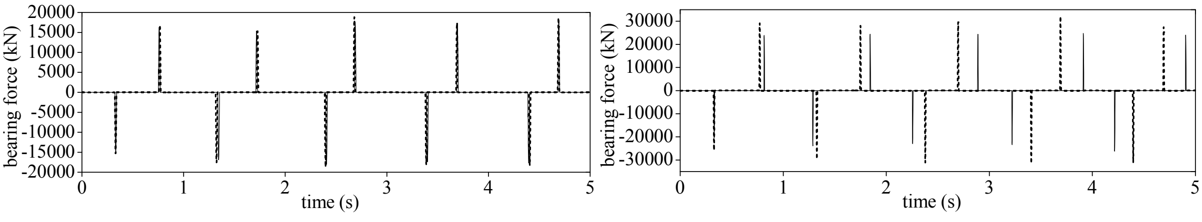
Force time histories of the expansion bearing of the two models are shown in Figure 3.2. The expansion bearing force is composed of the friction force and the collision force, and the latter is far larger than the former. In the figure, solid lines indicate the results of the whole structural model, and broken lines indicate the results of the SDOF model.



(a) β=0.0034 (b) β=0.0138



(c) β=0.034 (d) β=0.103



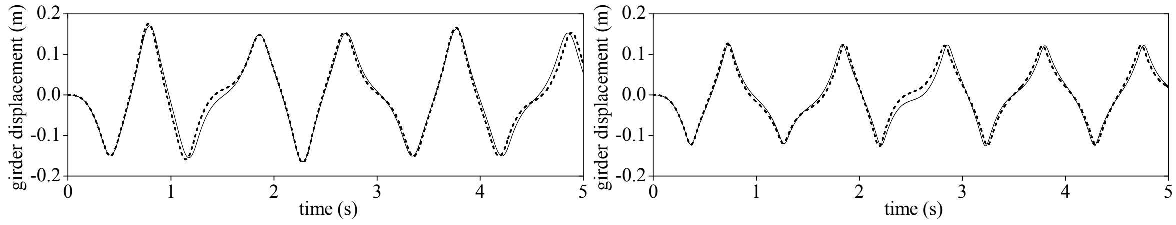
(e) β=0.345 (f) β=1.0

Figure 3.2 Force time histories of the expansion bearing

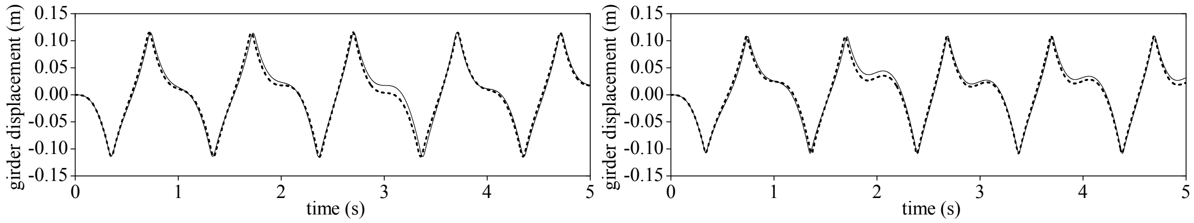
Every spike in Figure 3.2 indicates one collision between the girder and the pier. As shown, along with the increase of the pier stiffness, the collision duration becomes shorter and the collision force becomes larger. Differences between the two models are very small, except for β=1.0. The collision force and the collision time are almost the same for the first five cases. Therefore, the SDOF model can estimate the collision force fairly accurately for $\beta<0.345$ at least. All the peak collision forces calculated by the SDOF model are larger than those of the whole structural model, which is similar to the theoretical results. For β=1.0, differences of the collision force and the collision time between the two models are much larger than other cases. So the SDOF model is not applicable for the girder-pier collision issue of bridges with rigid piers.

Longitudinal displacement time histories of the girder of the two models are compared in Figure 3.3. In the figure, solid lines indicate the results of the whole structural model, and broken lines indicate the results of the SDOF model. As shown, along with the increase of the pier stiffness, girder displacement becomes smaller and approaches to 0.1m gradually.

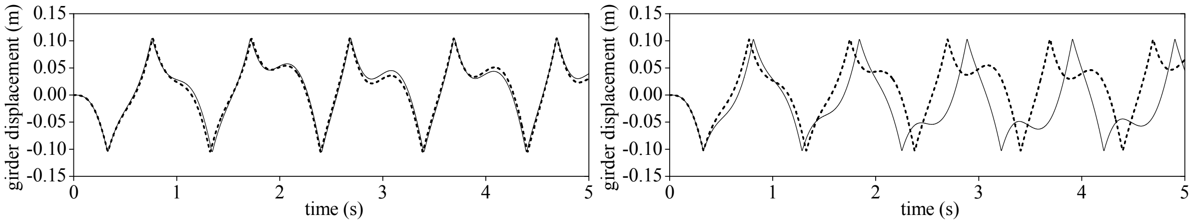
The differences between the two models are very small, except for β=1.0. Therefore, the SDOF model can estimate the girder displacement fairly accurately for β<0.345 at least.



(a) β=0.0034 (b) β=0.0138



(c) β=0.034 (d) β=0.103

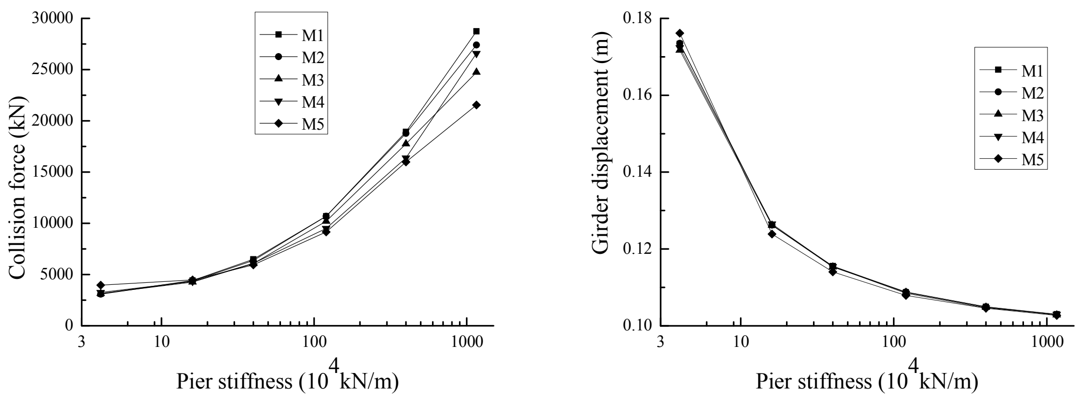


(e) β=0.345 (f) β=1.0

Figure 3.3 Longitudinal displacement time histories of the girder

3.3 Influences of pier stiffness and pier mass on the collision responses

The influence of pier horizontal stiffness on the collision responses was studied by altering the elastic modulus of piers as mentioned in section 3.2. The influence of pier mass on the collision responses was studied by altering the density of piers. Pier masses are designed to be 5ton, 10ton, 25ton, 50ton, 125ton respectively, which are named as M1 to M5. The maximum collision responses of the whole structural model for different pier masses and pier stiffnesses are shown in Figure 3.4.



(a) the maximum collision force (b) the maximum girder displacement

Figure 3.4 The maximum collision responses

As shown, the collision force will increase and the girder displacement will decrease with the increase of the pier stiffness. The influence of pier mass on the girder displacement is very small. For minor pier stiffnesses, the influence of pier mass on the collision force is also not too much, so the pier mass can be neglected and the SDOF model is applicable. However, the influence of pier mass on the collision force is bigger for major pier stiffnesses, so the pier mass is not negligible and the SDOF model is no more applicable.

4. CONCLUSIONS

In this paper, theoretical and finite element analysis were conducted to study the longitudinal girder-pier collision issue, the following conclusions are drawn:

(1) Based on theoretical results, it was found that the longitudinal girder-pier collision issue can be simulated as a SDOF model and the pier mass can be neglected if the pier stiffness is minor.

(2) It was found based on the results of finite element analysis along with the increase of the pier stiffness, collision duration becomes shorter and collision force becomes larger.

(3) The SDOF model can estimate the collision force and girder displacement fairly accurately if the pier stiffness is minor.

AKCNOWLEDGEMENT

This work was supported in part by the National Natural Science Foundation of China (No. 51278372) and the Ministry of Science and Technology of China, Grant No. SLDRCE 14-B-15.

REFERENCES

1. Zhu P, Abe M, Fujino Y. (2002). Modeling three-dimensional non-linear seismic performance of elevated bridges with emphasis on pounding of girders. *Earthquake Engineering and Structural Dynamics*, 31(11): 1891-1913.
2. Malhotra P K. (1998) Dynamic of seismic pounding at expansion joints of concrete bridges. *Journal of Engineering Mechanics, ASCE*, 124(7): 794-802.
3. Muthukumar S, DesRoches R A. (2006) Hertz contact model with non-linear damping for pounding simulation. *Earthquake Engineering and Structural Dynamics*, 35(7): 811-828.
4. Anagnostopoulos S A. (2004) Equivalent viscous damping for modeling inelastic impacts in earthquake pounding problems. *Earthquake Engineering and Structural Dynamics*, 33: 897-902.
5. Goldsmith. (2001) Impact. New York: Dover Publications, INC.
6. Watanabe G, Kawashima K. (2004) Numerical simulation of pounding of bridge decks. *13th World Conference on Earthquake Engineering*, Paper No. 884.
7. Kim S, Shinozuka M. (2003) Effects of seismically induced pounding at expansion joints of concrete bridges. *Journal of Engineering Mechanics*, 129(11): 1225-1234
8. Clough R, W. Penzien J. (2003) Dynamics of Structures (Third Edition). Computers & Structures, Inc.