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# The study of sliding displacement spectrum on sliding base isolation

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**ABSTRACT (10 pt)**
Sliding structure is promising in China. But the research on sliding structure is not perfect, and it needs to conduct in-depth study. The dynamic response of pure friction-sliding isolation system under seismic action is analyzed through the Matlab/Simulink, the MDOF calculation model of sliding isolation structure is set up. Sliding displacement spectrum and sliding displacement with time history analysis of curve to the SDOF equivalent rigid model, and the DDOF equivalent model and the MDOF equivalent model of the sliding isolation system are contrasted respectively. The results show that the dynamic response of the DDOF equivalent is closed to response of the MDOF equivalent model; the maximum amount of sliding isolation layer is only related with coefficient of friction and seismic wave which is put in, and nothing is related with the overall mass of the structure; when coefficient of friction of the isolation layer is less than 0.1, the SDOF equivalent rigid model is fitted with the DDOF equivalent model. Accordingly, the rationality and the feasibility of using sliding displacement spectrum which is determine to simulate actual sliding structure is confirmed by the equivalent rigid model, and a foundation for the spread of sliding isolation system is established.

***KEYWORDS:*** *sliding structure; dynamic response; response spectrum*

## INTRODUCTION

As one kind of aseismic structures which is different from the aseismic structure with the larger damping and longer period in general, sliding structure could isolate the seismic energy which is passed up effectively. It also could reduce earthquake and be more used for the low-rise building. Sliding structure has good prospects for development in China; At the same time, the construction process is simple and low-cost and the advantages of having a wide range of applications because of its simple structure, easy production. Using sliding base isolation technology in the real structure not only make the building more reliable under the earthquake, but also can limit the number of layers of the building structure specification required a breakthrough in high intensity areas. But now, the research base sliding isolation technology is not perfect, theoretical and computational methods needed to conduct in-depth study[1]. Usually, to determine the earthquake with isolation structure is mainly the time-history analysis method. But time-history analysis method in the calculation model, isolation technology with the input seismic wave in engineering practice, and there are deficiencies in determining the seismic isolation structure selection and the results of data processing. In order to promote the role and effect aspects，to establish a quick, more accurate and practical method to simplify the calculation for large damping ratio and long-period structure is needed.

Because response spectrum method for seismic design is more mature and relatively simple calculation, seismic design code in most countries in the world use it as a basic design method. Response spectrum theory considers the relationship between the structural and dynamic characteristics and ground motion characteristics. This method is based on statistical analysis, and it gave the maximum response of different natural cycles of single degree of freedom system under the influence the different types of seismic waves (different earthquake mechanisms, different distance and different venues )[2]. Development of computer technology makes the time-history analysis of complex structures have greater capacity, but it is not universal, of practical significance, and not conducive to conventional engineering. Clear Response spectrum method has a clear concept, its accuracy can meet the engineering requirements, and software operation easy to understand, easy to master engineering and technical personnel. So it became the earthquake engineering community the most widely used method.

Sliding base isolation response spectrum theoretical research in the past two or three decades is deepening. Hong Feng[3]and others used the equivalent linearization method, and they get maximum slip sliding isolation rigid structure of the mean and standard deviation by the stochastic model, then get the slip structure response spectrum maximum slip according to the mean response; According on research on sliding isolation structure system, MAO Li-jun and LI Ai-qun which are from University on 67 by 02 edition seismic characteristic period of the seismic response spectrum, made statistical analysis on two types of sliding venue spectral response of isolated structure system, and pointed out its statistical characteristics. By sliding base isolation structure analysis of the dynamic characteristics of the building, Lin Yong[4] who is from Xi'an university of Architecture and Technology used the FORTRAN programming language draws response spectrum sliding structure, and analyzed the factors affecting sliding structure response spectra. FAN Jian, TANG Jia-xiang[5] made continuous exponential friction model. Based on this model, they calculated the seismic response of sliding structure and draw the structure dynamic response coefficient and maximum slip base spectral response spectrum. From the above, we can see easily that the sliding base isolation response spectrum theory promote an in-depth study of the sliding isolation system. But this study is still under development. The sliding base isolation structure which is built in early stage is dominated multi-layer brick structure. So dynamic model is also based on simplified approximation brick structure[6]. The sliding isolation will be used in a growing number in structural system with the development of the sliding isolation, such as frame structure, shear wall structure. However, based on sliding isolation system dynamic model of such structures are few which needs further exploration and research. Although many scholars through a number of seismic waves plotted sliding structure response spectra statistics， but the lack of verification-related experiments and specific examples and the lack of systematic research and accumulation in terms of the use of the domestic construction industry.

Based on Coulomb's Friction, equivalent simulation was conducted by the Matlab / Simulink. The MDOF calculation model of sliding isolation structure is set up, and sliding displacement spectrum and sliding displacement with time history analysis of curve to the SDOF equivalent rigid model, the DDOF equivalent model and the MDOF equivalent model of the sliding isolation system are contrasted respectively. Then changes would be explored in the dynamic response of the isolation system in different coefficient of friction to make reference to the actual project.

## 1. ESTABLISH OF SLIDING DISPLACEMENT SPECTRUM ANALYSIS MODEL

In order to study the slip displacement spectrum better, the DDOF equivalent model is used in this paper, as shown in Figure.1.1.When this equivalent system is not sliding, and this is equivalent to the basic dynamic performance of a single degree of freedom system. And that method is used for the elastic response spectrum analysis. When this equivalent system is sliding, the DDOF equivalent model could simulate the feedback effect on the upper structure of isolation layer, relatively close to the multi-degree of freedom isolation system. In Fig.1.1, the weight of isolation layer is $m_0$, equivalent mass of the upper structure is $m_1$ , $k_1$ and $c_1$ are equivalent stiffness and equivalent viscous damping of the upper structure. $u$ is isolation layer friction coefficient and $F_f$ is friction of the isolation layer. $F_f$ is based on Coulomb's Friction, and $F_f = umg \cdot sign(\dot{x})$， $sign(\dot{x})$ is a Sign function， and $\dot{x}$ is velocity of the isolation layer.

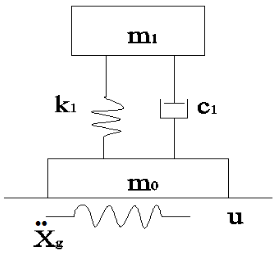



Fig.1.1 The DDOF equivalent model of the sliding isolation system

（1）Make stress analysis on $m_1$ :

$$m_1(\ddot{x}_1 + \ddot{x}_0 + \ddot{x}_g) + c_1\dot{x}_1 + k_1x_1 = 0 \tag{1.1}$$

In this formula, $x_1$ is super structural relative displacement from isolation layer, and $x_0$ is isolation layer displacement relative to the base.

（2）Make stress analysis on $m_0$ :

$$m_0(\ddot{x}_0 + \ddot{x}_g) + F_f = c_1\dot{x}_1 + k_1x_1 \tag{1.2}$$

（3）Make stress analysis on the whole system:

$$m_0(\ddot{x}_0 + \ddot{x}_g) + m_1(\ddot{x}_1 + \ddot{x}_0 + \ddot{x}_g) + F_f = 0 \tag{1.3}$$

Combine equations, then can be calculated:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = \frac{-m_1(-F_f - m_1\ddot{x}_1)}{m_0 + m_1} \tag{1.4}$$

（4）Make $R = \frac{m_1}{m_0 + m_1}$ , then make $1 - R = \frac{m_0}{m_0 + m_1}$ ,and divide $m_1$ （$1-R$）simultaneously on both sides of the formula， can be calculated：

$$\ddot{x}_1 + \frac{c_1}{(1-R)m_1}\dot{x}_1 + \frac{k_1}{(1-R)m_1}x_1 = \frac{ug \cdot sign(\dot{x}_0)}{1-R} \tag{1.5}$$

Make $\omega_0 = \sqrt{\frac{k_1}{m_1}}$ 、 $\zeta_0 = \frac{c_1}{2m_1\omega_0} = \frac{c_1}{2\sqrt{k_1m_1}}$ 、 $\omega_1 = \frac{\omega_0}{\sqrt{1-R}}$ 、 $\xi_1 = \frac{\xi_0}{\sqrt{1-R}}$ ,and let them into the above equation, can be calculated:

$$\ddot{x}_1 + 2\xi_1\omega_1\dot{x}_1 + \omega_1^2x_1 = \frac{ug \cdot sign(\dot{x}_0)}{(1-R)} \tag{1.6}$$

The above equation is similar to SDOF differential equation when the object is powered by forced vibration. As can be seen from equation, the dynamic response of the upper structure is related to the different site conditions, the input ground motion (spectrum, amplitude and duration time), the mass ratio and the friction coefficient of the dynamic characteristics of the structure itself (stiffness, mass and damping), the Mass ratio and the friction from upper part of the structure and isolation layer. Since the first vibration of isolation structure type reactions maximum, and the first mode period and damping as well as its seismic response of isolated structure is mainly determined by the characteristics of the isolation system which is regardless of the structural period and damping. So, mass of isolation layer and the upper structure can be equivalent to a single mass vibrator if less important modal form factor and the contribution of higher modes are negligible.

## 2. ESTABLISH OF EQUIVALENT SIMULATION MODEL

### 2.1. Establish isolation layer friction model

Due to the strong nonlinear on isolation layer, the response of sliding structure is strongly nonlinear. And this make the structure more difficult to analyze. Consider difficulty of sliding structure simulation analysis is that establishing isolation layer friction model accurately and breadth of applications, accuracy of the determination and convenience to use on Coulomb's Friction. So, discontinuous Coulomb's Friction is be used on slip surface friction model.

Mathematical equations on friction of isolation layer[7]:

$$F_f=\begin{cases} sign(\dot{x})uF_n, \left|F_{sum}\right|>uF_n \\ F_{sum}, \dot{x}=0\text{且}\left|F_{sum}\right|\le uF_n \end{cases} \tag{2.1}$$

Among them, $u$ is coefficient of friction， $F_n$ is positive pressure, and $F_{sum}$ is forced in a stationary state.

$$F_{sum}=-m\ddot{x}_g-c\dot{x}-kx \tag{2.2}$$

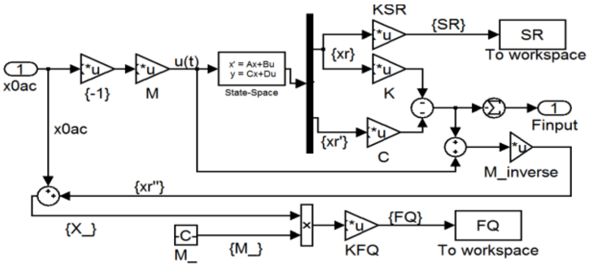
Friction is generally divided into static friction and dynamic friction, depending on whether the object movement:

$$uF_n=\begin{cases} u_{static}F_n, \dot{x}=0 \\ u_{sliding}F_n, \dot{x}\neq 0 \end{cases} \tag{2.3}$$

Among them, $u_{static}$ and $u_{sliding}$ are the coefficient of static friction and sliding friction coefficient. Considering the above two conditions formula, the following equation friction can be determined：

$$F_f=\begin{cases} sign(\dot{x})F_{sliding}, \dot{x}\neq 0 \\ F_{sum}, \dot{x}=0\text{且}\left|F_{sum}\right|<F_{static} \\ sign(F_{sum})F_{static}, \dot{x}=0\text{且}\left|F_{sum}\right|\ge F_{static} \end{cases} \tag{2.4}$$

Fig.2.1 is shown that a sub-model of friction by Matlab/Simulink.



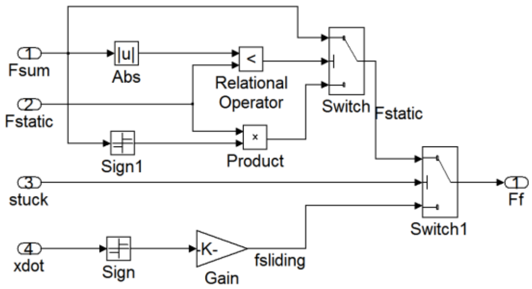




Fig.2.1 Sub-model of the friction Fig.2.2 The simulation model of the upper structure

## 2.2. Establish of Sliding structural MDOF dynamic analysis models

The sliding base isolation structure is divided into two parts, the upper part of the structure and the isolation layer. Among them, dynamic analysis model of isolation layer is:

$$F-F_f=m_0(\ddot{x}_0+\ddot{x}_g) \tag{2.5}$$

In this formula, $m_0$ is weight of the isolation layer, F is external input force isolation layer. $\ddot{x}_0$ and $\ddot{x}_g$ are isolation layer relative to the ground floor of the absolute acceleration and acceleration. $F_f$ is friction of the isolation layer,and this selects Coulomb's Friction.

Superstructure is shear with multiple degrees of freedom in series system, and the acceleration of seismic isolation layer is considered the upper structure of incentives.Dynamic analysis model of the ground floor relative to the isolation layer $m_0$ is:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=-[M][I]\{\ddot{x}_0+\ddot{x}_g\} \tag{2.6}$$

In this formula, $(\ddot{x}_0 + \ddot{x}_g)$ is absolute acceleration of isolation layer, and $[M]$, $[C]$, $[K]$ are mass, damping and stiffness matrices of the upper structure. $\{\ddot{x}\}$, $\{\dot{x}\}$, $\{x\}$ are acceleration, velocity and displacement vectors of superstructure relative to isolation layer.

The power differential equations of overall structure is：

$$\sum_{i=1}^{n} m_i(\ddot{x}_i + \ddot{x}_0 + \ddot{x}_g) + m_0(\ddot{x}_0 + \ddot{x}_g) + F_f = 0 \qquad (2.7)$$

Established by Matlab/Simulink something is obtained:

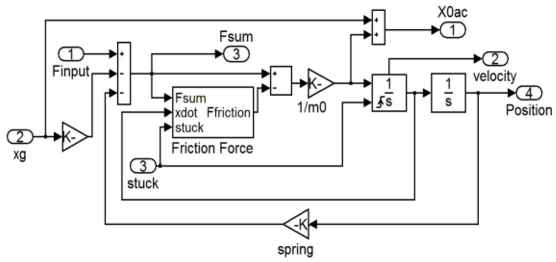



Fig.2.3 the simulation model of the isolation layer

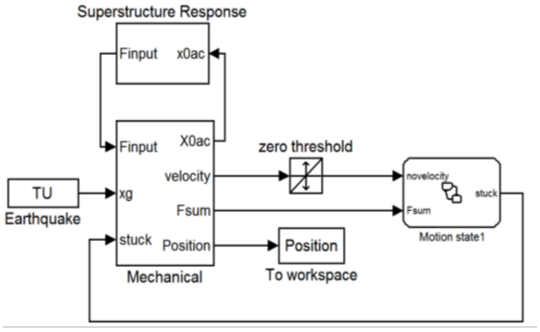



Fig.2.4 the master model of the sliding structure

## 3. DYNAMIC RESPONSE ANALYSIS OF SLIDING BASE ISOLATION STRUCTURE

In this paper, the actual engineering model[8] is a prototype scale mode from Shanxi Provincial Academy of Building Research sliding isolation tests. The model is a 5-story frame structure that the first floor column size is 750mm × 750mm,and 2-5 floor column size is 600mm×600mm, the girder is 300mm×600mm, the secondary beam is 200mm×400mm,the thickness of the board is 100mm,the thickness of the isolation layer is 100mm.The storey floor is 4.2m,the remaining layers storey is 3.6m,the column spacing is 6m.The isolation layer size is 15m × 15m,the horizontal and the vertical are two cross, the scale model test is shown as Fig.3.1.Detailed engineering model data is in Table 3.1, where the isolation layer instead by layer 0.

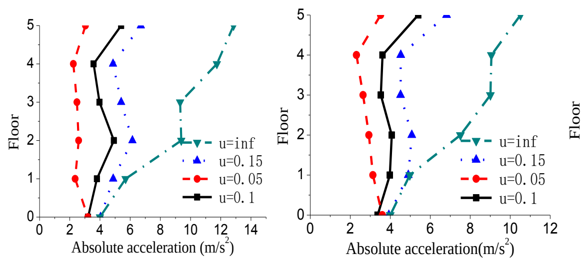
Fig.3.1 the scale model on test of the sliding base isolation

Table 3.1 the engineering model data

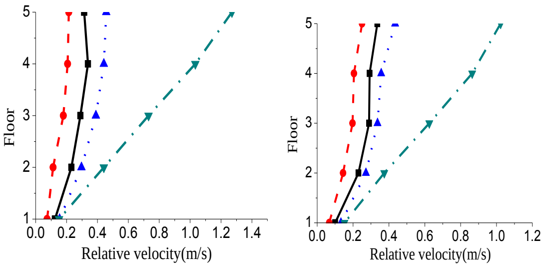
| Number of floors | Mass(kg) | Stiffness(kN/m) | | Height (m) |
|---|---|---|---|---|
| | | X Direction | Y Direction | |
| 5 | 146E+3 | 1.4029E+5 | 1.4029E+5 | 3.6 |
| 4 | 146E+3 | 1.5589E+5 | 1.5589E+5 | 3.6 |
| 3 | 146E+3 | 1.6184E+5 | 1.6184E+5 | 3.6 |
| 2 | 146E+3 | 1.8435E+5 | 1.8435E+5 | 3.6 |
| 1 | 171E+3 | 3.4395E+5 | 3.4395E+5 | 4.2 |
| the isolation layer. | 146E+3 | —— | —— | —— |

To analyze the effect and dynamic response of the isolation sliding base isolation structure, this selection two typical seismic waves that Elcentro and Taft on the basis of fixed and sliding base isolation two structures were loaded with Middleweight. According to a certain percentage ,the peak acceleration adjusted octave rare earthquake corresponding peak acceleration（400cm/s$^2$）.Using the Simulink simulation model which established in response to the above two structures in the rare case of power under the comparative analysis of the seismic action, then acceleration, velocity, displacement and shear values from each floor structure are obtained. Then, the isolation effect will be analyzed.

Using above models established by Simulink, make numerical simulation analysis on MDOF in different coefficient of friction when $u = 0.05、0.1、0.15、\infty$ under octave rare earthquake. The dynamic response of the structure is obtained as shown in Fig.8-9.Fig.10 is shown that the top curve acceleration process under different friction coefficients. $u = \text{inf}$ expresses infinite friction coefficient, the foundation is in a fixed state.



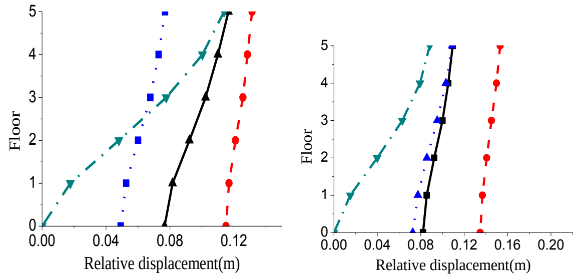
（a）Under the Elcentro wave （b）Under the Taft wave



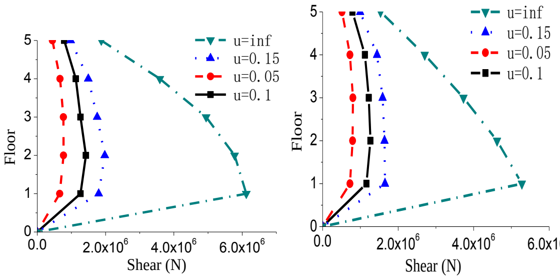
（a）Under the Elcentro wave （b）Under the Taft wave

Fig.3.2 The distribution of absolute acceleration in different coefficient of friction

Fig.3.3The distribution of relative velocity in different coefficient of friction



（a）Under the Elcentro wave （b）Under the Taft wave



（a）Under the Elcentro wave （b）Under the Taft wave

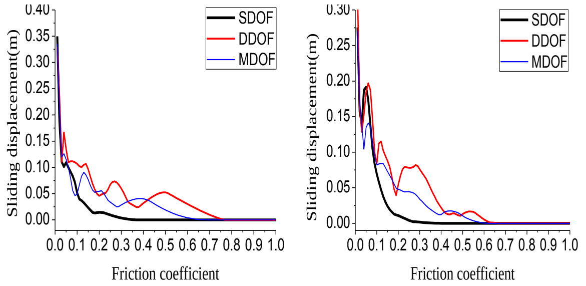
Fig.3.4 The distribution of relative displacement in different coefficient of friction

Fig.3.5 The distribution of shear in different coefficient of friction

As can be seen in Figure 3.2-3.5: Acceleration, velocity, shear friction from isolated structure in coefficient at different levels have different degrees of decrease, compared to traditional seismic structure. Isolated significant effect; With the decreasing of the coefficient of friction, acceleration is also smaller, its corresponding displacement of isolation layer is growing. The results of the original West Kenda more consistent test results, indicating that Simulink can simulate the isolation effect.

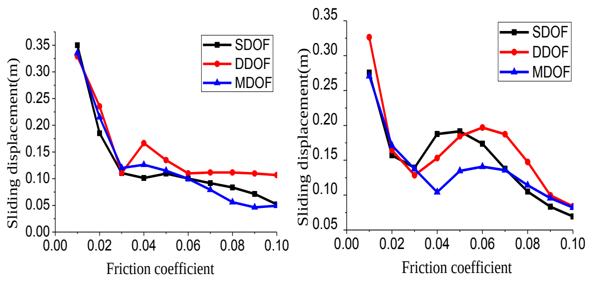
## 4. DISPLACEMENT SPECTRAL ANALYSIS OF SLIDING BASE ISOLATION STRUCTURE

In this paper, make the dynamic response of the MDOF as benchmark, and contrast it with the dynamic response of SDOF and DDOF under rare octave earthquake. Then see the three models under the dynamic response of the structure and analyze the similarities and differences between the two, such as the range of reasonable generation model and apply. Among them, number of floors N=5，Superstructure damping ratio is 5%.Single degree of freedom model cannot output dynamic response of the upper structure. So choose isolation layer slip displacement (maximum amount of slip) as indicators of discrimination.




（a）Under the Elcentro wave （b）Under the Taft wave

Fig.4.1 The sliding displacement spectrum( $\mu < 1$)




（a）Under the Elcentro wave （b）Under the Taft wave

Fig.4.2 The sliding displacement spectrum( $\mu < 0.1$)

Structure under seismic waves of different computing model gives response comparison from Figure 4.1-4.2, something can be seen:

（1）Under the El Centro wave and Taft wave, with the increase of the friction coefficient, slip displacement system of sliding structure under three different computational model of isolation layer gradually decreases and the spectral shape are basically the same. Slip displacement spectra with multiple degrees of freedom model fitting results of different models such as generation due to friction coefficient varies, which slip displacement DDOF model and MDOF model in the entire spectrum of the friction coefficient range is close.

（2）when $u < 0.1$, the isolation layer slip is larger, isolation results were quite good. Then the friction coefficient interval spectral line values fall faster, followed by a relatively small rate decreases; When $u > 0.6$, the slip displacement spectrum value under three different calculation models are 0.No sliding isolation layer, and the structure loses isolation capability; when $0.1 < u < 0.6$, isolation layer slippage is small and isolation effect is not very significant.

（3）From the above analysis, the isolation layer is not sliding and the structure lose the ability to isolate under octave rare earthquake and when $u > 0.6$; When $u \leq 0.1$, amount of slip isolation layer is large and isolation effect on the structure would be better. And in the friction coefficient range, slip displacement spectra fit very well under different degrees of freedom model. It shows isolation layer reaction from sliding structure is close under different models, and so is the upper structure. Equivalent rigid model can be used to simplify the multi-degree of freedom model to calculate reaction from isolation layer and the upper structure.

## 5. APPLICATION OF SLIP DISPLACEMENT SPECTRUM

Use slip displacement spectra and equivalent response spectrum to determine the actual engineering model of maximum amount of slip and the maximum shear from bottom of the upper structure which mentioned in herein. It is shown in table 5.1-5.2. As can be seen in table 5.1-5.2, results from time history analysis is not large with using slip displacement spectrum and the upper structural equivalent response spectrum, and it can meet the needs of the project to simplify the calculations and preliminary design.

Table 5.1 The contrast in dynamic responses with different degrees of freedom model under the Elcentro wave

| Basic cycle from superstructure T=0.5902 | | MDOF | | DDOF | | SDOF | | Error |
|---|---|---|---|---|---|---|---|---|
| | | The maximum amount of slip(m) | The maximum base shear(N) | The maximum amount of slip(m) | The maximum base shear(N) | The maximum amount of slip(m) | The maximum base shear(N) | The maximum base shear(N) |
| Sliding isolation system | u=0.05 | 0.11504 | 6.5839E5 | 0.13473 | 7.2907E5 | 0.10955 | —— | 10.74% |
| | u=0.08 | 0.05598 | 9.8671E5 | 0.11155 | 1.0362E6 | 0.08358 | —— | 5.02% |
| | u=0.10 | 0.04917 | 1.2668E6 | 0.1068 | 1.2447E6 | 0.05177 | —— | 1.74% |
| Fixed base | u= ∞ | —— | 6.1198E6 | —— | 6.2451E6 | —— | —— | 2.05% |

Table 5.2 The contrast in dynamic responses with different degrees of freedom model under the Taft wave

| Basic cycle from superstructure T=0.5902 | | MDOF | | DDOF | | SDOF | | Error |
|---|---|---|---|---|---|---|---|---|
| | | The maximum amount of slip(m) | The maximum base shear(N) | The maximum amount of slip(m) | The maximum base shear(N) | The maximum amount of slip(m) | The maximum base shear(N) | The maximum base shear(N) |
| Sliding isolation system | u=0.05 | 0.13478 | 7.2355E5 | 0.18434 | 7.6940E5 | 0.19143 | —— | 6.34% |
| | u=0.08 | 0.11411 | 9.7066E5 | 0.14753 | 1.0782E6 | 0.10500 | —— | 11.07% |
| | u=0.10 | 0.08213 | 1.1596E6 | 0.08379 | 1.3527E6 | 0.06936 | —— | 16.65% |
| Fixed base | u= ∞ | —— | 5.2774E6 | —— | 4.9046E6 | —— | —— | 7.06% |

## 6. CONCLUSION

Matlab/Simulink is combined in this paper, and equivalent rigid model of sliding Isolation is established. Make some analysis on DDDF and SDOF, then get slip displacement spectrum .Contrast this with the actual dynamic response of sliding structure. The main contents and results are as follows:

（1）Under rare earthquake, and when the friction coefficient is low（$u<0.1$）. Slip displacement from isolation layer is large and isolate has significant effect, the value from spectral lines of slip displacement decrease quickly; when the friction coefficient is large（$u>0.15$）, isolation layer slippage remained as a constant value. Changes in value of the spectrum are small and close to zero. That means isolate does not have has significant effect when the friction coefficient is large.

（2）when the friction coefficient is less than 0.1, the slip displacement spectra by SDOF fit better with that of MDOF, which are structurally unrelated to the overall mass. It verifies the rationality and feasibility of substituting rigid structure to simulate the actual sliding structure.

（3）In the preliminary design ,it can be used to estimate its maximum about amount of slip under given friction coefficient or select range of the coefficient friction under a given about maximum amount of slip. Using the equivalent response spectrum by DDOF can simplify and preliminary design calculations on maximum of base shear from sliding base isolation structure.

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