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Identification of Rotating Blades Using Rodrigues' Rotation Formula from A 3-D Measurement

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ABSTRACT
. This study was to conduct the 3-D position identification of a research-scale rotating blade in the laboratory using limited number of sensors. The turbine system was rotated by a controlled motor and one of the blades was instrumented with a single 3-dimensional accelerometer at the tip of one blade. Data acquisition was achieved using a prototype wireless sensing system. First, the Rodrigues' rotation formula was used to determine the blade rolling angle and pitching angle of the rotating blade system through optimization. From the residual signal between the recorded and the calculated data, the blade flap-wise natural frequency can be identified. To verify the result of identification, the covariance-driven stochastic subspace identification method (SSI-COV) was also used to construct the stability diagram from which the system natural frequencies can also be identified. It is proved that by using 3-D measurement from a single sensor and through Rodrigues' rotation formula one can identify the blade geometry setup as well as the blade flap-wise vibration frequency

KEYWORDS: *Signal processing, stochastic subspace identification, Rodrigues' rotation formula, blade flap-wise rotating frequency, wind turbine system.*

1. GENERAL INSTRUCTIONS

Two main types of vibration occur in wind turbine blades, flap-wise and edgewise. Flap-wise vibrations are vibration occurring out of the plane of rotation of the blades, whereas edgewise vibrations occur in the plane of rotation. In extreme cases these two vibration phenomenon may lead to the turbine blades colliding with the tower resulting in catastrophic failure of the structure. From the theoretical viewpoints the dynamic characteristics of a rotating blade are affected by two axial phenomena: the first being centrifugal stiffening and the second being blade gravity self-weight effects. Besides, the geometry setup of blade system may also influence the vibration characteristics of blades. The identification of the dynamics of operating wind turbines and its 3-D setup position require the use of appropriate techniques that can capture and describe effectively the dynamic behavior and cyclic effects of the natural vibration response and the structural modes. Simmermacher et al. (1997) proposed the structural health monitoring technique of wind turbines. Lading *et al.* (2002) proposed the technique of conditional monitoring techniques of turbine blade using embedded sensors. Ciang *et al.* (2008) has compiled a state-of-the-art review of SHM and damage detection methods specifically for wind turbine applications. Besides, many advanced signal analysis and damage detection algorithm have been developed. However, the majority of wind turbine condition monitoring and fault diagnosis techniques proposed to have used the Fourier Transform (FT) or the wavelet transform, which is less capable of solving the problem due to its shortcomings in dealing with complex response signals to the condition monitoring, as discussed by Ghoshal *et al.* (2000). Therefore, health monitoring of the rotor blades and timely identification of potential failure become an important issue to prevent failure of the entire horizontal axis wind turbine system.

In this paper, the vibration-based technique is used to identify the modal properties of blade/tower interaction and the geometry location of turbine system during rotation using experimental data. To analyze the vibration data and the geometry setup of wind turbine system (including pitch angle and rolling angles of blade) the optimization technique together with the Rodrigues' rotation formula are used. Dynamic characteristics brought about by the rotation of three blades (or beam-like structure) are also investigated using subspace identification.

2. EXPERIMENTAL SETUP OF ROTATING TURBINE SYSTEM AND TOWER

A rotating wind turbine system is used in the laboratory for the study of vibration monitoring and feature extraction. The sketch of the research-scale turbine system is shown in Figure 1. The height of the tower is 3.0 meter and the blade dimension is 80 cm in length and 20 cm in width. The diameter of the rotation blade is about 180 cm. In order to control the excitation (rotation of the turbine blade), a motor was used to spin the blades at controlled angular velocities. It is characterized by tower/blade interaction. In the experimental study, each blade was installed with two accelerometers (at the tip and middle of each blade). Besides, on one of the blade tip a 3-D accelerometer (Model 1221 with input range ±2g, Silicon Design, Inc.) was also used to replace the 1-D accelerometer. The wireless sensing system together with accelerometer as a sensing node is used for vibration monitoring of the wind turbine system. The sampling rate for the measurement data is 200 Hz.

Three different test cases on this research-scale turbine system are conducted in the laboratory. Data from Test-1 will be used for detecting the blade flap-wise vibration frequency. Data fromTest-2 and Test-3 will be used for identify the rolling angles of blade. Each test setup is described as follows

- Test-1: Two different test rotation frequencies (15 rpm and 50 rpm) were conducted; response measurement was taken both from one 3-D accelerometer at the tip of one rotating blade and from 6 accelerometers (two on each blade along the flapwise motion direction);
- Test-2: Wind turbine rotated with 15 rpm and one turbine blade was designed with three different types of connection angles (change γ_1 rolling angle only) between the blade and the hub (i.e. type-1: $\boldsymbol{\gamma_1}$=6°; type-2: $\boldsymbol{\gamma_1}$=10°, and type-3: γ_1=20°, as shown in *Figure 2a*); The other two rolling angles are set as γ_2=0°, γ_3=0°;
- Test-3: Same as Test-2 but one turbine blade was designed with three different types of connection (change both γ_1 and γ_2 rolling angles) between the blade and the hub (i.e. type-1:$\boldsymbol{\gamma_1}$**=6°, **$\boldsymbol{\gamma_2}$=12°,; type-2: γ_1=10°, γ_2=12°,; and type-3: $\boldsymbol{\gamma_1}$**=20°, **$\boldsymbol{\gamma_2}$**=12°**, as shown in Figure 2b);

(a) (b)

Figure 1. (a) Skech of the research-scale wind turbine system, (b) Photos of the wind turbine mounted on the shaking table.

Test-1 Test-2

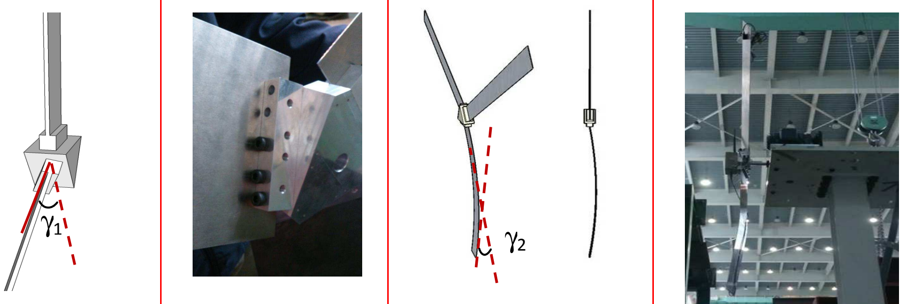


Figure 2: Sketch and photo show two setups of blade installation: Test-1 shows the change of rolling angle γ_1 of one blade at the connection between blade and hub; Test-2 shows the change of out-of-plane rolling angle γ_2 of one blade

3. IDENTIFICATION OF GEOMETRY SETUP OF TURBINE BLADE SYSTEM USING RODRIGUES' ROTATION

Due to complicated geometry setup of the turbine blade system, particularly the pitching and rolling angles of turbine blades during operating condition, the monitoring of these geometry parameters directly from measurement becomes a challenge. If the geometry setup can be identified during operating condition, the information may provide safety assessment of turbine blade system. Since a 3-D accelerometer was mounted on the rotating blade, and Rodrigues' rotation formula can provide an efficient algorithm to compute a rotation matrix from a rotating vector in space with a given an axis and angle of rotation, therefore, a robotic manipulation in mathematics from which the Rodrigues' rotation formula was introduced (Murray *et al.* ,1994). This formulation can be obtained from *MathWorld* by Belongie (1999).

Assumed the blade rotating plane will have a pitch angle of ϕ with respect to the direction of gravity (as shown in *Figure 3a*). Now consider a fixed rectangular coordinate system (defined as X-, Y-, and Z- axes on the blade rotation plane) which is defined as a fixed global coordinate system at the hub of the rotation turbine blade system. At the tip of one blade a 3-D sensor was placed and there are two forces acting on it: one is the centrifuge force ($F = mr\omega^2$) due to the rotation blade with a specific rotation frequency ω and the other is the gravity force. First, the centrifuge force can be separated into two forces along the fix global coordinate system (X, Y, and Z coordinate system) at any rotation angle θ_t: i.e. $a_{tx} = r\omega^2 \cos\theta_t$ *and* . $a_{ty} = r\omega^2 \sin\theta_t$ (as shown in Figure 3b). Second, coordinate transformation is applied to the gravity force and transforms the gravity force to the original fix global coordinate system (on the blade rotating plane and perpendicular to the blade rotating plane, as shown in Figure 3a). Therefore, the force vector (or in terms of acceleration, $\{\boldsymbol{a}_t(t,\phi,\theta_t)\}$ in the fix global coordinate system can be expressed as:

$$\{\boldsymbol{a}_t(t,\phi,\theta_t)\} = \begin{bmatrix} a_{tx} \\ a_{ty} \\ a_{tz} \end{bmatrix} = \begin{bmatrix} -r\omega^2 cos\theta_t \\ -r\omega^2 sin\theta_t \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & sin\phi & cos\phi \\ 0 & cos\phi & -sin\phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \tag{1}$$

where θ_t is the angle of rotation at any particular time (it is a function of time) and ϕ is the pitch or tilt angle, and *r* is the distance of between the sensor location and the hub center of the rotation axis.

Since the data vector $\{\boldsymbol{a}_t(t,\phi,\theta_t)\}$ is expressed in the fix global coordinate system and needs to be transformed to the local coordinate system at the tip of a blade (X'-, Y'- and Z'- coordinate system or radial, tangential and vertical coordinate system), and the transformed new data vector $\{\boldsymbol{a}_{tr}(t,\phi,\theta_t)\}$ can be expressed in the following equation (as shown in Figure 3b):

$$\{\boldsymbol{a}_{tr}(t,\phi,\theta_t)\} = \begin{bmatrix} -\cos\theta_t & -\sin\theta_t & 0 \\ \sin\theta_t & -\cos\theta_t & 0 \\ 0 & 0 & 1 \end{bmatrix} \{\boldsymbol{a}_t(t,\phi,\theta_t)\} \tag{2}$$

Besides, the installation of sensing node (at the tip of blade) may also have three different rotation angles, γ_{S1}, γ_{S2}, *and* γ_{S3}, (yaw angle, pitching angle and rolling angle) with respect to the local coordinate system (X''-, Y''- and Z''-axes), as shown in Figure 3c. Therefore, the data vector $\{\boldsymbol{a}_{tr}(t,\phi,\theta_t)\}$ also needs to be transformed to the real sensor coordinate system through the Rodrigues' rotation.

Consider a unit vector $\mathbf{n} = < n_x, n_y, n_z >$, where $n_x^2 + n_y^2 + n_z^2 = 1$, the matrix for a rotation by an angle of θ about an axis in the direction of **n** is

$$\boldsymbol{\mathcal{R}} = \begin{bmatrix} cos\theta + n_1^2(1 - cos\theta) & n_1 n_2(1 - cos\theta) - n_3 sin\theta & n_1 n_3(1 - cos\theta) + n_2 sin\theta \\ n_1 n_2(1 - cos\theta) + n_3 sin\theta & cos\theta + n_2^2(1 - cos\theta) & n_2 n_3(1 - cos\theta) - n_1 sin\theta \\ n_3 n_1(1 - cos\theta) - n_3 sin\theta & n_3 n_2(1 - cos\theta) + n_1 sin\theta & cos\theta + n_3^2(1 - cos\theta) \end{bmatrix} \tag{3}$$

This is the matrix form of Rodrigues' rotation formula. Since each blade in turbine system may have pitch angle γ_{S2}, yaw angle γ_{S1}, and rolling angle γ_{S3}, therefore, the data vector $\{\boldsymbol{a}_{tr}(t,\phi,\theta_t)\}$ also needs to be transformed to the real sensor coordinate system through the Rodrigues' rotation. It is assumed that the estimated 3-dimention acceleration response at any instant of time, $\boldsymbol{a}_e(t,\overline{\boldsymbol{\beta}})$, can be expressed as:

$$\boldsymbol{a}_e(t, \overline{\boldsymbol{\beta}}) = \mathfrak{R}(-\gamma_{s1}, -\gamma_{s2}, -\gamma_{s3}) \ \{\boldsymbol{a}_{tr}(t,\phi,\theta_t)\} \tag{4}$$

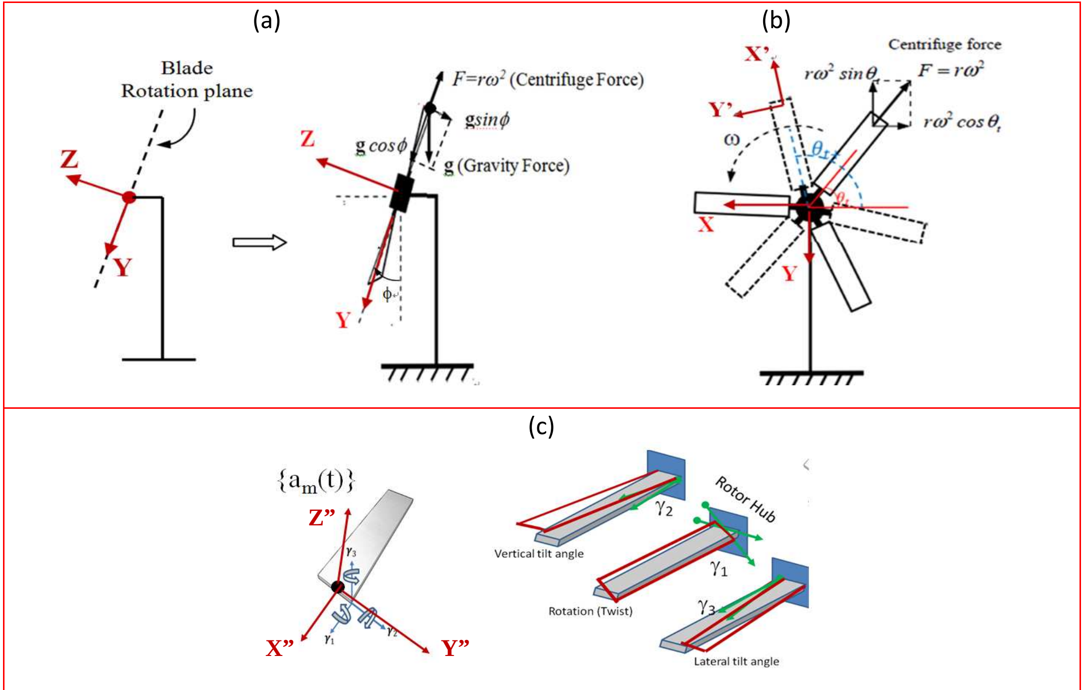


Figure 3: (a) Side view of a rotating wind turbine which shows he global coordinate system X-, Y- and Z-axes (plane of rotating blades). The gravity force at the sensor location is shown.
(b) Front view of the turbine blade system which indicates the local coordinate system at the sensing node and the centrifuge force.
(c) Three rolling angles on sensor coordinate system with respect to blade rotation plane.

where $\mathfrak{R}$ is Rodrigues' rotation formula corresponding to rotation with respect to the three rolling angles of blade and $\bar{\boldsymbol{\beta}} = \langle \theta \ \gamma_{s1} \ \gamma_{s2} \ \gamma_{s3} \ \phi \rangle$. Combine Equations (1), (2), and (4) one can estimate the 3-dimension motion at the sensor location if suitable parameters are given correctly. The estimated response $\boldsymbol{a}_e(t,\bar{\boldsymbol{\beta}})$ was developed only with the consideration of rotation-induced vibration (blade flap-wise vibration is not included).

To estimate the parameters $\bar{\boldsymbol{\beta}}$ from the measurements, an objective function is defined which minimize the error (norm) between the estimated motion $\boldsymbol{a}_e(t,\bar{\boldsymbol{\beta}})$ and the measurement $\boldsymbol{a}_m(t)$:

$$J(\bar{\boldsymbol{\beta}}) = \left\| \boldsymbol{a}_e(t,\bar{\boldsymbol{\beta}}) - \boldsymbol{a}_m(t) \right\| \tag{5}$$

In Eq.(5) there are five parameters need to be determined: θ_t (blade rotation angel at time t), ϕ (pitch angel), γ_{S1}, γ_{S2}, *and* γ_{S3} (rolling angels of blade with respect to the sensor coordinate system). To start the optimization, an initial assumption of a data vector $\{\boldsymbol{a}_e(t,\bar{\boldsymbol{\beta}})\}$ with an initial set of $\bar{\boldsymbol{\beta}}$ is calculated together with a set of 3-dimensional sinusoidal signal (with turbine rotation frequency) is assumed. Then through minimization of the objective function $J(\bar{\boldsymbol{\beta}})$ to obtain the best estimate of modal parameters and the best estimate response time series, $\{\boldsymbol{a}_e(t,\bar{\boldsymbol{\beta}})\}$. The identified parameters $\bar{\boldsymbol{\beta}} = \langle \theta \ \gamma_{s1} \ \gamma_{s2} \ \gamma_{s3} \ \varphi \rangle$ can provide information on the installation of blade pitching and rolling angles. The signal $\{\boldsymbol{a}_e(t,\bar{\boldsymbol{\beta}})\}$ represented the reconstructed vibration signal of blade with respect to sensor coordinate system due to blade rotation only without considering the blade flexible vibration. Once the response signal was estimated, then the residual signal (i.e. $\boldsymbol{a}_e(t,\bar{\boldsymbol{\beta}}) - \boldsymbol{a}_m(t)$) can be calculated which reflected the flap-wise vibration of the blade. The residual signal can be used for estimating the flap-wise vibration frequency of turbine blade.

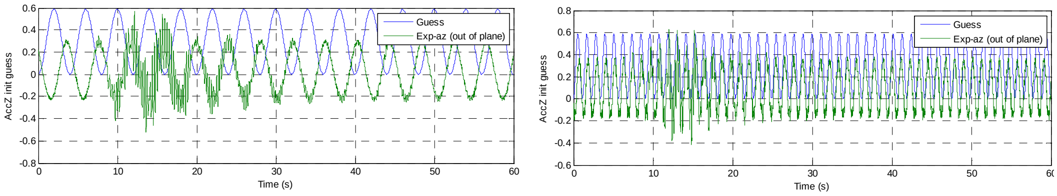
4. VIBRATION-BASED IDENTIFICATION OF TURBINE BLADE

Data collected from the experimental test of wind turbine system subject to constant rotation was used to identify the blade system pitching angle and blade rolling angles.

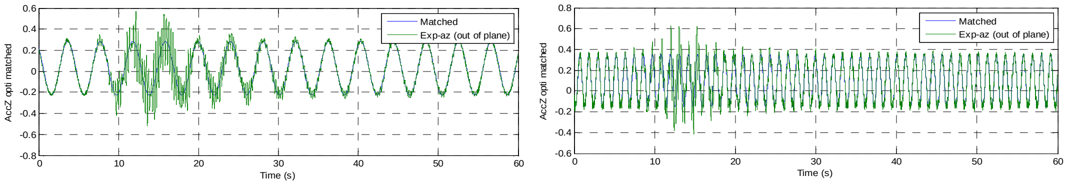
4.1 System Identification Using Rodrigues' Rotation Formula

The test data from Test-2 and Test-3 are used. First, to estimate the parameters of Eq. (5), a dataset with a designated rotation frequency (e.g. in the case of 15 rpm rotation a sinusoidal signal with dominant frequency of 0.25 Hz is assumed) and with arbitary initial phase *θ(t)* are assumed. Through the minimization procedure by adjusting the phase *θ(t)* the best estimate of the response vector $\{\boldsymbol{a}_e(t,\bar{\boldsymbol{\beta}})\}$ can be identified from which the parameter vector $\bar{\boldsymbol{\beta}} = \langle \theta\ \gamma_{s1}\ \gamma_{s2}\ \gamma_{s3}\ \varphi \rangle$ in the objective function of Eq. (5) can also be identified. Figure 4a shows the comparison on the recorded acceleration and the initial guess time history (with designated rotation frequency) of the blade flap-wise direction of motion for case of Test-2 for 15 rpm and 50 rpm tests. Figure 4b shows the comparison between the final results of the estimated time response data and the recorded acceleration after optimization. Figure 4c shows the extracted residual signal, in blade flap-wise direction of motion, which shows the difference between the recorded and the estimated acceleration responses at the tip of one rotating blade. The result of optimization can also provide the best estimate of three blade rolling angles and the pitching angle of the blade rotating plane. The identified rolling angles (γ_1, γ_2 and γ_3) for Test-2 and Test-3 is shown in Table 1a and 1b. The identified rolling angels of turbine blade and pitching angle of turbine system are very consistent with the initial setup.

(a) Recorded & initial assumption of flap-wise wave forms (Left:15 rpm; Right:50 rpm)



(b) Recorded and estimated flap-wisewave form (Left:15 rpm; Right:50 rpm)



(c) Residual signal of flap-wise wave form (Left:15 rpm; Right:50 rpm)

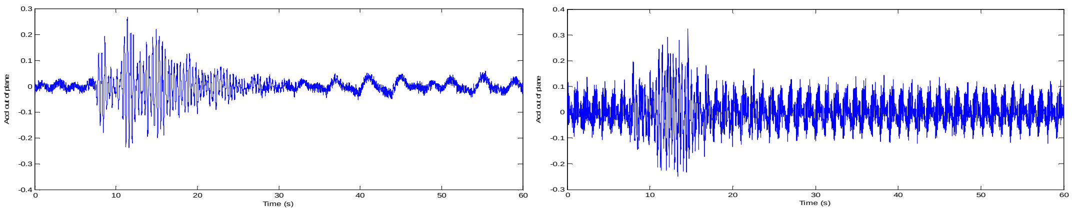


Figure 4. Plot the comparison of the out-of-plane mtion of turbine blade (test case of 15 rpm and 50 rpm) between; (a) Recorded and initial waveform, (b) Recorded and waveform after Rodrigues' rotation, (c) Residual signal and (d) Fourier amplitude spectrum of residual signal.

Table 1a: Identified wind turbine pitching and rolling angles from Test-2

Test-2	ϕ	γ_1	γ_2	γ_3
15 rpm: Case 1	97.62 (90.0)	-6.78 (-5.0)	-0.35 (0.0)	-0.01 (0.0)
15 rpm: Case 2	89.93 (90.0)	-10.16 (-10.0)	-0.16 (0.0)	0.17 (0.0)
15 rpm: Case 3	96.25 (90.0)	-20.43 (-20.0)	-0.4 (0.0)	-0.15 (0.0)

Table 1b: Identified wind turbine pitching and rolling angles from Test-3.

Test-3	ϕ	γ_1	γ_2	γ_3
15 rpm: Case 1	89.96 (90.0)	-6.58 (-5.0)	12.81 (10.0)	0.1 (0.0)
15 rpm: Case 2	90.18 (90.0)	-9.6 (-10.0)	12.05 (10.0)	0.63 (0.0)
15 rpm: Case 3	85.1 (90.0)	85.1 (90.0)	10.69 (10.0)	-0.84 (0.0)

Note: (*) indicate the blade rolling angle in its original setup.

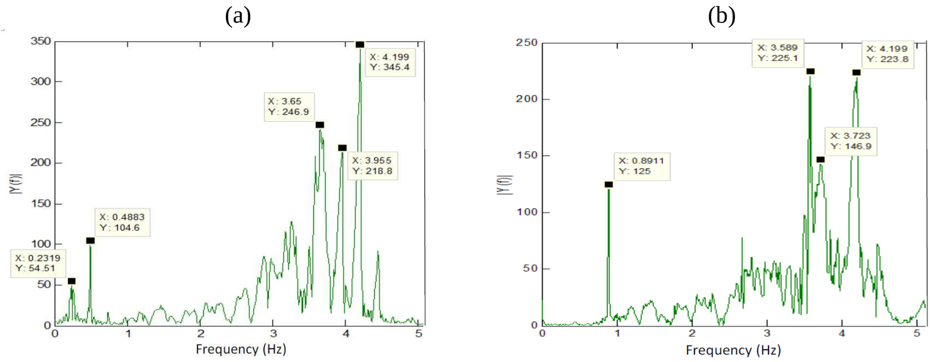


Figure 6: Fourier amplitude specrum of residual signals of blade flapwise vibration signal; (a) test case of 15 rpm, (b) test case of 50 rpm.

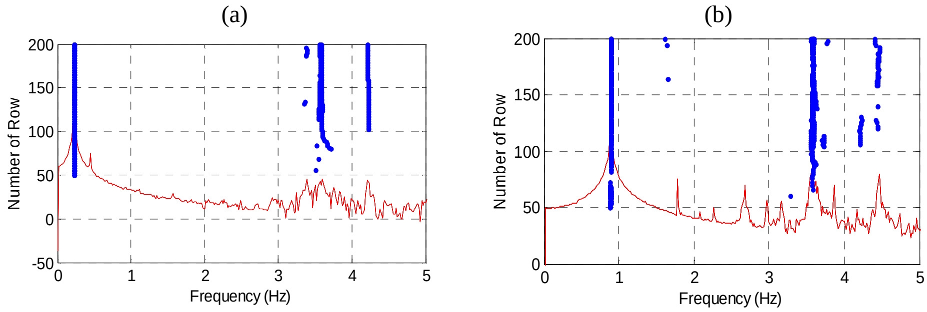


Figure 7: Plot the stability diagram using SSI-COV method on blade flap-wise response measurement; (a) test case of 15 rpm, (b) test case of 50 rpm.

It is important to point out that the residual signal in the flap-wise direction of motion shows more significant response information than other direction of motion because of the flapping frequency of the rotating blade is much lower than the edgewise blade frequency and longitudinal directions. Fourier amplitude spectrum of the residual signal of blade flap-wise direction of motion is calculated for case of 15 rpm and 50 rpm and is shown in Figure 6. Four distinct frequencies can be identified from the Fourier amplitude spectrum (i.e. 0.2319 Hz, 3.66 Hz, 3.95 Hz and 4.199 Hz for case of 15 rpm; and 0.8911 Hz, 3.589 Hz, 3.723 Hz, and 4.199 Hz for case of 50 rpm). Discussion on the Fourier amplitude spectrum of residual spectrum will show in next section.

4.2 Identification of Blade Flap-wise Vibration Frequency Using Subspace Identification

The above mentioned analysis was focus on using a single sensor (3-D accelerometer) and adopted the Rodrigues' rotation formula to identify the turbine flap-wise vibration frequencies and the geometry setup of turbine blade system. To verify the dominant frequencies identified from the residual signal after Rodrigues' rotation, a multivariate signal processing and system identification technique, covariance-driven stochastic subspace identification (SSI-COV), is used to identify the flap-wise vibration frequencies of turbine blade. First, consider the acceleration response measuement of the flapwise motion of the three rotating turbine blades (with two measurements on each blade) for the two test cases with rotation frequency of 15 rpm and 50 rpmfrom. The reference-based SSI-COV method, proposed by Bart et al. (1999), stems from the need to solve the problem through identifying a stochastic state-space model (matrices **A** and **C**) from output-only data. The first step is to establish the data Hankel Data matrix and then form the block Toeplitz matrix by a multiplication between future and transpose of past measurements. The Toeplitz matrix can be factorized into the extended observability matrix and the reversed extended stochastic controllability matrix. Singular Value Decomposition (SVD) is used to perform the factorization on Toeplitz matrix. The observation matrix can be obtained from which the system matrix **A** can be computed by exploiting the shift structure of the extended observability matrix $\mathbf{O}_i$. Finally, the modal frequencies and effective damping ratios can be computed by conducting eigenvalue decomposition of the system matrix **A**, and the corresponding eigenvectors multiplied by the output matrix **C** are used to observe mode shapes.

The SSI-COV method is applied to identify the system vibration frequencies (Huang et al. 2014). From the stability diagram of SSI-COV analysis, as shown in Figure 7, the rotation frequency (0.232 Hz and 0.891 Hz for 5 rpm and 50 rpm test cases, respectively) and the blade vibration frequency (f=3.60 Hz) can be clearly identified for both test cases. Besides, the tower vibration frequency (f=4.20 Hz) can also be identified from the

15 rpm test because the wind turbine was errected at the top of a tower, while in the case of 50 rpm the tower frequency is not so clear to be identified because of the contribution of bouble frequency. Comparison between the identified system frequencies from SSI and the dominant frequencies of the Fourier spectrum of the residual signal, the blade vibration frequency, tower vibration frequency and the rotation frequency are very consistent (. It demonstrated that by using a single sensor (3-D acceleometer) mounted on the tip of a rotating blade, through Rodrigues' rotation formulation, one can easily identify the vibration frequency of the blade, the tower/blade interaction and the rolling algles of the blade installation.

5. CONCLUSIONS

In this study, a research-scale turbine blade system is used to study the dynamic characteristics of the turbine blade under rotation through data collected directly from the vibration measurement. The research-scale turbine is rotated by a controlled motor. The wind turbine blades were instrumented with a 3-dimensional accelerometer at the tip of one blade, and data acquisition was achieved using a prototype wireless sensing system. The raw data from the test was analyzed for identifying the flap-wise natural frequencies of the blade. First, the Rodrigues' rotation formula was used to determine the blade rolling angle and pitching angle of the turbine blade system through optimization. With different test setup of blade system, the proposed method can identify the blade geometry setup. The residual signal of blade flapping response can be used to determine the vibration frequency of turbine blade. It demonstrated that through the response measurement the vibration frequency of rotating turbine blade and the rolling angles of blade installation can be identified.

It is also important to point out that during the operating frequency of a shaft, synchronous vibration will occur (rotation frequency). Synchronous vibration, ω, is the vibratory frequencies that are related to the operating frequency of the shaft. All these phenomena need to be careful examine from the measurement of rotating wind turbine. From the proposed method (using a single sensor and adopt the Rodrigues's rotation) as well as the SSI method, the signal due to rotational frequency is still exist. To remove this rotational frequency a pre-processing technique, singular spectrum analysis (SSA), as mentioned in reference Loh et al. (2013), needs to be applied.

Application of Rodrigues's rotation on the test data can extract the blade flapwise vibration signal from its recorded data during operation and also can identify the geometry setup of blade such as pitching angle and blade rolling angles. These identified parameters can provide information for turbine blade damage detection through continuous monitoring of the turbine blade system. The advantage of using this method is only a 3-D sensing data is required which can minimize the cost of instllation and reduce the computation effors. Verification of the proposed method on the identification of blade flapwise vibration frequency, a different approach by using multivariate sensing data through the stability diagram of SSI-COV method, indicated that the identified blade vibration frequency in the flapwise direction of motion is quite accurate.

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