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Nonlinear Analysis of Reinforced Concrete Bridges under Earthquakes

Hong-Nan Li[1], Yu Zhang[2], Gang Li[3] and Suyan Wang[4]

1 *Professor, Faculty of Infrastructure Engineering, Dalian University of Technology, Dalian, Liaoning Province, China. E-mail: hnli@dlut.edu.cn*
2 *Ph. D., College of Ocean and Civil Engineering, Dalian Ocean University, Dalian, Liaoning Province, China. E-mail: zhangyu1985@dlou.edu.cn*
3 *Professor, Faculty of Infrastructure Engineering, Dalian University of Technology, Dalian, Liaoning Province, China. E-mail: gli@dlut.edu.cn*
4 *Professor, Faculty of Infrastructure Engineering, Dalian University of Technology, Dalian, Liaoning Province, China. E-mail: suyanw@dlut.edu.cn*

ABSTRACT
The nonlinear analysis method is proposed based on the fundamental concept of the Force Analogy Method (FAM) for reinforced concrete (RC) bridges in this paper. The governing equations for both nonlinear dynamic and static analysis are derived based on the FAM. Since the stiffness matrices are constant throughout the whole computational process, the advantages of the FAM, such as efficient and stable, are all retained in the presented nonlinear analysis methods. The presented nonlinear dynamic analysis for a RC bridge is carried out to illustrate its application for engineering structures.

KEYWORDS: *Force Analogy Method, reinforced concrete bridges, nonlinear analysis method.*

1. INTRODUCTION

To reasonably evaluate the seismic performance of reinforced concrete (RC) bridges is an issue of urgent needs according to the framework of Performance Based Earthquake Engineering (PBEE).

The seismic performance assessment should undoubtedly be considered in the demand and capacity evaluations at various performance levels of the structure. Clearly, a nonlinear dynamic analysis can be used to predict with sufficient reliability the forces and cumulative deformation demands in each element of the structure. In addition, conventional nonlinear dynamic analysis methods incorporating numerical differentiation formulations or numerical integration formulations, are time consuming and unstable since the stiffness matrices are changing throughout the whole computing progress, especially when the nonlinear response is large and the structural strength is degrading. The nonlinear static analysis, particularly the pushover method, is a simplified nonlinear analysis method, which can be viewed as a method for predicting the structural capacities, such as the global drift or interstory drift. Compared with the nonlinear dynamic analysis, the pushover analysis needs less computing effort and is easier for implementation in the engineering practice.

The fundamental concept of the Force Analogy Method (FAM) was firstly proposed by Lin [1] for an application of stress-strain space in the continuum mechanics with the inelastic behavior defined by the plastic strain, and was applied firstly in building structures by Wong and Yang [2]. It is a nonlinear analytical method by formulating each plastic deformation of the structural member as a degree of freedom to depict the global structural nonlinear behaviors through constant stiffness matrices. Therefore, coupling with the basic concept of the FAM, the nonlinear problems can be solved with efficiency and stability [2-4].

In this paper, the basic concept of the FAM is applied for nonlinear dynamic and static analysis of RC bridges and the corresponding governing equations are derived. In the FAM, the nonlinear structural responses are generated from local plastic mechanisms, which are assigned to structure members. In this paper, typical local plastic mechanisms are introduced for nonlinear dynamic ad static analysis. Coupling above local plastic mechanisms with the FAM, the nonlinear dynamic analysis of RC bridges subjected to bidirectional earthquakes is also efficient and stable. A numerical example of nonlinear dynamic analysis for a RC bridge is carried out to illustrate the application of this study.

2. NONLINEAR DYNAMIC ANALYSIS WITH THE FAM

2.1. Governing Equations

As shown in Figure 2.1, in which the mass of the RC bridge is discretized to m_i and m_{abut} that are concentrated on top of each pier and the abutment, respectively. The earthquake load on X axis is resisted by intermediate bent piers only, while the earthquake load on Z axis is resisted by the intermediate bent piers and the abutments with stiffness K_{abut}. The nonlinear behaviors of the bridge are generated by the piers only and are simulated by local plastic mechanisms, rotation hinge (RH) and slide hinge (SH), which are functions of defining internal force in terms of plastic deformation.

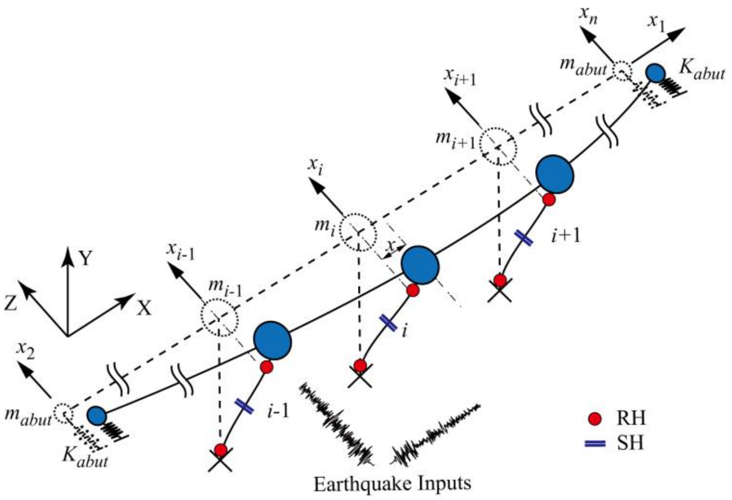


Figure 2.1 Numerical model of RC bridges.

Assuming there are totally $n=n_X+n_Z$ translational degrees of freedom for X and Z axes, q_f RHs and q_s SHs assigned to the piers together with $2(q_f+q_s)$ local plastic deformation degrees of freedom. According to the basic concept of the FAM, the total displacement vector $\mathbf{X}(t)$ can be written by:

$$\mathbf{X}(t)=\mathbf{X}'(t)+\mathbf{X}''(t) \tag{2.1}$$

where $\mathbf{X}'$(t) and $\mathbf{X}''$(t) represent the elastic displacement and plastic displacement vectors, respectively. Then, the restoring force vector $\mathbf{F}_s(t)$ of the bridge can be expressed in matrix form as [2]:

$$\mathbf{F}_s(t)=\mathbf{K}\mathbf{X}(t)-\mathbf{K}'\mathbf{\Lambda}''(t) \tag{2.2}$$

where $\mathbf{\Lambda}''(t)$ is the local plastic deformation vector. The internal force vector $\mathbf{f}(t)$ can be obtained as [2]:

$$\mathbf{f}(t)=\mathbf{K}'^{\mathrm{T}}\mathbf{X}(t)-\mathbf{K}''\mathbf{\Lambda}''(t) \tag{2.3}$$

Eqs.(2.2) and (2.3) are the governing equations for the RC bridge shown in Figure 2.1. For a given input total displacement $\mathbf{X}(t)$, the restoring force vector $\mathbf{F}_s(t)$ (n unknowns), local plastic deformation vector $\mathbf{\Lambda}''(t)$ ($2(q_f+q_s)$ unknowns), and the internal force vector $\mathbf{f}(t)$ ($2(q_f+q_s)$ unknowns) make up the $n+4(q_f+q_s)$ unknowns. However, the aforementioned governing equations by themselves only provides $n+2(q_f+q_s)$ equations, and $2(q_f+q_s)$ relationship of biaxial local plastic mechanisms of RHs and SHs expressed in Eq.(2.4) must be incorporated to solve the unknowns uniquely.

$$\begin{cases}\mathbf{M}(t)=f\left[\mathbf{\Theta}''(t)\right]\\ \mathbf{V}(t)=g\left[\mathbf{\Delta}''(t)\right]\end{cases} \tag{2.4}$$

2.2. Equations of Motion

The equations of motion for the RC bridge subjected to ground acceleration $\ddot{a}_g(t)$ can be written by:

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{F}_s(t)=-\mathbf{M}\boldsymbol{\iota}\ddot{a}(t)_g \tag{2.5}$$

where **M** is the mass matrix; C denotes the structural damping matrix constructed based on the Rayleigh damping; $\boldsymbol{\iota}$ represents the influence coefficient vector; $\mathbf{X}(t)$, $\dot{\mathbf{X}}(t)$, and $\ddot{\mathbf{X}}(t)$ are the displacement, velocity and acceleration vectors, respectively. Substituting Eq.(2.2) into Eq.(2.5) yields:

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=-\mathbf{M}\boldsymbol{\iota}\ddot{x}_g+\mathbf{K}'\boldsymbol{\Lambda}''(t) \tag{2.6}$$

As the stiffness matrix **K** on the left side of Eq.(2.6) remains constant throughout computing process, it can be solved using the state-space formulation which is a significant advantage since computational efficiency is greatly increased comparing to a formulation that requires recalculation of the stiffness matrix at each time step. The term $\mathbf{K}'\boldsymbol{\Lambda}''(t)$ added on the right side of Eq.(2.6) as the external force vector can be computed at any instant with the complementary equations shown in Eq.(2.4).

3. NONLINEAR STATIC ANALYSIS WITH THE FAM

As shown in Figure 3.1, the lateral load **F**, which may be invariant or variant, is used to represent the distribution of inertia forces due to a design earthquake:

$$\mathbf{F}=\left[\mathbf{F}^1\cdots\mathbf{F}^k\cdots\mathbf{F}^l\right] \tag{3.1}$$

where superscript l implies that the whole loading process is divided into l steps, and superscript k denotes the load pattern at step k:

$$\mathbf{F}^k=\left[F_1^k \quad \cdots \quad F_i^k \quad \cdots \quad F_n^k\right]^T \tag{3.2}$$

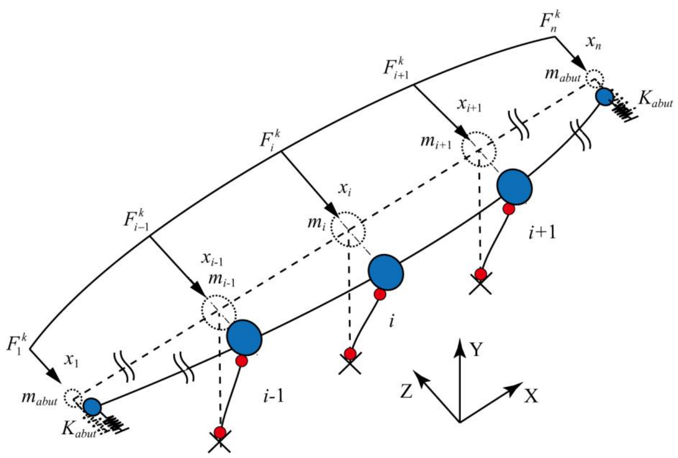


Figure 3.1 Numerical model of RC bridges.

Define the incremental plastic rotations $\boldsymbol{\Lambda}''(t)$ at step k:

$$\Delta\boldsymbol{\Lambda}''=\boldsymbol{\Lambda}''^k-\boldsymbol{\Lambda}''^{k-1} \tag{3.3}$$

Eq.(3.3) is substituted into Eq.(2.2) and solved for the displacement at step k to obtain:

$$\mathbf{X}^k=\mathbf{K}^{-1}\left(\mathbf{F}^k+\mathbf{K}'\boldsymbol{\Lambda}''^{k-1}+\mathbf{K}'\Delta\boldsymbol{\Lambda}''\right) \tag{3.4}$$

Substituting Eqs.(3.3) and (3.4) again into Eq.(2.3) and solving for the internal force at step k yields:

$$\mathbf{f}^k=\mathbf{K}'^T\mathbf{K}^{-1}\left(\mathbf{F}^k+\mathbf{K}'\boldsymbol{\Lambda}''^{k-1}+\mathbf{K}'\Delta\boldsymbol{\Lambda}''\right)-\mathbf{K}''\boldsymbol{\Lambda}''^{k-1}-\mathbf{K}''\Delta\boldsymbol{\Lambda}'' \tag{3.5}$$

Eqs.(3.4) and (3.5) are the governing equations for the pushover analysis with the FAM. In order to obtain the unknowns when given the lateral load, complementary equations shown in Eq. (2.4) must be incorporated.

4. NUMERICAL EXAMPLE

The geometry of the prototype bridge and the design of piers are illustrated in Figure 4.1, the bridge is three-span with a continuous concrete box-girder supported by two-column bent piers and monolithic abutments. The numerical model of the bridge is expressed in Figure 4.2. Eight biaxial RHs and four biaxial SHs assigned to the piers. The internal force versus plastic deformation relationships of RHs and SHs are established based on the modified compression field theory (MCFT) [5], the empirical equations [6] and the aforementioned hysteretic rules.

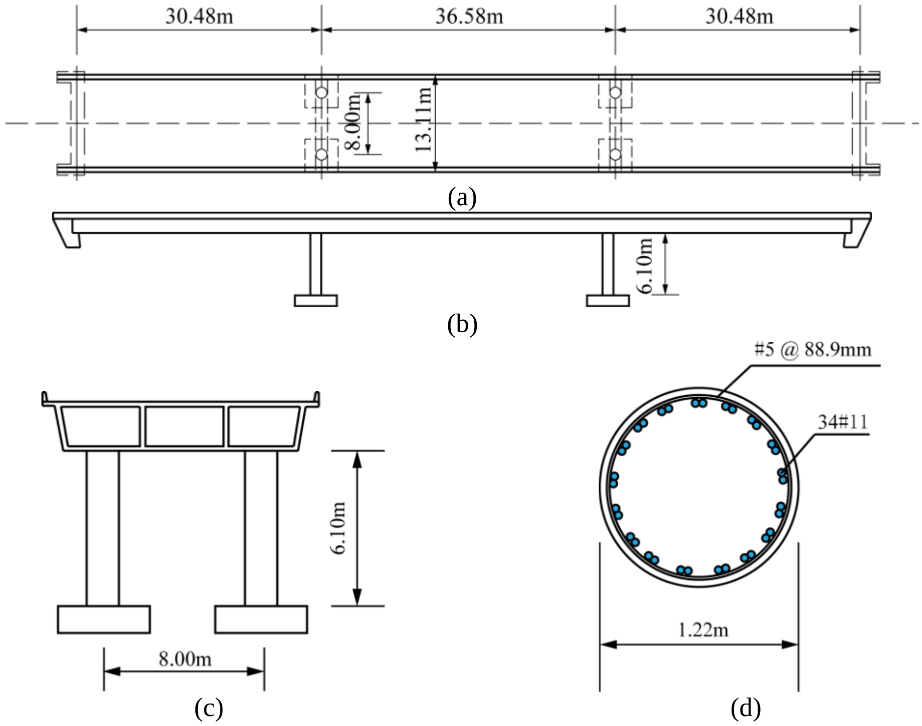


Figure 4.1 Schematic of the RC bridge and pier reinforcement details: (a) plan view; (b) elevation view; (c) transverse cross section of the bridge; (d) pier cross section.

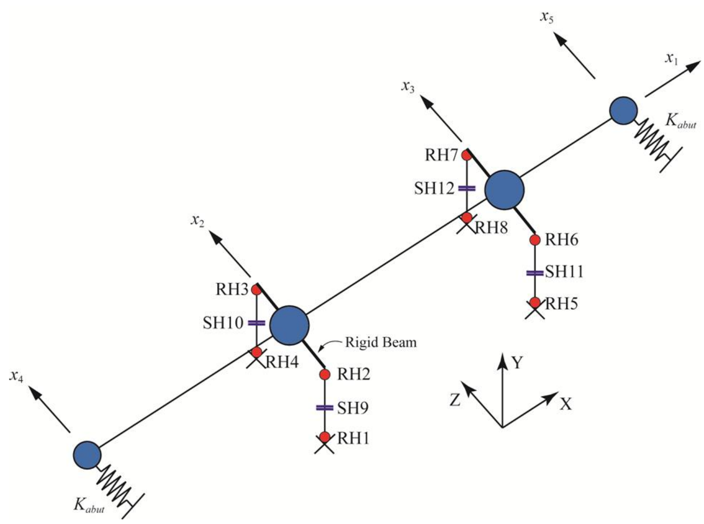
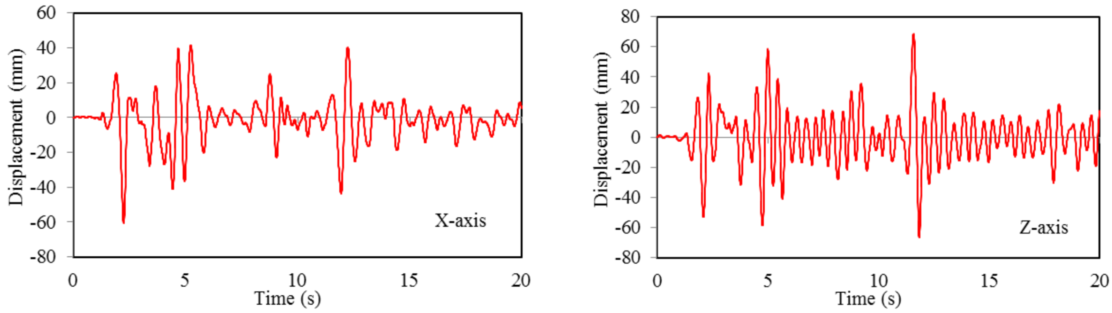


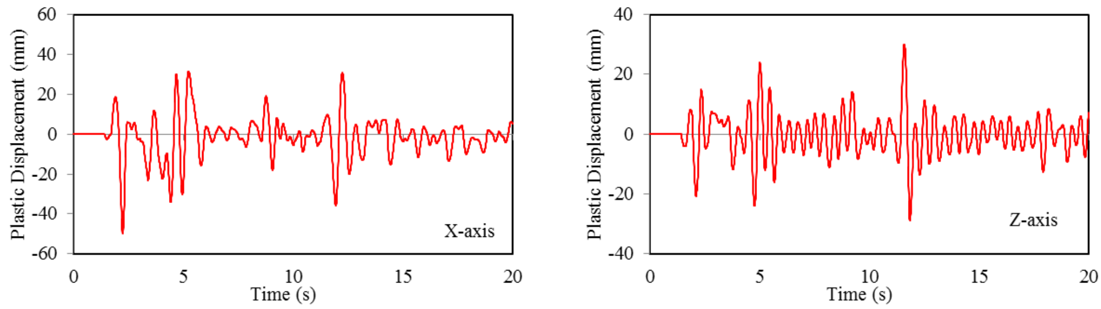
Figure 4.2 Numerical model of the RC bridge.

The El Centro earthquake ground motion is selected as excitation and the peak acceleration is scaled to 0.6g. The global responses of the X and Z directions of the bridge are shown in Figure 4.3, while Figure 4.4 shows the corresponding plastic responses. As illustration, the maximum displacement is 60mm, the peak plastic displacement on the X axis is larger than that on the Z axis. The hysteretic behaviors of selected RHs and SHs are illustrated in Figure 4.5, the piers are in the plastic domain and are dominated in flexural behavior since the RHs behave on their yielding platforms.



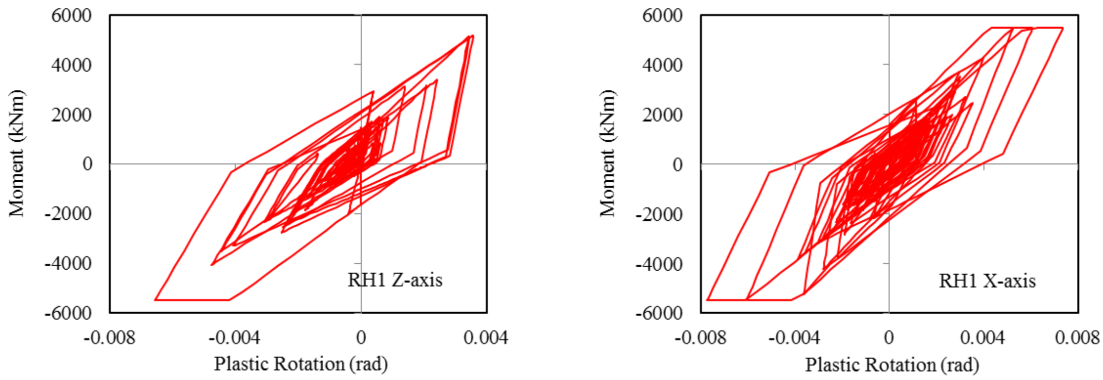
(a) (b)

Figure 4.3 Global displacement responses of the RC bridge: (a) X axis (b) Z axis.

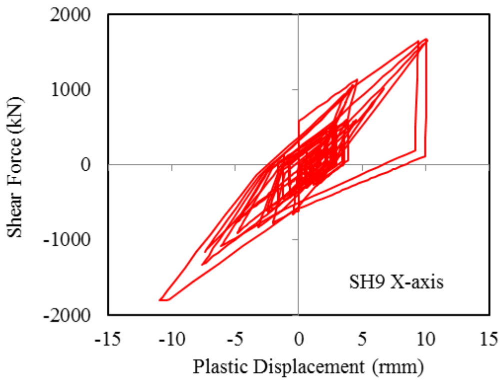


(a) (b)

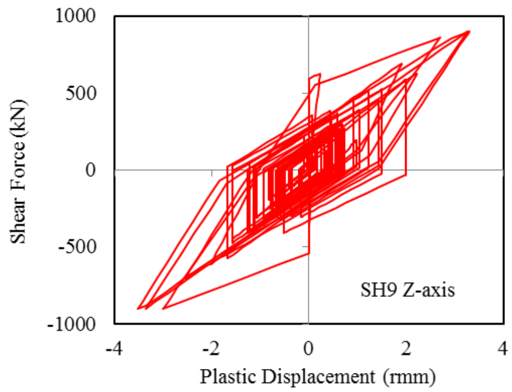
Figure 4.4 Plastic displacement responses of the RC bridge: (a) X axis; (b) Z axis.



(a) (b)



(c)



(d)

Figure 4.5 Hysteretic behaviors of selected hinges: (a) hysteretic behavior of RH1 around Z axis; (b) hysteretic behavior of RH1 around X axis; (c) hysteretic behavior of SH9 in X axis; (d) hysteretic behavior of SH9 in Z axis.

5. SUMMARY

This paper focuses on the implementation of the FAM for the nonlinear analysis of RC bridges. The governing equations of the nonlinear dynamic and static analysis with the FAM are derived. Since stiffness matrices are unchanging throughout the whole computing process, the computing process is efficient and stable. The proposed approach is implemented in a RC bridge to demonstrate its application on simulating inelastic behaviors of RC bridges.

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