



Effective Viscous Damping Ratio in Seismic Response of Reinforced Concrete Structures

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ABSTRACT

It is common practice in structural design to assume that reinforced concrete (RC) buildings have constant viscous damping ratio, typically 2% or 5%, regardless the level of drift. Using a calibrated empirical technique that allows extracting restoring force in structural elements from actual dynamic response records, we carried out a detailed study of nineteen laboratory test specimens dynamically tested on an earthquake simulator. The results show that in a low to mid-rise RC building responding to strong ground shaking in the inelastic range and at its primary characteristic mode, which is analogous to the fundamental mode in linear elastic systems, the equivalent viscous damping coefficient stays nearly constant. The corollary to this observation is that the corresponding equivalent viscous damping ratio varies linearly with the equivalent period of the primary characteristic mode of lateral response.

KEYWORDS: *effective damping ratio, damping coefficient, viscous damping, reinforced concrete, buildings.*

1. INTRODUCTION

The response of buildings to earthquake ground shaking is a complex process. During strong shaking, the structural system of the building may be forced to behave inelastically. In addition to the hysteretic energy dissipation due to inelastic behavior, the structure will dissipate energy through means currently not well understood. In the simplest of approaches, linear viscous damped models with constant modal damping ratios are used to represent structures. More elaborate models may use Rayleigh damping (Caughey, 1960), i.e., combination of mass and stiffness proportional damping, but not necessarily with justification for the use of Rayleigh damping (Wilson, 2010). Compared to assuming a constant value for the viscous damping ratios, say, 2% or 5% of the critical damping, using equivalent damping ratios estimated from the dynamic response of actual building structural systems would be more realistic. Such an empirical approach may also help to identify the structural system and response parameters the damping ratio correlates well with. Here, an empirical method based on the procedure proposed by Dowgala (2013) is proposed. The method has so far been applied to nineteen laboratory test specimens dynamically tested on the University of Illinois Earthquake Simulator (Fig. 1), an experimental setup designed to subject small-scale structures to unidirectional base motions.

2. BACKGROUND

Based on tests by Cecen (1979), Schultz (1980) and others, Algan (1982) observed that the peak inelastic displacement of a reinforced concrete (RC) structure subjected to a strong ground motion (GM) correlates with the elastic fundamental period. Different methods have been proposed to estimate the maximum inelastic deformation using equivalent periods and equivalent damping ratios (for example, Lepage 1996) or displacement modification factors (Newmark 1982; Miranda 2000). The key parameters (equivalent period, damping ratio, displacement modification factors) are expressed as functions of displacement ductility ratio (Rosenblueth 1964; Gulkan 1974; Iwan 1980;

Kowalsky 1994; Priestley 1996). Assuming velocity proportional linear viscous damping, the equation of motion for a structure subjected to ground acceleration can be written as in Eq. (2.1).

$$M[\ddot{x}(t) + \ddot{x}_g(t)] + C\dot{x}(t) + Fs(t) = 0 \quad (2.1)$$

where,

M	:	Mass of the structure
C	:	Equivalent viscous damping coefficient of the structure
$Fs(t)$:	Restoring force provided by the structural system; if the structure remains linear-elastic, $Fs(t) = Kx(t)$ with K being the stiffness of the structure
$x(t)$:	Displacement relative to ground
$\dot{x}(t)$:	Velocity relative to ground
$\ddot{x}(t)$:	Acceleration relative to ground
$\ddot{x}_g(t)$:	Ground acceleration

Taking a mass-normalized formulation approach, the acceleration measured on the structure becomes the mass-normalized inertial force. The mass-normalized damping force can then be written as:

$$C_m \dot{x}(t) = -[\ddot{x}(t) + \ddot{x}_g(t)] - F_{sm}(t) \quad (2.2)$$

where,

C_m	:	Mass-normalized equivalent viscous damping coefficient
$C_m \dot{x}(t)$:	Mass-normalized equivalent viscous damping force
$\ddot{x}(t) + \ddot{x}_g(t)$:	Mass-normalized inertial force
$F_{sm}(t)$:	Mass-normalized restoring force

The procedure proposed by Dowgala (Dowgala, 2013; Dowgala and Irfanoglu, 2013, 2014) is used to estimate the mass-normalized damping coefficient. The procedure is based on the principle that a peak in restoring force should occur at the time of a zero-crossing of relative velocity. The mass-normalized damping force that enforces this principle yields the mass-normalized damping coefficient. Averaging the results for each peak in restoring force, after fitting a probability distribution to the extracted values, one can obtain a single damping coefficient for the duration of the GM. The mass-normalized velocity proportional equivalent viscous damping ratio can be written in terms of the damping coefficient and the natural period of a sub-critically structure as

$$\beta = \frac{C_m}{4\pi} T_n \quad (2.3)$$

where,

β	:	equivalent viscous damping ratio of the structure
C_m	:	mass-normalized equivalent viscous damping coefficient
T_n	:	natural period of the structure (i.e., period of the dominant mode of response)

The proposed method of estimating the damping ratios of buildings (Dowgala, 2013) has so far been applied to two 10-story 3-bay MDOF RC small-scale frames with yielding girders (Cecen, 1979), two 9-story 3-bay MDOF soft-story RC frames with yielding columns (Schultz, 1980) and fifteen SDOF RC test structures with different stiffness and mass (Bonacci, 1989). The details of the observations are given in the next section.

3. OBSERVATIONS FROM LABORATORY TEST DATA

The proposed empirical method for estimating the mass-normalized equivalent viscous damping coefficients and the change in the damping ratio of softening structural systems has been applied for the following three structural systems:

- Fifteen SDOF RC test structures with different stiffness and mass (Bonacci, 1989)
- Two 10-story 3-bay MDOF RC frames with yielding girders (Cecen, 1979)
- Two 9-story 3-bay MDOF soft-story RC frames with yielding columns (Schultz, 1980)

3.1 MDOF RC frames with yielding girders

Two identical 10-story 3-bay test structures were tested by Cecen (1979). The girders in the models were flexurally weaker than the columns. He observed nearly identical peak inelastic displacements in both series when the GMs were identical. The GMs were sorted in increasing intensity, with different numbers of excitations for the two structures. Identical maximum intensity GMs were applied last on both structures. Estimates from free-vibration response before and after each sequential test run showed that the equivalent viscous damping ratios increased from 2 percent to 10 percent between the first and the last test in both series. Fig. 3.1 illustrates the setup of the test structures.

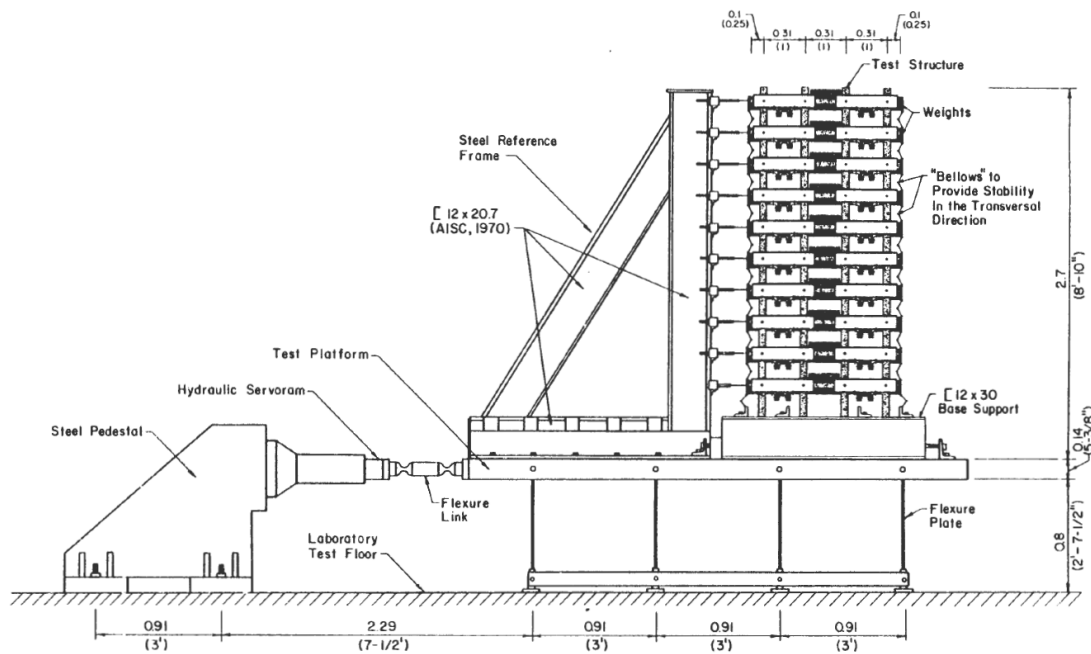


Figure 3.1 General view of the Cecen test setup (Cecen, 1979)

Sequential earthquake ground motions were applied to each test structure. The test specimen H1 and the test specimen H2 were subjected to three and seven sequential GMs, respectively, with increasing peak ground acceleration (PGA). El Centro N-S 1940 GM (Fig. 3.2) was scaled and used as the input excitation. H1 and H2 were designed to resist PGA of 0.35 g. The data for Cecen's test runs are available at the NEEShub Project Warehouse under Project No. 1072 (Sozen, 2011).

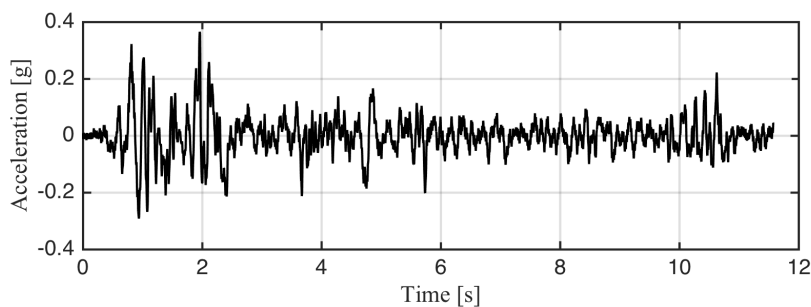


Figure 3.2 Scaled and compressed 1940 El Centro earthquake motion

Figs. 3.3 shows the base PGAs and PGVs for the sequential test runs H1 and H2.

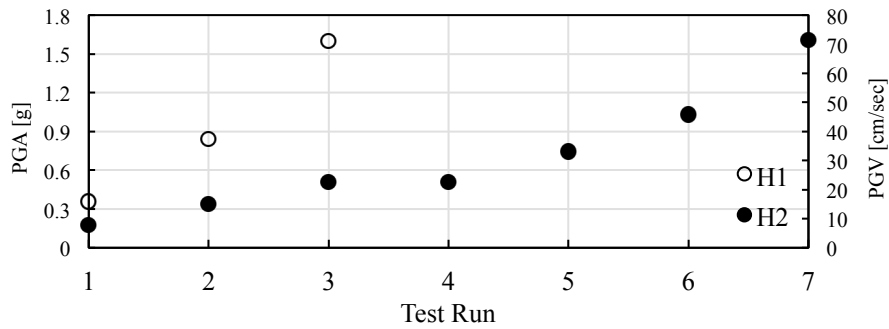


Figure 3.3 PGAs and PGVs of Cecen test runs

Fig. 3.4 shows the first story absolute acceleration response for the first run of H1.

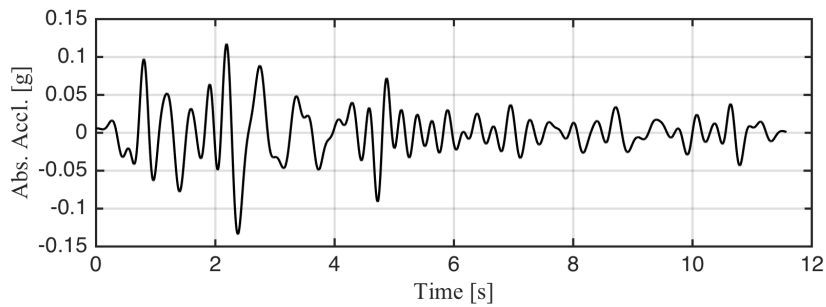


Figure 3.4 First story acceleration response from run#1 of Cecen test structure H1

Absolute velocity and absolute displacement responses can be estimated from the acceleration response through direct integration. Velocity and displacement relative to the ground for the first run of H1 are shown in Figs. 3.5 and 3.6, respectively.

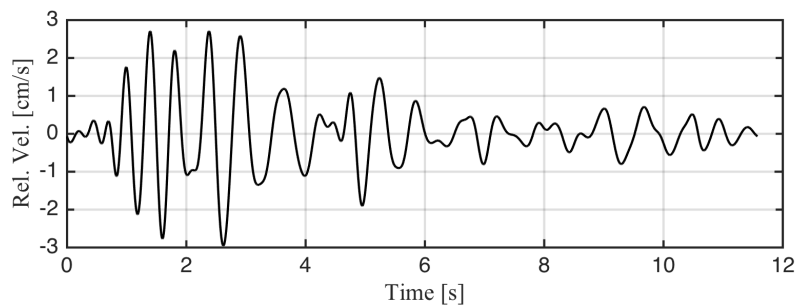


Figure 3.5 First story relative velocity response from run#1 of Cecen test structure H1

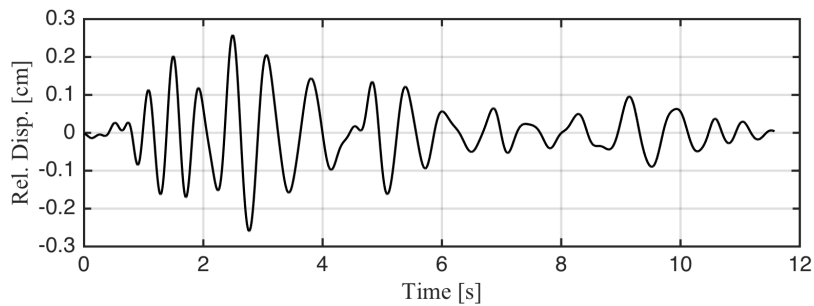


Figure 3.6 First story displacement response from run#1 of Cecen test structure H1

Fig. 3.7 shows the variation of the mass-normalized inertial force in the first run of H1.

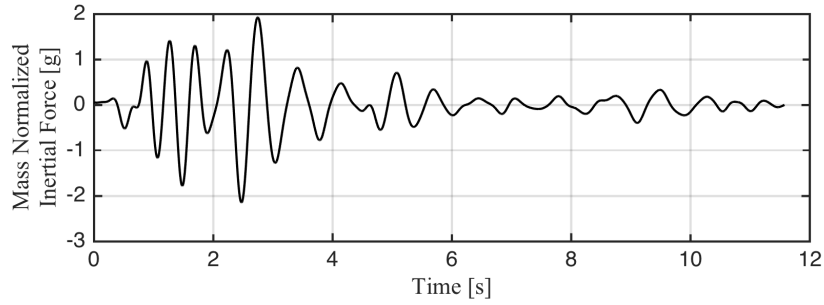


Figure 3.7 Mass-normalized inertial force from run#1 of Cecen test structure H1

Mass-normalized equivalent damping coefficient extracted from the H1 first run is shown in Fig. 3.8.

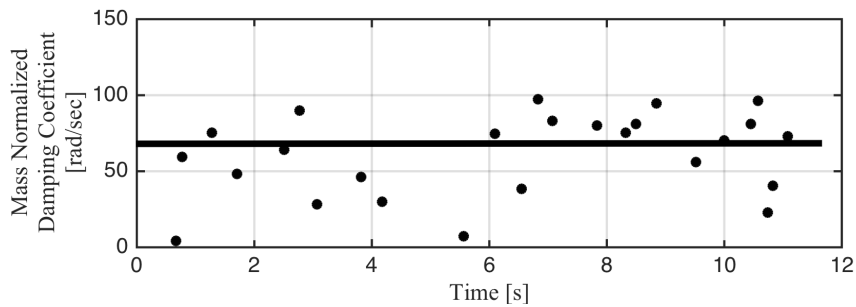


Figure 3.8 Mass-normalized damping coefficient from run#1 of Cecen test structure H1

Analysis of all of the sequential test runs of H1 and H2 shows that the mass-normalized damping coefficient remained nearly constant during the sequential test runs (see Fig. 3.9).

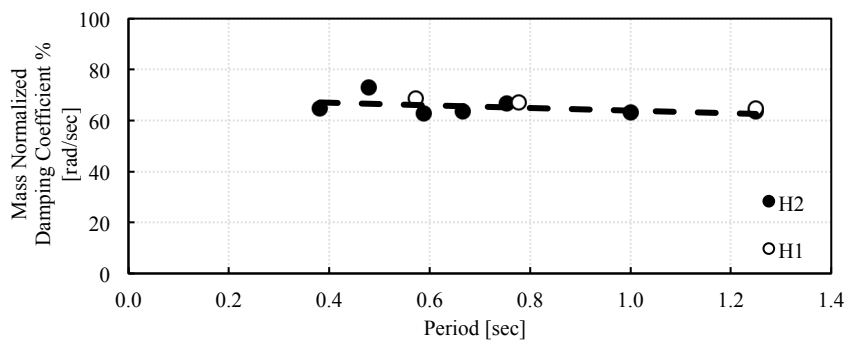


Figure 3.9 Mass-normalized damping coefficient for Cecen test structures

Based on Eq. 2.3, there is a linear relationship between the damping ratio and the period of the H1 and H2 test specimens. Four different periods are estimated: 1) low amplitude free vibration period obtained from the free-vibration response before and after each test run; 2) moderate amplitude vibration period obtained from the tail end of displacement response; 3) high amplitude vibration period obtained from the maximum displacement response region; and, 4) apparent period obtained from the Fourier Spectrum of the response. Fig. 3.10 illustrates that the apparent periods and the high amplitude vibration periods are very similar.

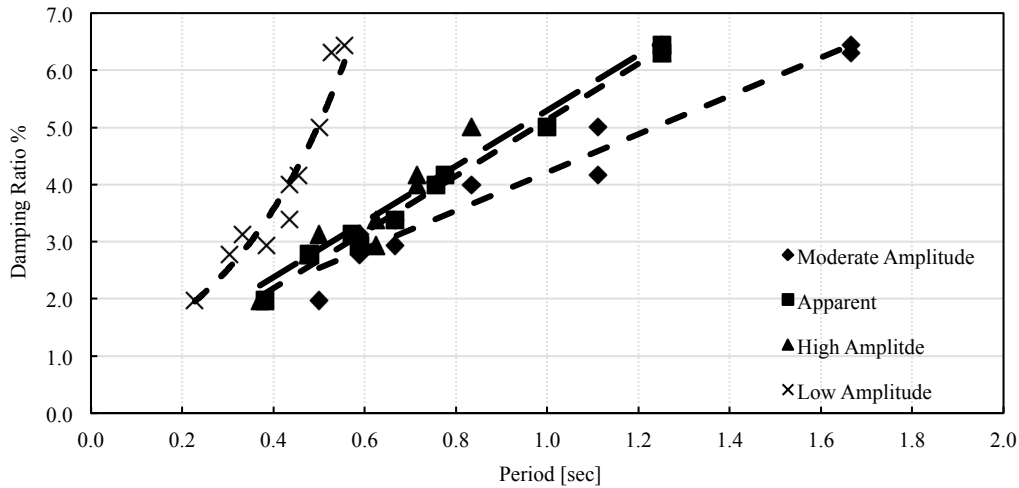


Figure 3.10 Damping ratio vs. period for Cecen test structures

Similar analyses have been carried out on the data from tests by Schultz (1980) and Bonacci (1989). Schultz (1980) tested two 9-story 3-bay RC test specimens with yielding columns. Bonacci (1989) tested fifteen SDOF RC test structures. Like Cecen (1979) and Schultz (1980), Bonacci (1989) used a series of sequential earthquake ground motions with increasing PGA. The data for Schultz and Bonacci test runs are available at the NEESHUB Project Warehouse under Project No. 1063 (Sozen and Schultz, 2011) and Project No. 1065 (Bonacci and Sozen, 2011), respectively. The results are similar to what has been found in study of the data from Cecen (1979) tests: equivalent viscous damping coefficient remained nearly constant for a given test structure throughout the GM series in Schultz (Fig. 3.11) and Bonacci (Fig. 3.12) tests.

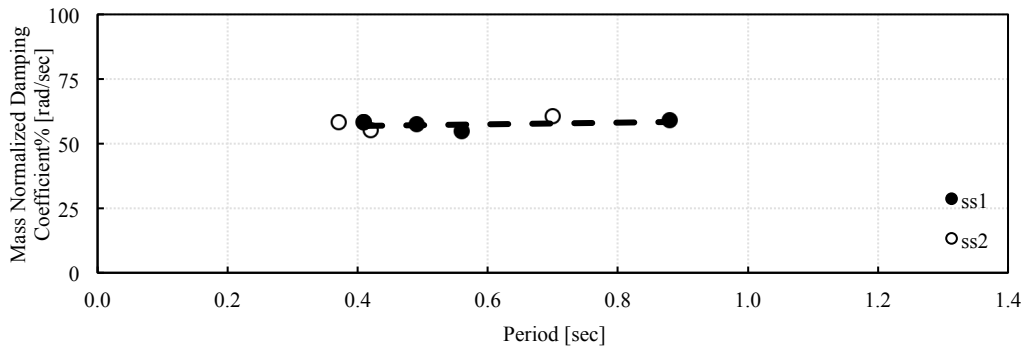


Figure 3.11 Mass-normalized damping coefficient for Schultz (1980) test structures

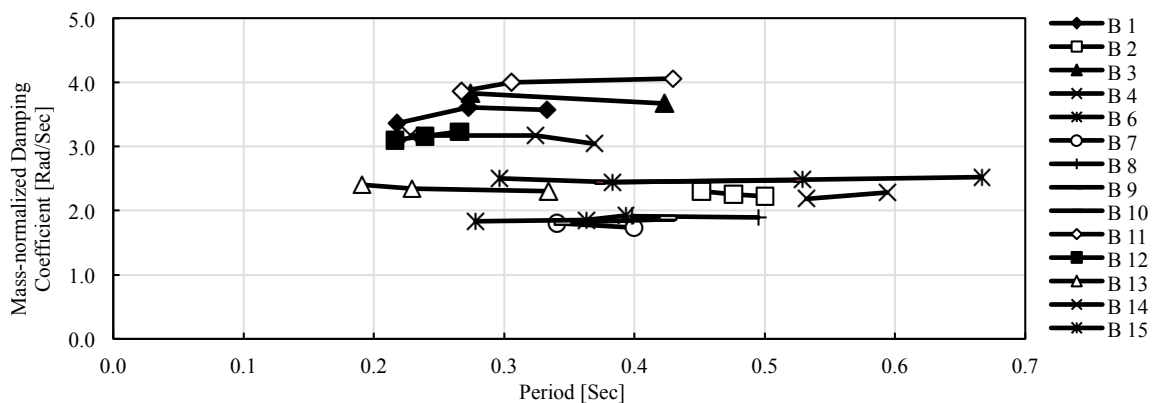


Figure 3.12 Mass-normalized damping coefficient for Bonacci (1989) test structures

4. CONCLUSIONS

Our study of laboratory test data indicate that the equivalent viscous damping coefficient for a low to mid-rise RC building structural system responding at its dominant mode (equivalent of fundamental mode in linear elastic systems) may be assumed to be nearly constant during response to ground shaking. The corollary is that the corresponding equivalent viscous damping ratio varies linearly with the equivalent period of the dominant mode. If the above observation is shown to hold true for a wide range of RC building structural systems, more realistic dominant mode (fundamental mode) damping ratios can be identified to estimate the internal forces in RC structural elements. This damping ratio can either be smaller or larger than 2% or 5% commonly assumed in design of RC buildings. The common approach of assuming a constant viscous damping ratio regardless of the elongation in period of the structure is not an accurate approach. Proper modelling of damping ratio would allow more accurate estimation of the seismic response of a building. For preliminary design and rapid evaluation purposes, the inelastic response spectra for SDOF systems with displacement-dependent damping ratio can be generated once the relationship between damping ratio and drift ratio is identified. Using this tool, an average constant damping ratio can be recommended for use in estimating the peak response during different levels of base acceleration. Likewise, a constant average stiffness (corresponding to equivalent period) may be recommended. That way one could check not only whether use of 2% or 5% or some other constant damping ratio would be more reasonable but also whether stiffness reduction factors for RC elements recommended in seismic design, for example per ACI-318 (2011), are appropriate.

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