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Analysis of Time-Frequency Energy for Environmental Vibration Induced by Metro

Guangzhen Li[1], Xiaosong Ren[2], Bin Zhang[3] ,Gang Zong[4]

1 *Graduate student, Research Institute of Structural Engineering and Disaster Reduction, Tongji University, Shanghai 200092, China.*
E-mail: 1332857@tongji.edu.cn
2 *Professor, Research Institute of Structural Engineering and Disaster Reduction, Tongji University, Shanghai 200092, China.*
E-mail: rxs@tongji.edu.cn
3 *Graduate student, Research Institute of Structural Engineering and Disaster Reduction, Tongji University, Shanghai 200092, China.*
E-mail: 972373707@qq.com
4 *Lecturer, Research Institute of Structural Engineering and Disaster Reduction, State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, Shanghai 200092, China.*
E-mail: zong@tongji.edu.cn

ABSTRACT
Time and frequency domain analysis processing are two conventional methods for vibration analysis. Despite the advantages in the time or frequency domain, the cluttering of non-stationery signal cannot be effectively distinguished by time domain analysis, and the characteristics in certain frequency segments may be neglected in the frequency domain analysis.
By the adjustment of the length of time window, time-frequency energy method can effectively show the characteristics of energy in time domain and do the component analysis in the specific frequency range. Hence the local characteristics of signals in the useful range can be got. Although this method was put forward early, it is not widely used for the restriction of the hardware and software for numerical analysis.
The ground vibration induced by metro is neither a kind of stationary vibration nor a transient impact vibration. The energy is concentrated in the frequency range above 50Hz. This paper presents the signal processing using time-frequency energy method. The signals collected in 8 points along the direction vertical to the metro line are chosen as examples. By comparative study of signal duration, frequency band recognition and energy characterization, the vibration signal induced by non-metro aspects can effectively be separated and the propagation law for metro-induced vibration can be found.

KEYWORDS: *Time-frequency energy; Environmental vibration induced by metro, Signal analysis processing, Local characteristic*

Signal analysis processing is usually done in time domain and frequency domain. From the analysis in time domain, the characteristics, such as time duration, the peak value, the RMS (root-mean-square) and the attenuation curve can be easily got, and the cross-correlation function can reveal the linear dependence between different signals. On the other hand, the signal analysis in frequency domain can proceed by using the Fourier Transform. The frequency components can be got and hence modal identification can be done by the combination of auto-power spectrum and cross-power spectrum.

For non-stationary signals, time history analysis and frequency spectrum analysis seem not effective and convincible. For example, the RMS as an energy representation is a mean value along time axis which may not reflect the reality for the reason of determination of duration for weak signal, while Fourier Transform is also not accurate enough for analyzing local characteristic of the non-stationary signal.

In the 40s of last century, the concept of time-frequency energy was put forward for the treatment of the non-stationary signals. Compared with the traditional processing method, time-frequency energy shows advantage in analyzing local characteristic of frequency domain because the length of time-window and frequency-window can be adjusted to get a balance of accuracy between time domain and frequency domain. The time-frequency energy is the direct reflection of signal energy and makes the energy characterization more close to the reality, which overcomes the scattering of signal energy by using traditional method. The references as [1-5] show that so far the signal processing is still separated in time domain or frequency domain. The main reason is the restriction of the hardware and software for numerical application of time-frequency energy.

As a typical environmental vibration signal with evident local characteristics, the metro-induced ground vibration is neither a kind of stationary vibration nor a transient impact vibration. Due to the relative gradual variety of envelope in the whole process of signal, the time domain analysis is still suitable for signal evaluation. While from the view on signal detail, it is more like a transient impact vibration signal and the parameters in time domain, such as the peak value, RMS, vibration acceleration level by weighted RMS cannot accurately describe the characteristics of signal. In this paper, the ground vibration induced by metro is analyzed by the means of time-frequency energy and through the comparison to the traditional method, the advantages of time-frequency energy is presented.

1. INTRODUCTION OF TIME-FREQUENCY ENERGY

For the signal $x(t)$, the frequency representation can be obtained by the Fourier transform,

$$X(f)=\int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \tag{1.1}$$

where f is the signal frequency .The energy E_x can be calculated through the integral as follows.

$$E_x=\int_{-\infty}^{\infty} |x(t)|^2 dt=\int_{-\infty}^{\infty} |X(f)|^2 df \tag{1.2}$$

where $|x(t)|^2$ and $|X(f)|^2$ are used as the representation of energy in time domain and frequency domain. Similarly, quadratic time-frequency distribution is used for the instantaneous signal energy in this paper,

Among the classic quadratic time-frequency distribution functions, the Cohen bilinear time-frequency distribution function and Affine bilinear time-frequency distribution function are most commonly used for expression of time-frequency transform [6]. As the real part of Rihaczek time-frequency distribution, which is one kind of Cohen bilinear time-frequency distribution, Margenau-Hill-Spectrogram distribution is different from other time-frequency distribution for dealing with signal energy. It is the direct description of the discrete signal energy and has advantages to treat the cross-term, minus energy and edge effect optimization [7]. In this paper, the time-frequency energy analysis is based on Margenau-Hill-Spectrogram distribution. The time-frequency distribution is defined as follows.

$$MHS_x(t,f)=R\left\{\frac{1}{K_{gh}}F_x(t,f;h)F_x^*(t,f;g)\right\} \tag{1.3}$$

where $MHS_x(t,f)$ is the time-frequency distribution coefficient; $R\{\ \}$ represents the real part of an imaginary number; $F_x(t,f;h)$ is Rihaczek distribution in time domain with window g ; $F_x^*(t,f;g)$ is conjugate of Rihaczek distribution in frequency domain with window h ; $K_{gh}=\int_{-\infty}^{\infty} h(u)g^*(u)du$ is the integration for time window h and conjugate of frequency window g , which is used to adjust the additive energy caused by the window.

The instantaneous frequency estimation, group delay estimation, effective duration (abbreviated as TE) and effective frequency band (abbreviated as BF) and so on, are used as evaluation index of time-frequency characteristics, as stated in Ref. [8-10]. The instantaneous frequency estimation is to describe the instantaneous frequency feature on the local time point, while the instantaneous frequency on the whole duration reflects the time-dependent law of frequency. The group delay demonstrates the phase rate of changes due to the frequency change rate, which is intuitively time delay of signal waveform envelope. The effective duration and the effective frequency band are two parameters to show the duration range and frequency band that contribute most to the signal energy. The effective duration is used to calculate the concentration degree of the signal and the effective frequency band is used to calculate the bandwidth of the signal frequency [10].

$$TE=2\sqrt{\frac{\pi}{E_x}\int_{-\infty}^{+\infty}(t-t_m)^2|x(t)|^2 dt} \tag{1.4}$$

$$BF = 2\sqrt{\frac{\pi}{E_x}\int_{-\infty}^{+\infty}\left(f - f_m\right)^2 \left|X(f)\right|^2 df} \tag{1.5}$$

where t_m and f_m are the time center and frequency center, respectively.

$$t_m = \frac{1}{E_x}\int_{-\infty}^{+\infty} t\left|x(t)\right|^2 dt \tag{1.6}$$

$$f_m = \frac{1}{E_x}\int_{-\infty}^{+\infty} f\left|X(f)\right|^2 dv \tag{1.7}$$

The signal can be characterized in the time-frequency plane by its center position (t_m, f_m) and a domain of time-bandwidth product as $TE \times BF$.

The energy value can be calculated in any time and frequency range for concern. Here, the energy value is calculated just in the effective duration and effective frequency band. This kind of calculation eliminates the problem of scattering of signal energy over duration and the influence of other frequency.

The local energy E can be calculated through the integral of time-frequency distribution coefficient as follows.

$$E = \int_{t_m-\Delta t}^{t_m+\Delta t}\int_{f_m-\Delta f}^{f_m+\Delta f} P(t, f)dtdf \tag{1.8}$$

where $P(t, f)$ is the time-frequency distribution coefficient, the same as Eq. (1.3) in this paper; $\Delta t = \frac{TE}{2}$ $\Delta f = \frac{BF}{2}$.

For the discrete signal, the formula can be expressed as follows.

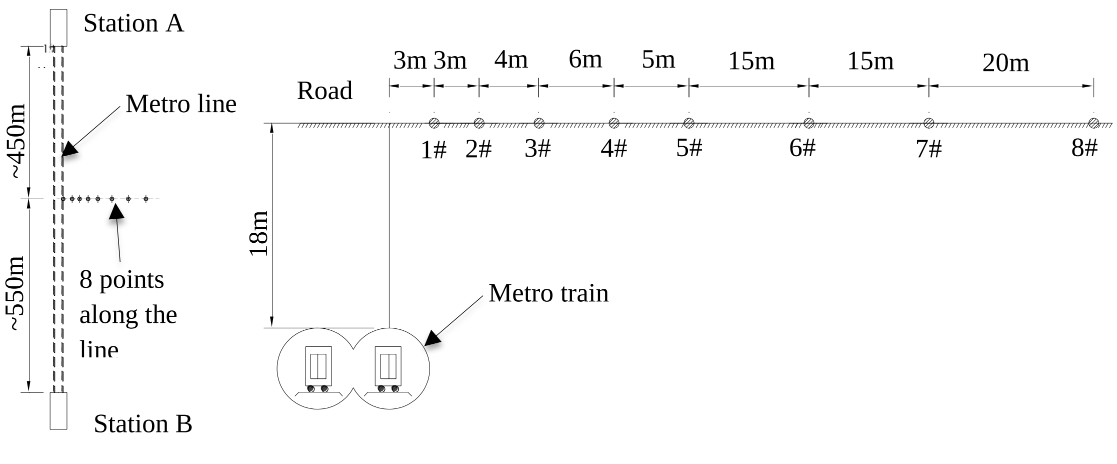
$$E = \sum_{t_i}\sum_{v_i} P_i(t_i, v_i) \tag{1.9}$$

where $P_i(t_i, v_i)$ is the discretized value of time-frequency distribution $P(t, f)$ at position (t_i, f_i) .

It is seen that the time-frequency energy can be used for the analysis of energy distribution of the transient impact signal in the concerned time length and frequency range.

2. ANALYTSIS FOR METRO INDUCED GROUND VIBRATION

The site test was done at 8 points on the ground, which is along a line vertical to the metro line as shown in Figure 2.1.



(a)Plan of site (b) Section of test site and metro

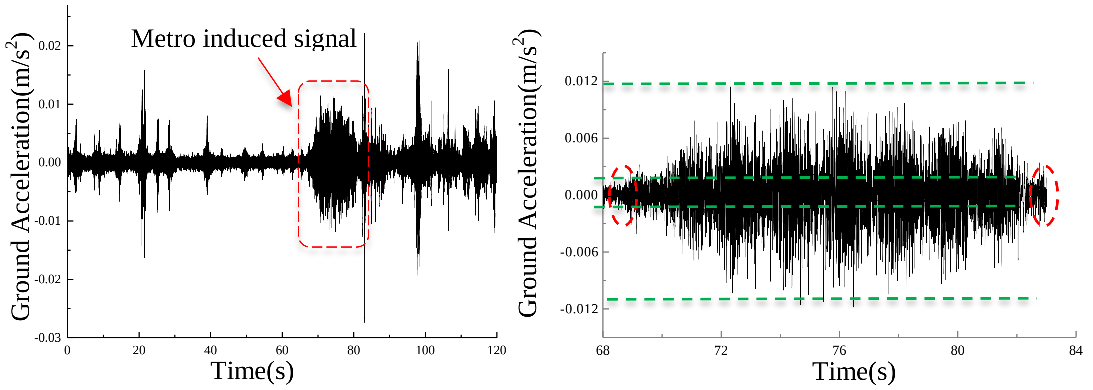
Figure 2.1 Location of test points

The test points are located in one side of the metro line as it is an empty construction site, and many buildings are at the opposite side of these test points. The metro line is beneath the road with depth of about 18 meters. The total length of site test is 4 fours in the morning. The distance between two metro stations is about 1km and the test points are nearly located in the middle. Although the metro is moving on two ways, the test points are mainly influenced by the metro train of the near side in one way.

As the metro is passing away about every 2 minutes, the signals including the metro and non-metro aspects are measured. The time duration for metro influence is found to be about 15 seconds. In order to get the influence of metro on the ground vibration, it is necessary to distinguish the signals induced by the metro and non-metro aspects.

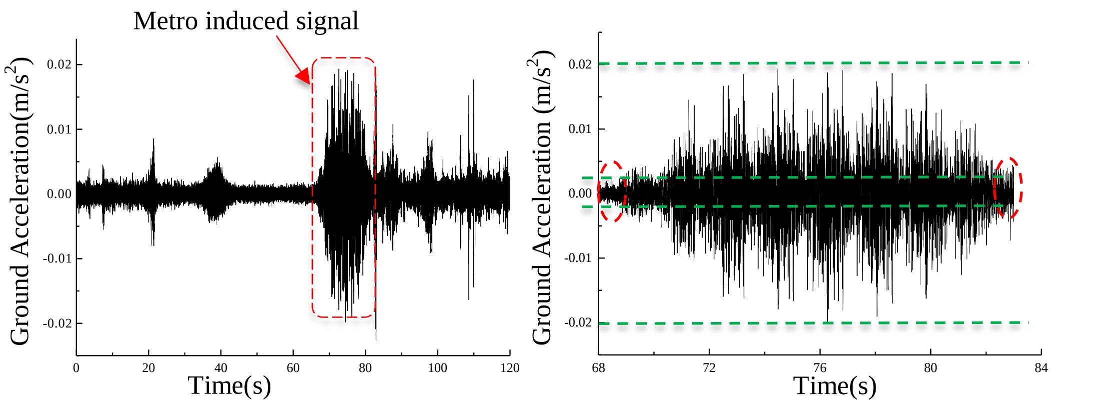
2.1 Sample curve of ground acceleration

As the traffic vehicles moves on the road, the signals of ground vibration caused by these aspects are got from site test and should not be considered for analysis on the influence of metro. According to the timetable of metro, the signals are separated into many segments of about 2 minutes and the influence of metro is found to be about 15 seconds. The typical curves in test point 1 and 4 are given in Figure 2.2 and 2.3. The left figures in Figure 2.2 and 2.3 represent the whole signals in one interval of the metro train, and the length of time is 120 seconds. And the right figures in Figure 2.2 and 2.3 are part of the whole signals, which is marked as red block in left figures and represent the signals with obvious influence of only one metro train in about 15 seconds (from 68 second to 84 second in the whole signal).



(a)in one interval of the metro train (b)with obvious influence of only one metro train

Figure 2.2 Typical time history of vibration signal at test point 1



(a)in one interval of the metro train (b)with obvious influence of only one metro train

Figure 2.3 Typical time history of vibration signal at test point 4

10 sample curves with duration of about 15 seconds are chosen for analysis in this paper. The typical curves involve a short period of the ground-borne vibration with peak value as 0.002m/s^2 at the beginning and the end of the signal, which are marked by red dashed circles in the graph. From the site test in other quiet periods, the typical curves of the ground borne vibration is found with peak value as 0.002 m/s^2, which is consistent with the test signals at the beginning and the end.

It can be found that the peak acceleration in test point 1 and 4 are about 0.012 m/s^2 and 0.020 m/s^2, which includes the influence of the ground borne vibration. Although the ground borne vibration cannot be eliminated directly by subtracting it from the test signals, the peak value of ground vibration at test point 4 is much larger than that at test point 1. It is an interesting phenomenon of metro-induced ground vibration and will be demonstrated by other methods later.

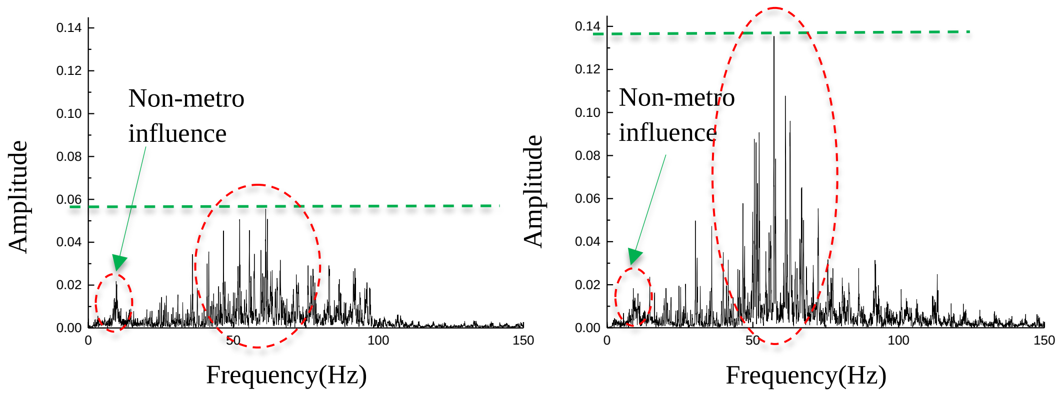
2.2 Analysis by traditional method

The average value of the ten signal duration turns to be 15.91 seconds, as shown in Table 2.1. For the reason of not restrict way for truncating data from the whole record, the time duration of signal may be shorter or longer when the ground borne vibration is less or more considered for analysis.

Table 2.1 Statistic value of metro vibration signal duration

Sample	1	2	3	4	5	6	7	8	9	10	average
Duration(s)	16.87	15.04	15.21	15.21	17.29	17.96	17.54	15.12	13.85	15.00	15.91

Applying Fourier transform, the analysis in the frequency domain can be done and the energy distribution in frequency can be found. In Figure 2.4, the vertical axis represents the energy by Eq. (1.2). The predominant frequency band is found to be in the range of 50Hz-70Hz. It is consistent with the general conclusion that predominant frequency for the metro induced vibration is usually above 50Hz. By comparing the amplitude of Fourier transform near 60Hz, the value for test point 4 is about twice the value for test point 1, which means the amplification phenomenon of the ground vibration in certain area as found in time history. The influence of low frequency as below 50Hz in the frequency spectrum of the signal can also be found in the figures, which represents the influence of non-metro aspect, such as the traffic vehicles with influence in the range of 10-20Hz. As the influence of low frequency (less than 40Hz) not easy to be eliminated, frequency domain analysis is not clear and efficient for analysis of metro-induced ground vibration.



(a) Test point 1 (b) Test point 4

Figure 2.4 Frequency spectrum of typical signal at test point 1 and test point 4

2.3 Analysis of time-frequency energy

Local characteristics of time-frequency energy at test point 1 and 4 are summarized in Table 2.2 and 2.3. The effective duration, the time center and the frequency center of the vibration signal at point 1 and 4 is nearly the same, while the effective frequency band at point 1 is large than that at point 4, which means the energy is more concentrated at the frequency center as about 62Hz. The time center and the effective duration are nearly the half and two thirds of the value in time domain respectively.

Table 2.2 Local characteristics of time-frequency energy at test point 1

Sample	1	2	3	4	5	6	7	8	9	10	average
t_m (s)	8.55	8.01	6.87	7.8	7.66	8.59	10.06	8.04	7.34	7.04	8.00
TE (s)	11.30	11.55	10.34	11.05	12.06	12.51	14.63	11.12	11.84	10.33	11.67
f_m (Hz)	66.17	68.3	61.56	56.11	70.01	64.2	42.99	66.04	60.68	65.14	62.12
BF (Hz)	69.00	67.01	52.20	93.81	65.20	76.85	66.35	58.61	51.86	52.81	62.57

Table 2.3 Local characteristics of time-frequency energy at test point 4

Sample	1	2	3	4	5	6	7	8	9	10	average
t_m (s)	7.79	7.37	6.38	8.36	7.66	8.55	10.06	8.69	7.85	7.35	8.00
TE (s)	11.10	10.95	9.46	11.08	12.03	11.08	13.05	10.63	11.78	9.85	11.10
f_m (Hz)	61.62	59.41	63.12	60.92	62.11	59.20	58.31	61.21	63.42	63.11	61.24
BF (Hz)	44.01	44.11	27.90	34.61	37.65	44.70	40.80	42.15	31.85	35.75	38.35

The time-frequency energy is illustrated in Figure 2.5, while the left figure is the power spectral density function by Fourier transform and the upper figure is the time history of ground vibration. From the bar on the left, the red and blue colour represents relative large and low energy concentration on the time-frequency energy distribution. The energy is mainly distributed in the domain by its center position (t_m, f_m) and a time-bandwidth product as $TE \times BF$ .The time history curve is got from the site test and the frequency domain analysis curve as the amplitude represents the energy on the whole process of signal. The time-frequency energy reflects the influence of time axis and the frequency axis along with the quantity of energy by different colour in the figure. It means three dimensional aspects of the signal of ground vibration, as the time duration, the frequency range and the energy distribution.

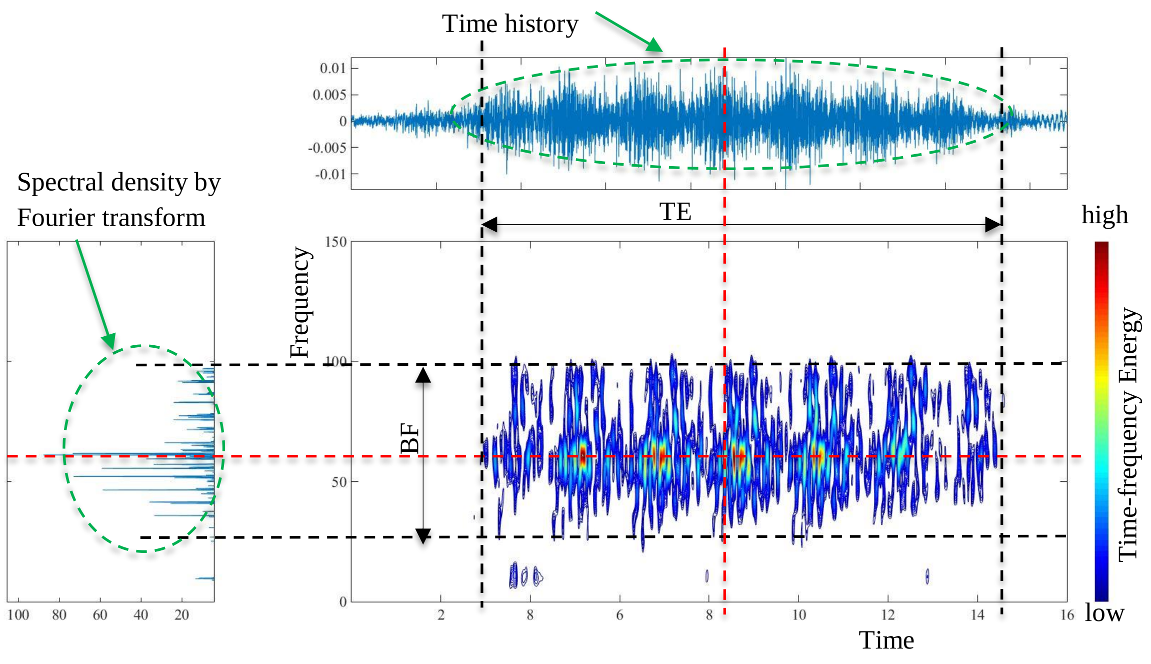


Figure 2.5 Illustrative time-frequency energy vs. the time history and Fourier spectrum

The time-frequency energy of typical signal at point 1 and 4 are shown in Figure 2.6 and 2.7, which is mainly in the range of 70 second to 82 second on the time axis while the effective frequency band is different with nearly the same frequency center at about 62Hz. It is deduced the energy distributed in the domain as $TE \times BF$ is mainly induced by the metro.

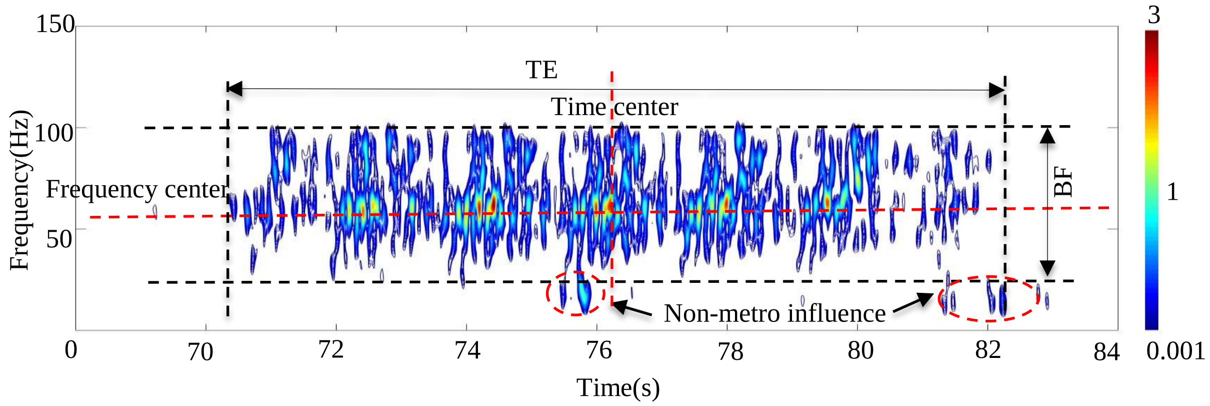


Figure 2.6 Time-frequency energy of test point 1

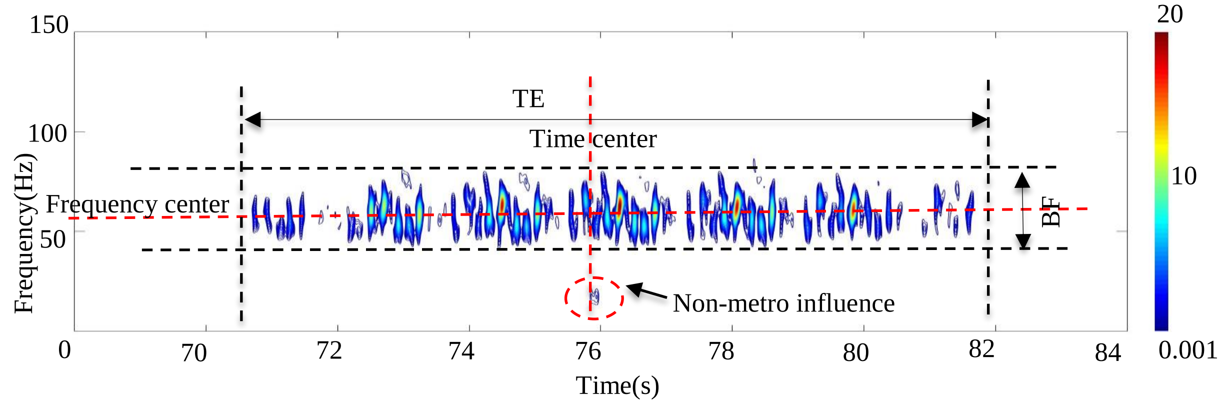


Figure 2.7 Time-frequency energy of test point 4

By comparison of the data on the left bar, which represents the relative magnitude of time-frequency energy, the vibration level at test point 4 is much larger than that at test point 1. This phenomenon of metro-induced ground vibration is also found by the time domain analysis and frequency domain analysis as described previously in this paper, which means the vibration in point 4 is more influenced by the metro. Besides the image in the domain for research interest, some images are also found below the lower dashed frequency line, which are marked by red dashed circles on the time-frequency energy. The small amount signals are distributed in 10-20Hz frequency, which represent the influence of non-metro aspects as the traffic vehicles moving through the line section by further investigation on the road. Comparatively, by Fourier transform the 10-20Hz frequency band can be found in the spectrum graph but these signals cannot be located on the time axis.

For the non-stationary signal, the effective duration and effective frequency band are more accurate and important to recognize the influence of vibration than the traditional method of analysis in time and frequency domain. The energy of this kind of signal concentrates in the effective duration rather than the whole time length, and in the specific frequency range of metro influence as the frequency band. The time-frequency energy shows the combination of time domain and frequency domain analytical result in one figure. This will be important for signal recognition and signal separation.

2.4 Analysis of vibration propagation

For an actual vibration signal, the energy value by RMS is a mean value over the whole time range. But for an individual or a structure, the response for the vibration often depends on the concentrated energy pack in a short duration, which is different from the peak value of the signal. Here it can be called as local energy feature, which is defined as the time distribution of the energy on the concern frequency range.

The curve of the RMS and time-frequency energy as MHS for 8 test points are plotted in the left and right part of Figure 2.8.The relative value of point 1 is set to be 1. The three curves marked by different symbol color represent the condition of minimum, maximum and average for 10 samples. The general attenuation tendency matches well with difference in value of the two curves. In the attenuation curve of time-frequency energy, the relative value for point 4 is about 6.5 and is large than 2.6 in the left figure. Comparing with the traditional index RMS, the time-frequency energy is more evident to show the local amplification phenomenon.

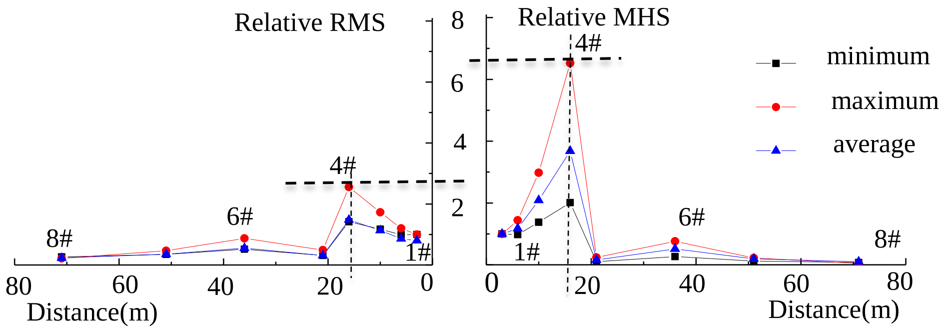


Figure 2.8 The relative value of RMS and MHS for vibration signals in 8 test points

3. CONCLUSION

This paper presents the analysis of time-frequency energy for metro induced environmental vibration by Margenau-Hill-Spectrogram distribution. The signal can be characterized in the time-frequency plane by its center position (t_m, f_m) and a domain of time-frequency band product as $TE \times BF$. The evaluation index for time-frequency energy is calculated and further discussed. Comparing with the traditional methods in the time domain and the frequency domain, the time-frequency energy can directly show the characteristics of the signal contents with convenience and reliability. By analysis of the time-frequency energy, the influence of the non-metro aspects can be removed and the propagation law for environmental vibration induced by metro can be got effectively. The environmental vibration is usually attenuated with the distance except local amplification not far away. Using the time-frequency energy as the evaluation index rather than the RMS, the amplification phenomenon in certain area is more evident with low variance of the data.

4. ACKNWLEDGMENT

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