6th International Conference on Advances in Experimental Structural Engineering
11th International Workshop on Advanced Smart Materials and Smart Structures Technology
August 1-2, 2015, University of Illinois, Urbana-Champaign, United States

Nonlinear Effects of Energy Harvesting Circuit Topology on a Structure-harvester System

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ABSTRACT
It is predicted that energy consumption will increase by 56% between year 2010 and 2040. To cope with this upcoming energy demand, innovative technologies have been developed in recent decades to harvest energy from the environment such as wind, solar, tide and vibration. Harvesting energy from structural vibration presents two major challenges: efficiency and vibration control. In particular, vibration could be of low and potentially time-varying frequency for large-scale structures. Harvesting energy effectively under such circumstances requires high efficiency transducers and electrical circuits to reduce losses during energy conversion. Most of the previous research however adopted a resistance-inductance model for the electrical circuits which cannot portray the inherent nonlinearity of these electrical circuits. In this study, the nonlinear behavior of a typical energy harvesting circuit, the Standard Energy Harvesting Circuit (SEHC) is modeled. The effect of electromechanical coupling between this energy harvesting circuit and a single-degree-of-freedom structure is then analyzed with the aid of dimensional analysis (DA). The energy dissipation capability and the energy harvesting efficiency of the coupled system are then discussed. An analytical solution for a weakly coupled system under steady-state harmonic excitation is further derived to provide a quick suggestion on the design of an efficient energy harvester for civil engineering structures.

KEYWORDS: *Vibration energy harvesting, electromechanical coupling, energy harvester design, dimensional analysis, nonlinear dynamic.*

1. INTRODUCTION

It is predicted that energy consumption will increase by 56% between year 2010 and 2040 (U.S. Energy Information Administration, 2013). To tackle this challenge, harvesting energy from the environment such as wind, solar, tide, as well as structural vibration is investigated in these recent decades. Elvin et al. (2006) studied the possibility of using piezoelectric harvester to scavenge energy from various loading conditions. He showed that piezoelectric harvester with a volume of 20 to 200 cm^3 is capable to supply energy for a 5 cm^3 wireless sensor under dynamic wind and traffic loading. Ali et al. (2011) investigated the energy harvested from highway bridges under moving vehicle load. The feasibility of harvesting energy under wind load excitation from the Vincent Thomas Bridge was studied by Miao (2013) using a linear resonant device. Shen and Zhu (2012) developed a self-power vibration control and monitoring system (SVCM) capable of harvesting sufficient energy for wireless sensor applications while mitigating the vibration of the structure at the same time. They studied the application of a dual function electromagnetic (EM) damper for the stay cables of a cable-stayed bridge under wind excitation (Shen and Zhu (2014)). The aforementioned studies show that harvested power from regular events is potentially sufficient to provide energy for a structural health monitoring system.

Harvesting energy from structural vibration however presents two major challenges (Zuo and Tang 2013): efficiency and vibration control. Harvesting energy requires high efficiency transducers and electrical circuit interface in order to reduce losses during energy conversion. For large-scale structures, vibration could be of low and time-varying frequency which makes energy harvesting inefficient. To improve the efficiency and flexibility of the circuit interface, abundant research studies related to electronic circuit topology have been reported (Calió et al. 2014, and Li et al. 2013b). Guyomar and Lallart (2011) developed the Series Synchronized Switch Harvesting on Inductor Circuit to improve the harvested power from piezoelectric energy harvester. A much more advanced energy harvesting circuit topology, the Double Synchronized Switch Harvesting was developed by Lallart et al. (2008). Lefeuvre et al. (2005), Arroyo and Badel (2012) proposed the Synchronous Electric Charge

Extraction circuit to improve the harvested power from the piezoelectric energy transducer. Pros and cons of these harvesting circuits can be found in the review by Guyomar and Lallart (2011).

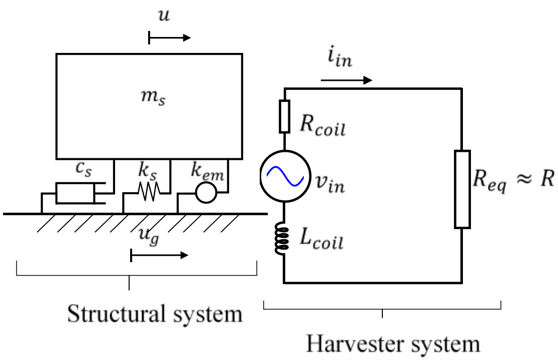

Figure 1.1 Schematic Drawings of Simplified Structure-harvester System: Linear Elastic SDOF Structure and Simplified Circuit (SC) Interface.

The aforementioned circuit topologies could generate strong nonlinear electromechanical behavior for the harvester. Some researchers chose to model the circuit interface as a simplified circuit (SC), as shown in Fig. 1.1 (Li et al. 2014, Dai et al. 2015, Jung et al. 2011). In that case, the nonlinearity of the circuit interface is neglected to simplify the analysis. This simplified circuit cannot portray the nonlinear behavior and could cause problems if the response of the structure-harvester system cannot be predicted accurately. In this paper, the nonlinearity relating to the electromechanical coupling of a typical energy harvesting circuit topology, the Standard Energy Harvesting Circuit (SEHC) (see Fig. 2.1), is studied.

This paper is organized as follows: first, Section 2 presents a comprehensive mathematical modeling of the coupled structure-harvester system. The response of the coupled system is then analyzed with the aid of dimensional analysis (DA) and the results are shown in Section 3. Section 4 provides an analytical method to investigate a weakly coupled system under steady state condition. Comparison between the SEHC and the SC modelling is presented with some discussion in Section 5. The important findings of the previous sections are summarized in Section 6.

2. MODELING OF STRUCTURE-HARVESTER SYSTEM

2.1 Structure Model

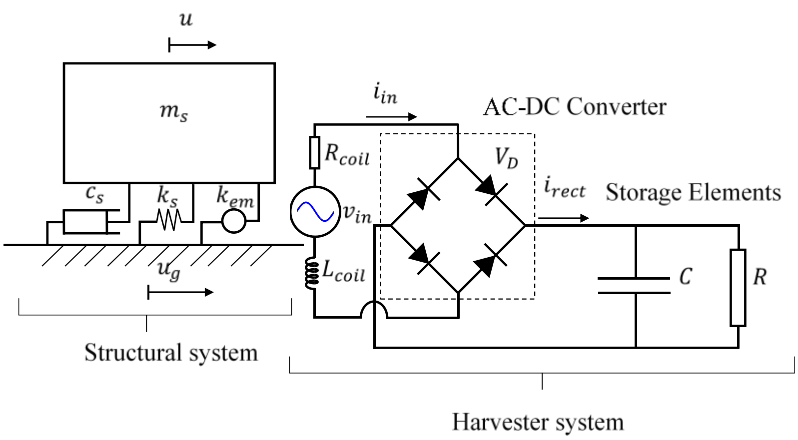

Figure 2.1: Schematic Drawings of Structure-harvester System: Linear Elastic SDOF Structure and Standard Energy Harvesting Circuit (SEHC).

Assume that the structural system can be effectively modelled as a Single Degree of Freedom (SDOF) oscillator, as shown in Fig. 2.1. An EM harvester is attached to the structure in parallel with a linear elastic spring and a linear viscous damper. The equation of motion is:

$$m_s\ddot{u} + c_s\dot{u} + k_s u + F_{em} = -m_s\ddot{u}_g \tag{2.1}$$

where u is the relative displacement of structure with respect to ground, u_g is the absolute ground displacement, m_s, c_s and k_s are the mass coefficient, the inherent viscous damping coefficient and the stiffness coefficient of the structure, respectively. According to the Faraday's Law, when there is a change of magnetic flux ($\dot{u} \neq 0$), an induced current i_{in} and a back electromotive force (emf) v_{in} are generated. Simultaneously, according to Lorentz's Law, an electromagnetic force F_{em} is produced and acts on the structural system, which can be calculated as follows:

$$F_{em} = k_f i_{in} \tag{2.2}$$

where k_f (N/A) is the force constant which depends on the design of harvester.

2.2 Electrical Model

The SEHC (see Fig. 2.1) consists of an AC-DC converter and a storage element connected in parallel with a resistor (an electrical load) (Zhu et al. 2012). This particular circuit interface is chosen as it represents the basic configuration of the state of art of advanced circuit interfaces (Calió et al. 2014). Thus it can portray most of the important nonlinearity effects of the harvesting circuit interface. We assume that the EM harvester is modelled as a voltage source that connect the resistance of coil R_{coil} and the inductance of coil, L_{coil} in series (Shen et al. 2012). Due to the low frequency content of the external excitation in civil engineering application (normally smaller than 10 Hz), the inductance of the coil is assumed to be negligible (Zhu et al. 2012, Shen and Zhu 2012). This assumption is valid when $L_{coil}/R_{coil} \ll 1/f$ where f is the frequency of excitation. Assume that the Kichoff's Current Law holds, the governing equation is:

$$C\dot{V}_c + \frac{V_c}{R} = i_{rect} \tag{2.3}$$

where V_c is the voltage flows through the capacitor, $i_{rect} = |i_{in}|$ is the rectified current, C is the capacitance of the capacitor, and R is the electrical load. The AC-DC converter (full-bridge rectifier), which is made by four ideal forward biased diodes, is used to rectify the induced current, and its forward-biased voltage, V_D is a constant (Ioinovici 2013). Based on the Kichoff's Voltage Law, the induced current i_{in} can be evaluated as follows (Zhu et al. 2012):

$$i_{in} = i_{in}(\dot{u}, V_C) = \begin{cases} \dfrac{v_{in} - (2V_D + V_C)}{R_{coil}} & v_{in} > 2V_D + V_C \\ 0 & |v_{in}| \leq 2V_D + V_C \\ \dfrac{v_{in} + (2V_D + V_C)}{R_{coil}} & v_{in} < -(2V_D + V_C) \end{cases} \tag{2.4}$$

where the induced back emf., v_{in} can be calculated as follows (Tang and Zuo 2011, Zhu et al. 2012):

$$v_{in} = k_v\dot{u} \tag{2.5}$$

where k_v (V/m/s) is a voltage constant depending on the design of the harvester. Assume that the harvester is ideal and the internal heat loss is neglected, hence $k_v = k_f = k_{em}$ (Zhu et al. 2012, 2013). Substituting Eqs. (2.4) and (2.5) into (2.1), together with (2.3), and dividing both side of the equations by m_s and Ck_{em} respectively, we obtain the following equation:

$$\begin{gathered} \ddot{u} + 2\xi_s\omega_s\dot{u} + \omega_s^2 u + \frac{k_{em}^2}{m_s R_{coil}} g_1\left(\dot{u}, \frac{V_D}{k_{em}}, \frac{V_c}{k_{em}}\right) = -\ddot{u}_g \\ \frac{\dot{V}_c}{k_{em}} + \frac{V_c}{k_{em}CR} = \frac{1}{CR_{coil}}|g_1| \end{gathered} \tag{2.6}$$

where $\xi_s = c_s/2m_s\omega_s$ and $\omega_s = \sqrt{k_s/m_s}$ are the structural damping ratio and natural frequency, respectively. g_1 is a function that relates to i_{in}.

3. DIMENSIONAL ANALYSIS

3.1 Dimensional Analysis

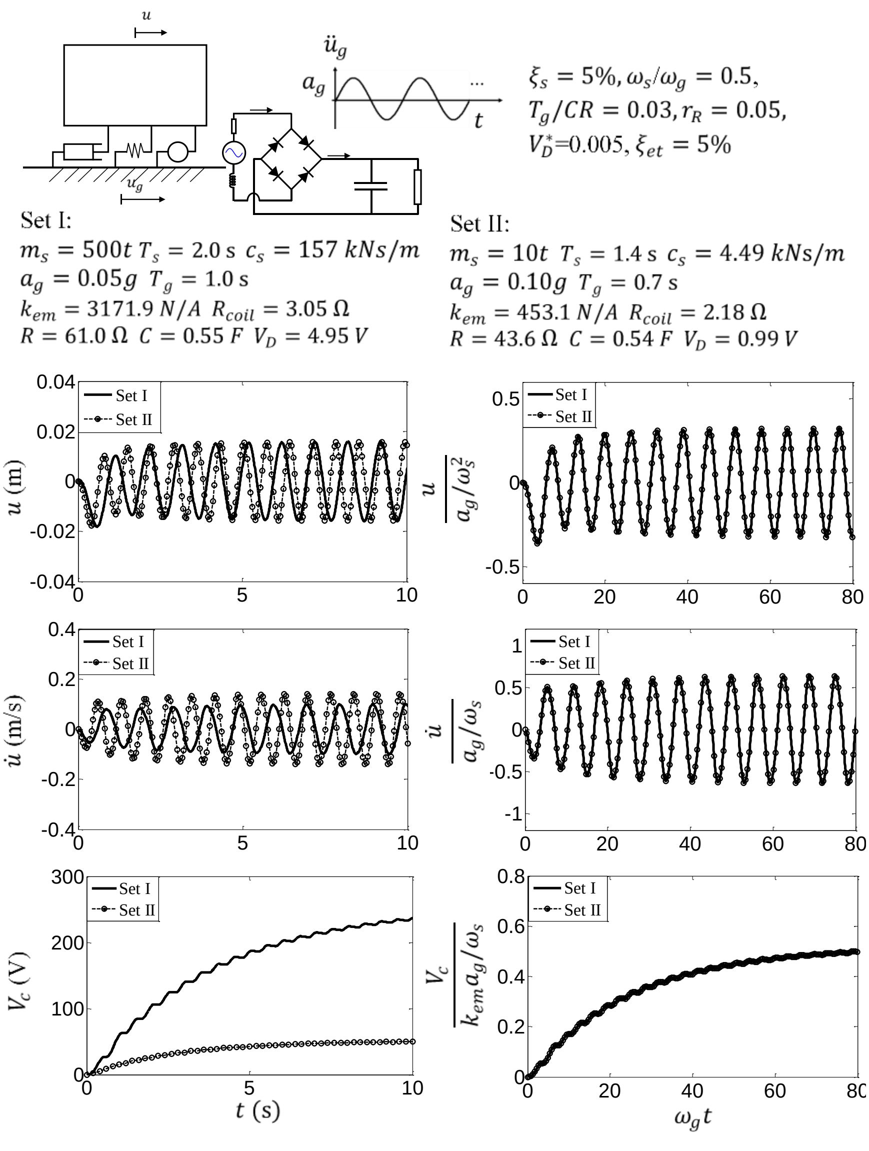

Figure 3.1 (Left) Time history response of two structure-harvester systems with different dimensional quantity, but with same dimensionless quantity; (Right) Self-similar response of the structure-harvester systems.

This study employs dimensional analysis to characterize the nonlinear dynamic system of Eq. (2.6). From the second equation of Eq. (2.6), we can observe that V_c/k_{em} is a function that depends on $1/CR$, $1/CR_{coil}$ and V_D/k_{em}. Consider a simple harmonic ground excitation with peak ground acceleration a_g and period T_g, i.e. $\ddot{u}_g = a_g \sin \omega_g t$, where $\omega_g = 2\pi/T_g$. The response quantity of interest (i.e. relative displacement, u) of the structure (the first equation of Eq. (2.6)) could be expressed as:

$$u = f\left(\xi_s, \omega_s, a_g, \omega_g, 1/CR, 1/CR_{coil}, k_{em}^2/m_s R_{coil}, V_D/k_{em}\right) \tag{3.1}$$

A response variable (left hand side of Eq. (3.1)) depends on the 8 independent variables in the right hand side of Eq. (3.1) (totaling 1+8=9 variables) that involve two reference dimensions, length L and time T. According to Buckingham's Π theorem (Barenblatt 1996), the number of dimensionless terms is = 9 (variables) – 2 (reference dimensions) = 7. Applying the theory of dimensional analysis, the dimensionless response quantity Π is a function of =6 independent dimensionless variables. Accordingly, Eq. (3.1) reduces to:

$$\Pi = \Phi\left(\frac{\omega_s}{\omega_g}, \xi_s, \frac{T_g}{CR}, r_R, V_D^*, \xi_{ec}\right) \tag{3.2}$$

where the (dependent) response is

$$\Pi = \frac{u}{a_g/\omega_s^2}, \frac{\dot{u}}{a_g/\omega_s}, \frac{V_c}{k_{em}a_g/\omega_s}, \text{or } \frac{E_{out}}{m_s a_g^2/\omega_s^2}$$

and independent variables are

$$\frac{\omega_s}{\omega_g}, \xi_s, \frac{T_g}{CR},\ r_R = \frac{R_{coil}}{R}, V_D^* = \frac{2V_D}{k_{em}a_g/\omega_s} \text{and } \xi_{ec} = \frac{k_{em}^2/R_{coil}}{2m_s\omega_s}$$

Π is the dimensionless response quantity of interest and T_g/CR indicates the stability of the capacitor voltage. A typical value of T_g/CR is less than 0.1 to reduce the fluctuation of the capacitor voltage; the value of r_R is typically less than 0.5 and V_D^* less than 0.5. ξ_{ec} is the electrical damping ratio provided by the coil. In practice, the value of ξ_{ec} alone does not represent the total damping provided by the harvester. It is recommended to consider the total damping (Stephen 2005, Tang and Zuo 2011 and Zhu et al. 2012) provided by the coil and the electrical load of the harvester, which is defined as $\xi_{et} = k_{em}^2/2m_s\omega_s(R_{coil} + R)$. Therefore, Eq. (3.2) becomes:

$$\Pi = \Phi(\omega_s/\omega_g, \xi_s, T_g/CR, r_R, V_D^*, \xi_{et}) \tag{3.3}$$

In the following, we will use ξ_{et} (instead of ξ_{ec}) to quantify the damping provided by the harvester. In fact, ξ_{et} is a combination of ξ_{ec} and r_R. There is no additional independent variable in the dimensional analysis. The total number of independent dimensionless variable remains six. Note that the ratio between ξ_{et} and ξ_s, (ξ_{et}/ξ_s) indicates the degree of electromechanical coupling between the two subsystems.

Fig. 3.1 demonstrates the time history response of the structure-harvester system with two sets of different dimensional term but same set of dimensionless quantities. When the six independent dimensionless quantities are the same, responses of the two sets of structure-harvester systems collapse to one curve. It means that the dynamic response of the structure-harvester system becomes indifferent to the intensity and the frequency content of the excitation. In other words, the system exhibits the property of physical similarity (Dimitrakopoulos et al. 2010).

3.2 Complete Similarity of Self-similar Response

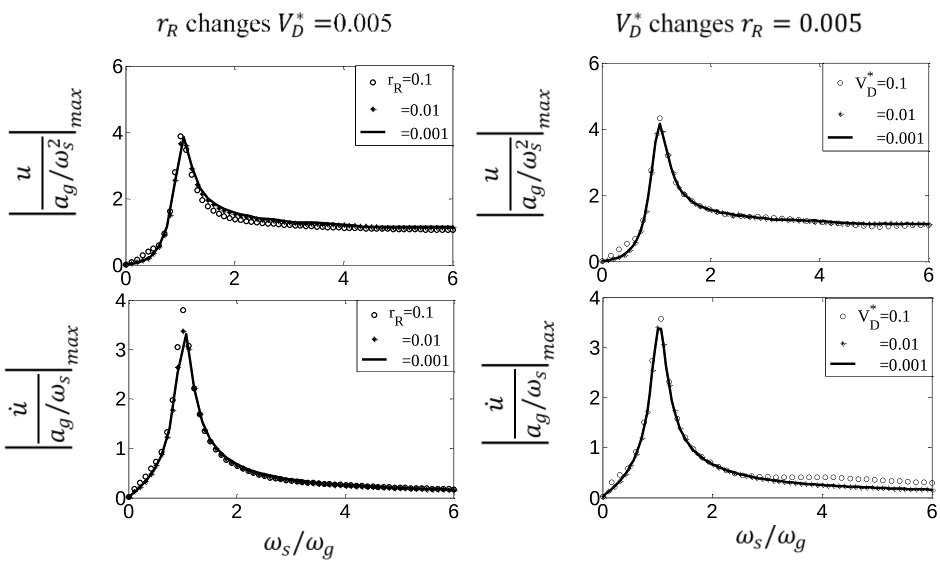

Figure 3.2 Complete similarity response of structure-harvester systems with dimensionless terms, r_R and V_D^*. ($T_g/CR = 0.06, \xi_s = 5\%, \xi_{et} = 5\%$)

Complete similarity response is sometimes found in nonlinear dynamics system (Barenblatt 1996). The self-similar response curves collapse to a "universal" curve when a particular dimensionless term tends to a small or large value (e.g. zero). For the structure-harvester system discussed in this paper, complete similarity or (similarity of first kind) for the dimensionless products, arises for small values of r_R and V_D^* (Barenblatt 1996). Fig. 3.2 shows the similar response spectrum of the coupled system under different values of r_R and V_D^*. As the values of r_R or V_D^* approach to zero, the response of the structure collapse to a finite value. It means that these two variables (r_R and V_D^*) are immaterial to the peak response of the structure. In other words, if a large load resistance is used or the forward biased voltage drop of the diodes are small, we can ignore these two variables in the design. This finding not only reduces the number of analysis in parametric study but also facilitates the design in practice. Note that the response spectra for $V_C/(k_{em}a_g/\omega_s)$ is not shown in this paper for brevity.

4. ANALYTICAL METHOD

This section provides an analytical method to estimate the response of the structure. Assuming that the nonlinearity of the harvester is not significant and the damping ratio ξ_{et} of the harvester is small, the steady-state solution of the displacement becomes sinusoidal, as shown in Fig. 3.1:

$$u = -u_m \sin(\omega_g t - \varphi) \tag{4.1}$$

where u_m and φ are the amplitude of vibration and the phase difference of the relative displacement with respect to the excitation. In order to obtain a stable output voltage, a supercapacitor can be used such that $RC \gg T_g$. This allows us to assume that the variation of capacitor voltage over one cycle of excitation is negligible (Lefeuvre et al. 2005). Define the capacitor voltage as the nominal voltage (i.e. $\dot{V_c} \approx 0$ and $V_c = V_{c,n}$), integrating both sides over one period of excitation, one can obtain:

$$V_{c,n} = \beta V_{max}$$
$$\text{where } V_{max} = k_{em}u_m\omega_g,\ \beta = \frac{2(\sqrt{1-a^2} - a\cos^{-1}a)}{\pi R_{coil}/R},\ \text{and } a = \frac{2V_D + V_{C,n}}{V_{max}} \tag{4.2}$$

To estimate the displacement amplitude of the structure, we first expand F_{em} in Fourier series:

$$F_{em}(t) = \frac{1}{2}a_o + \sum_{n=1}^{\infty} a_n \cos\left(n(\omega_g t - \varphi)\right) + b_n \sin\left(n(\omega_g t - \varphi)\right) \tag{4.3}$$

where a_i *and* b_i are the Fourier series coefficient. Assume that the first order term of Eq. (4.3) governs the response (only retain a_1 as $a_o = b_1 = 0$), Eq (4.3) then becomes :

$$F_{em}(t) \approx a_1 \cos(\omega_g t - \varphi) \tag{4.4}$$

Substitute Eq. (4.4) into (2.1) gives

$$u_m = \frac{a_g/\omega_s^s}{\sqrt{\left(1 - (\omega_g/\omega_s)^2\right)^2 + \left(2\xi_s\omega_g/\omega_s - a_1/m_s u_m \omega_s^2\right)^2}} \text{ and } \varphi = \tan^{-1}\frac{2\xi_s\omega_g/\omega_s - a_1/m_s u_m\omega_s^2}{1 - (\omega_g/\omega_s)^2} \tag{4.5}$$

Substituting Eq. (4.5) into (4.4), a_1 can be obtained as

$$a_1 = -\frac{2k_{em}V_{max}}{\pi R_{coil}}\left(\pi - \cos^{-1}a - a\sqrt{1-a^2}\right) \tag{4.6}$$

After solving the above transcendental equation for a_1, u_m and φ can be obtained from Eq. (4.5).

5. COMPARISON BETWEEN DIFFERENT MODELLING METHODS

As mentioned before, the simplified circuit (SC) shown in Fig. 1.1 have been used to analyze the coupled system.

The electromagnetic force provided by the harvester on the structure is:

$$F_{em} = c_{eq}\dot{u} \tag{5.1}$$

with $c_{eq} = k_{em}^2/(R_{eq} + R_{coil})$ is the equivalent damping coefficient provided by the harvester. Assume that a large resistance load is used, such that $R_{eq} \approx R,$ and $c_{eq} \approx k_{em}^2/(R + R_{coil})$. In the following, the difference between two modelling methods is discussed. Fig. 5.1 shows the response of the structure under different values of ξ_{et}. The difference in the transient state response between two modelling methods is relatively significant when compared to that of the steady state response. This implies that full scale dynamic of the circuit topology should be considered for examining the response of the structure under short duration pulses (e.g. impulse excitation). When the damping coefficient is small (Fig. 5.1 for $\xi_{et} = 5\%$), the differences between two modelling methods can be neglected under steady state condition. When the coupling coefficient is large ($\xi_{et}/\xi_s = 4$) (Fig. 5.1 for $\xi_{et} = 20\%$), the differences between two modelling methods are relatively significant. The nonlinearity of the harvester has more significant effect on acceleration than on displacement or velocity response. The SC model provides an upper bound estimation for all the response parameters of the structure-harvester system. In other words, it leads to a conservative design from the structure's viewpoint, however this overestimation is not beneficial, from the viewpoint of energy harvesting.

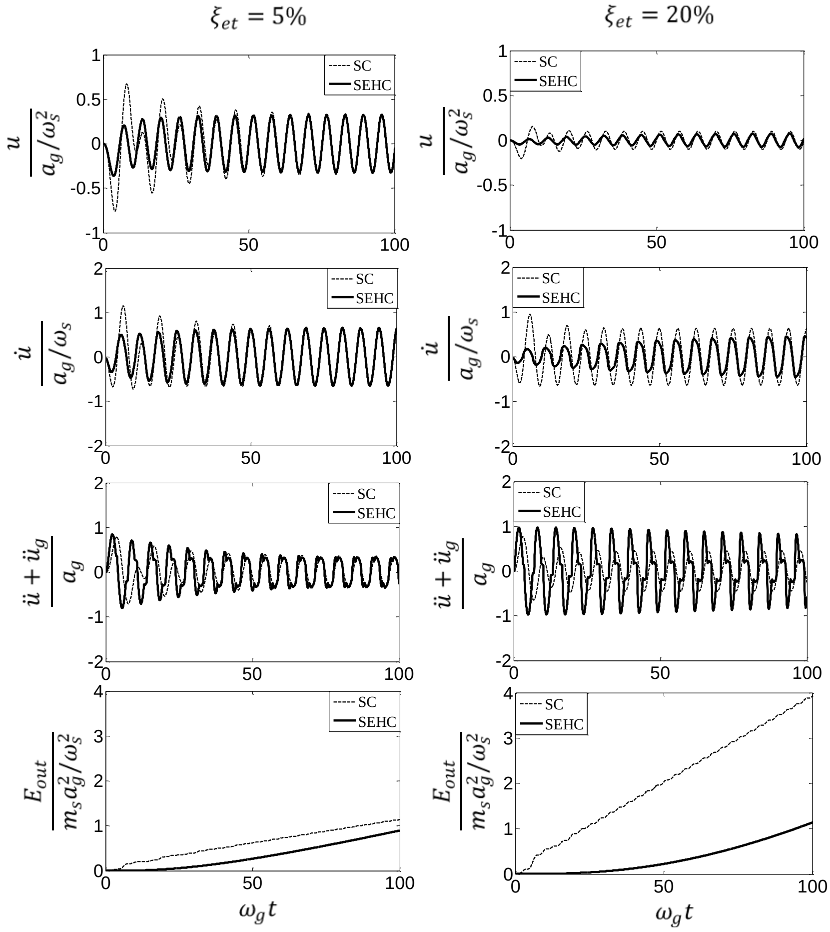

Figure 5.1 Time history response of structure-harvester systems with different modelling methods under simple harmonic excitation ($T_g/CR = 0.03, \xi_s = 5\%, V_D^* = 0.01. r_R = 0.05$).

6. CONCLUSION

This paper presented the nonlinearity effect of the circuit topology on the response of the coupled structure-harvester system under simple harmonic ground excitation. An EM harvester was attached to a SDOF structure and connected to a simplest yet realistic SEHC circuit. The behavior of the coupled system was characterized using dimensional analysis. Six independent dimensionless quantities were used to simulate the self-similar response of the system. Complete similarity was found in two of the dimensionless products (r_R and V_D^*). An analytical solution was obtained to predict the displacement response of the structure due to the effects of SEHC. Furthermore, a comparison between the results obtained with that using a SC circuit was presented. Results show that if the damping coefficient ξ_{et} is low, the SC circuit provides reasonable estimation on the displacement response of the structure under steady state condition. However, the SC circuit cannot predict the response under transient state accurately.

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