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Damped Hysteretic Resistance Identification of Bouc-Wen Model Using Data-Based Model-Free Nonlinear Approach

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ABSTRACT
Most of the currently available structural damage detection approaches are primarily applicable to linear systems in which damage develops slowly. For yielding inelastic systems, new approaches that examine the nonlinear hysteretic restoring force consisting of viscous and stiffness contributions (we call it damped hysteretic resistance (DHR) from now on) are more appropriate to assess damage. In addition, modelling the true inelastic behaviour of different types of structural components or substructures in a parametric way is still challenging. Consequently, it is desirable to establish general model-free inelastic behaviour identification approaches that do not rely on assumed parametric nonlinear hysteretic models *a priori*. In this paper, we present a data-based model-free damped hysteretic resistance (DMDHR) identification method for evaluating such nonlinear systems. A 6-story lumped-mass numerical linear building structure with Bouc-Wen nonlinear elements at two floors is used for illustration. The structural DHR is identified without prior knowledge of the structural mass distribution or a specific numerical model for the structure. In the simulation example, the nonlinear structure is excited by a random vibration applied to selected DOF (e.g., only 3rd floor is excited). Using records of the excitation and the response with a consideration of noise effects, the nonlinear DHR of the structure is identified. The results show the identified DHRs match the corresponding theoretical (numerical) ones well.

KEYWORDS: *Damped Hysteretic Resistance Identification, Model-Free Identification, Bouc-Wen Model*

1. INRODUCTION

In recent decades, to address the critical need to enhance the sustainability of our infrastructure, damage detection and system identification have been active research areas. Damage detection focuses on the detection, localization, and characterization of damage in structural systems. Identifying the level of damage is very closely connected with remaining service life prognosis for existing structures. However, several challenges exist for the full realization of damage detection and prognosis.

Traditionally, in many available vibration-based structural damage detection techniques, damage is evaluated in terms of decrease in structural stiffness or change in elastic modulus across entire structural components [1-3]. These modal property based estimations, such as extracted eigenvalue, modal shapes, and/or their derivatives, are not appropriate to represent the remaining load-carrying capacity of structures, a key index for operational service life prognosis. In addition, changes in elastic modulus do not describe the effects of geometric nonlinearities such as P-Δ effects. Overall, it is considered that such approaches are most applicable to linear systems with slowly developing damage. Nevertheless, in real structures with damaged components, nonlinearity always exists. Moreover, there has been increasing demands for understanding damage initiation, growth and distribution. Unfortunately, the identification theory for nonlinear systems is less advanced than for linear systems. Such information would contribute to our understanding of structural failure patterns, progressive collapse and energy dissipation mechanisms under severe dynamic loadings.

A second challenge in the realization of damage detection is in the numerical modeling of the structure. With inelastic systems, modelling the true inelastic behaviour of different types of structural components or substructures in a parametric way is still challenging despite the plethora of hysteretic models currently available. There are numerous parameters to identify, and the resulting model may be limited in its applicability to a particular amplitude of motion. Thus, it is desirable to establish general model-free inelastic behaviour identification approaches that do not rely on assumed parametric nonlinear hysteretic models a priori.

To address some of these challenges, Yang et al. (2006) proposed a parameter identification approach for nonlinear systems using recursive least-squares estimation (LSE) techniques and extended Kalman filter (EKF). A nonlinear parameter identification method to determine the coefficients for the Bouc-Wen model using EKF and LSE without the information of input excitation forces was proposed by Lei et al. (2011). Recently, on the basis of time-domain response and incomplete excitation information, an identification methodology to capture nonlinear restoring force was proposed [6,7]. He et al. (2012) extended this approach for nonlinear restoring force and dynamic loadings identification. Lastly, Xu et al. (2014) proposed a new identification methodology, known as the data-based model-free damped hysteretic resistance (DMDHR) identification approach, which is more appropriate to assess damage in actual structures by examining the damped hysteretic resistance (DHR). The method can capture structural DHR without a specific numerical model for the structure. It also has the ability to identify mass distribution of an engineering structure.

This paper investigates the application of DMDHR identification approach to identify the nonlinear DHR in a 6-story shear building numerical simulation model with MR dampers. The MR dampers are modelled as nonlinear Bouc-Wen elements. In simulation, this nonlinear system is excited with a random disturbance applied to selected DOF (e.g., only 3rd floor is excited). Using records of the excitation (force) and the responses (acceleration, velocity, and displacement) with a consideration of noise effect, the nonlinear DHRs associated with the MR dampers are identified. Consequently, it is shown that the identified DHRs are in good agreement with corresponding theoretical (numerical) ones.

2. DMDHR IDENTIFICATION APPROACH

DMDHR identification approach starts with formulating the governing equation of motion for a subject structure. For a linearly elastic structure subjected to an external force excitation $F(t)$, the state of this system is described by displacement $x(t)$, velocity $\dot{x}(t)$ and acceleration $\ddot{x}(t)$ as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t) \tag{2.1}$$

where M, C and K are, respectively, mass, damping and stiffness matrices.

To extend this equation to nonlinear systems, the last two terms in the left-hand-side of the above equation should be replaced by the DHR $R(x,\dot{x},q)$. For such systems, therefore, the equation of motion becomes

$$M\ddot{x}(t) + R(x,\dot{x},q) = F(t) \tag{2.2}$$

where q represents the system-specific parameters such as stiffness, damping coefficients, and so on.

Notice that there is no mass nonlinearity in typical engineering structures under dynamic excitations. So, an equivalent mass matrix M_E is introduced in place of nonlinear mass matrix M in Eq. 2.2 to avoid confusion. Then the equation of motion of the equivalent linear system for the nonlinear system can be described as

$$M_E\ddot{x}(t) + C_E\dot{x}(t) + K_E x(t) = F(t) \tag{2.3}$$

From these relationships, the DHR $R(x,\dot{x},q)$ can be expressed as the sum of an equivalent damping force $C_E\dot{x}(t)$ and an equivalent stiffness force $K_E\dot{x}(t)$ as follows:

$$R(x,\dot{x},q) = C_E\dot{x}(t) + K_E x(t) \tag{2.4}$$

The coefficients for each equivalent linear matrix in Eq. 2.4 can be easily acquired by solving its inverse problem. For easy understanding, the equation is re-written in the context of a least-squares problem.

$$H\theta = P \tag{2.5}$$

where H is response matrix composed of structural response vectors of displacement, velocity and acceleration, θ is unknown matrix composed of coefficients for each equivalent linear matrix, and P is input matrix composed of external excitation forces.

This expression, of course, is appropriate as it is applied to the case where the number of equations are greater than the number of unknowns. This is unsolvable in practice, except the special case where θ is in the column space of H. Thus, the estimated unknown coefficients matrix $\hat{\theta}$, which is the θ that minimizes an average error in all equations, can be identified by means of simple least-square solution as follows:

$$\hat{\theta} = (H^T H)^{-1} H^T P \tag{2.6}$$

As realistic constraint, exerting forces on all DOFs is difficult without performing a shake-table test when we have a discrete n-DOFs structure. So if only a part of the system is excited, the values at the remaining DOFs except the DOFs related to that part in input matrix will be filled with zeros, resulting in the rank of input matrix defined in Eq. 2.5 being less than n. Consequently, it is impossible for the whole unknown coefficients to be estimated by virtue of Eq. 2.6. As a means to resolve this problem, the equation of motion for the ith DOF where the external force is nonzero and available is considered. Then, Eq. 2.3 can be modified as follows:

$$\sum_{j=1}^{n} m_{E,i,j}\ddot{x}_j(t) + \sum_{j=1}^{n} c_{E,i,j}\dot{x}_j(t) + \sum_{j=1}^{n} k_{E,i,j}x_j(t) = F_i(t) \tag{2.7}$$

where the coefficients of $m_{E,i,j}$, $m_{E,i,j}$, and $m_{E,i,j}$ are the elements of mass, damping, and stiffness matrices of the corresponding equivalent linear system of the nonlinear system, respectively (i.e., the element in the ith row and jth column). $x_j(t)$ and $\dot{x}_j(t)$ are the displacement and velocity vectors of the jth DOF, respectively.

Because the input force is not zero and all responses are in use, the unknown mass, damping, and stiffness coefficients of the ith DOF can be identified by the direct use of Eq. 2.6. Then, the equation of motion of the jth DOF where the input force is zero (no external force is applied) is considered. Per Maxwell's Theorem of Reciprocity of Displacements:

$$k_{E,i,j} = k_{E,j,i} \quad (i, j = 1, 2, \ldots, n) \tag{2.8}$$

Using Eq. 2.8, the equation of motion for the jth DOF, where no input force is applied, can be re-written as

$$\sum_{s=1}^{n} m_{E,j,s}\ddot{x}_s(t) + \sum_{s=1}^{n} c_{E,j,s}\dot{x}_s(t) + \sum_{s=1, s\neq i}^{n} k_{E,j,s}x_s(t) = -k_{E,j,i}x_i(t) = -k_{E,i,j}x_i(t) \tag{2.9}$$

Because the coefficients of the ith DOF were previously identified, the right-hand-side in this equation is treated as known. In the same way, the unknown coefficients in Eq. 2.9 can be identified using Eq. 2.6.

Finally, all mass, damping, stiffness coefficients on all DOFs can be determined. So the DHR for a subject structure is determined through the modified Eq. 2.4. In short, the DMDHR identification method is able to capture nonlinear DHR without a specific numerical model for a subject structure. It also has an ability to identify mass distribution of such structure.

3. ILLUSTRATIVE EXAMPLE OF DHR IDENTIFICATION

3.1. Numerical Model of a Multi-Story Structure with MR dampers

As a numerical model for demonstration of applicability of DMDHR identification method, a 6-story lumped-mass nonlinear structure consisting of a linear MDOF structure with magnetorheological (MR) dampers represented by Bouc-Wen model is introduced (Fig. 3.1). The MR dampers are used to produce nonlinearity for the structure. Each story has one DOF in the horizontal direction. The lumped-mass corresponding to each DOF, damping coefficients of the linear numerical model, and story stiffnesses are set to be $m_i = 10$ kg, $c_i = 150$ N·s/m, and $k_i = 1.2\times10^5$ N/m ($i = 1,2,\ldots,6$), respectively. To observe variation of DHR with respect to locations of MR dampers or applied force and to examine the applicability of DMDHR identification method, three different cases are considered, which are summarized in Table 3.1.

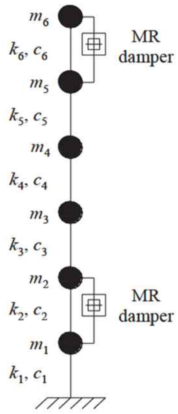

Figure 3.1 A 6-DOF lumped-mass numerical nonlinear structure with MR dampers

Table 3.1 MR damper and applied force locations according to different cases

	T1	T2	T3
Force Location (floor)	4th	3rd	3rd
Damper Location (floor)	2nd and 6th	2nd and 6th	2nd and 5th

3.2. MR Dampers with Bouc-Wen Model

MR dampers are semi-active devices that use controllable magnetorheological fluids whose properties can change with electric current when exposed to a magnetic field. Due to their applicability, MR dampers are widely used in many engineering areas. The simple mechanical model shown in Fig. 3.2 was developed and shown to predict the behavior of a prototype shear-mode MR damper over a wide range of inputs [10].

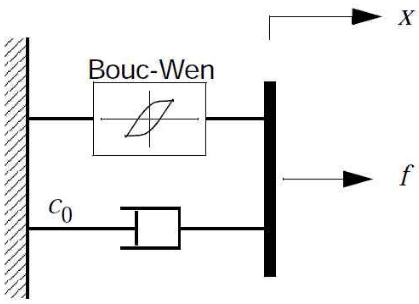

Figure 3.2 Mechanical model of MR damper

The equations governing the force produced by this model are given by

$$f = c_0\dot{x} + \alpha z \tag{3.1}$$

$$\dot{z} = -\gamma|\dot{x}|z|z|^{n-1} - \beta\dot{x}|z|^n + A\dot{x} \tag{3.2}$$

where z is an evolutionary variable that accounts for history dependence of the response.

The model parameters depend on the voltage to the current driver as follows:

$$c_0 = c_{0a} + c_{0b}u \quad \text{and} \quad \alpha = \alpha_a + \alpha_b u \tag{3.3}$$

where u is given as the output of the first-order filter.

The current driver circuit of the MR damper causes dynamics in the system, which can be described as the first order time delay in the response of the device to changes in the command input. To consider this effect, the following control input is given by

$$\dot{u} = -\eta(u - v) \tag{3.4}$$

where v represents command voltage applied to the control circuit and $1/\eta$ is time constant of the first order filter.

The parameters of the MR dampers used are selected as follows: $c_{0a} = 44.0\ \mathrm{N \cdot s/m}$, $c_{0b} = 440.0\ \mathrm{N \cdot s/(m \cdot V)}$, $\alpha_a = 1.0872e6\ \mathrm{N/m}$, $\alpha_b = 4.9615e7\ \mathrm{N/(m \cdot V)}$, $n = 1$, $A = 1.2$, $\gamma = 2500\ \mathrm{m}^{-1}$, and $\beta = 25\ \mathrm{m}^{-1}$. These parameters are based on the identified model of a shear-mode prototype MR damper tested at Washington University in St. Louis [11] and scaled up to have maximum capacity of $500\ \mathrm{N}$ under 1.0 Hz sinusoidal displacement with an amplitude of 0.05m (Fig. 3.3).

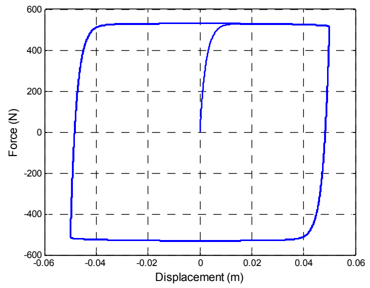

Figure 3.3 Response of MR damper under sinusoidal force

3.3. Evaluation of the Proposed DMDHR Identification Approach

Numerical validation on DMDHR identification approach is examined through different cases described in Table 3.1. To consider the effect of measurement noise existed in real test on DHR identification results, the noisy output measurements (displacement, velocity, and acceleration) are generated by adding a 5% of root-mean-square (RMS) of the clean signal, which is calculated by the RMS value times a zero mean Gaussian noise, to the noise free signal. Due to the practical constraints in the real world, exerting forces on all DOFs is difficult without performing a shake-table test. So to simulate a simple disturbance such as an isolated shaker, the random excitation forces, which follow a normal distribution with the mean magnitude of $1000\ \mathrm{N}$, are only applied to one floor between MR dampers for every case.

The performance of DMDHR identification methodology is evaluated by comparing the identified MR damper force with that determined by numerical simulation. According to the procedures mentioned in Section 3, all the identified coefficients related to mass, damping, and stiffness terms are those in corresponding matrices of the equivalent linear system for the nonlinear system. Then, the identified DHRs at DOFs where MR dampers are installed are obtained with Eq. 2.10. It is important to note here is that the MR forces should be determined by excluding the elastic restoring force and damping force provided by the linear structure itself from the identified DHR.

Fig. 3.4 shows the identified MR forces versus the story displacement under a dynamic excitation for Case T1. Based on the observations of the identified MR forces on the 1st, 3rd, 4th, and 5th floors, it is shown that the forces at those floors are close to zero, as expected. Given the identified results on the 2nd and 6th floors, where MR dampers are installed, the identified DHRs seem to track the MR damper forces with slight deviation. To observe its accuracy in an alternate way, the identified MR damper force with respect to time on those floors are provided in Fig. 3.5. Similarly, it is shown to closely follow the exact MR damper forces for both floors. Here the performance of DMDHR identification method is measured by the relative error (estimate-true)/max(|true|) of obtained MR damper force. The average of relative errors on the 2nd floor and 6th floor show 10.8% and

27.1%, respectively.

Similar identification results are obtained for Case T2 and Case T3 (Fig. 3.5 and Fig. 3.6). For Case T2, the error is 20.2% and 9.1% for each MR damper in ascending order of floor. Case T3 results in relatively low errors of 15.4% and 6.9%.

Overall, the level of accuracy for identification is reasonable, even though measurements used are contaminated with 5% simulated noise. Aside from this identification ability, it is interesting that MR damper force pattern in the lower floor is quite different from that in the upper floor. This implies that a considerable portion of implied force is dissipated in the MR damper introduced in the lower floor.

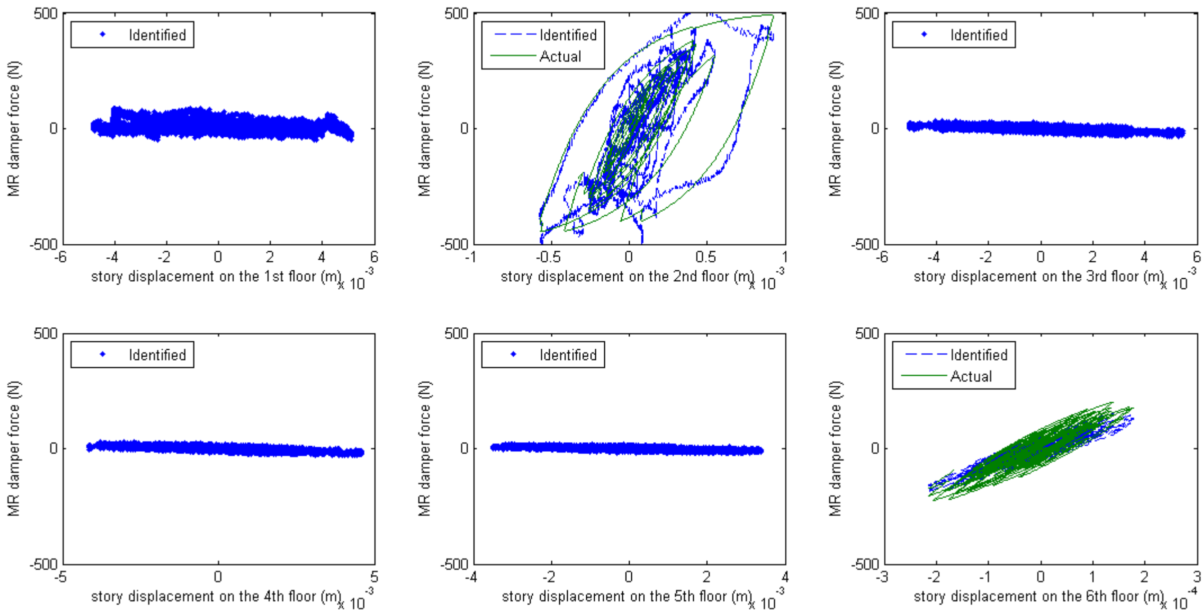

Figure 3.4 Identified DHR of MR damper (Case T1)

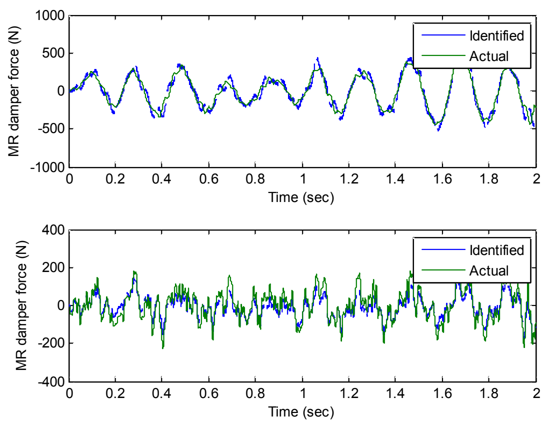

Figure 3.5 Identified MR damper force with respect to time (Case T2)

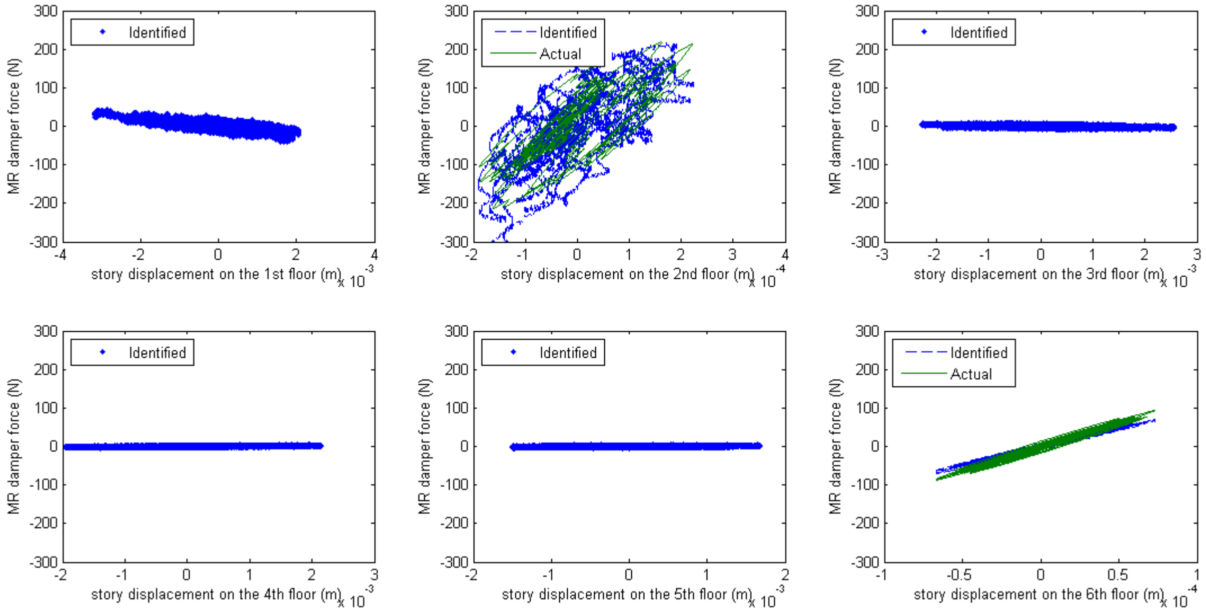

Figure 3.6 Identified DHR of MR damper (Case T2)

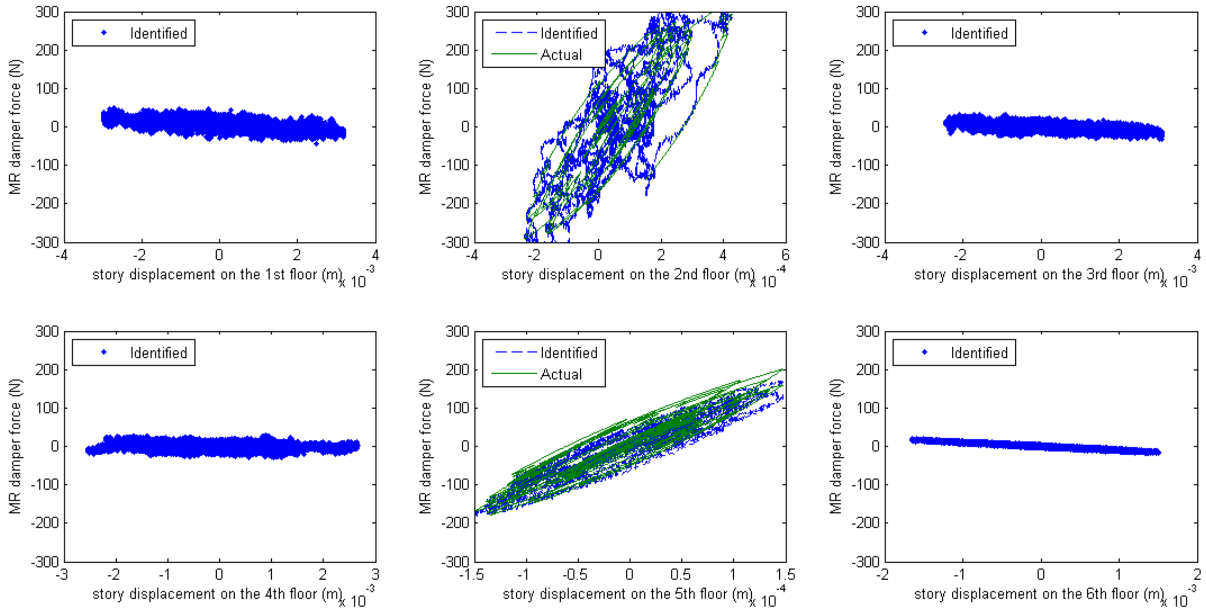

Figure 3.7 Identified DHR of MR damper (Case T3)

4. CONCLUSIONS

In this study, the applicability of data-based damped hysteretic resistance (DMDHR) identification approach for MR damper force of Bouc-Wen model is demonstrated and evaluated. A 6-story lumped-mass nonlinear structure consisting of a linear MDOF structure with hysteretic dampers (Bouc-Wen model) introduced at two locations is used for evaluation. To observe variation of DHR with respect to locations of MR dampers or applied force, three different cases are considered. Even with measurements containing 5% simulated noise, the DHRs of MR damper are successfully identified using the method and compared with actual forces. The results indicate that the identified DHR has good agreement with the corresponding theoretical (numerical) ones. Specifically, the average of relative errors of MR damper forces for each case account for (i) 10.8% and 27.1%; (ii) 20.2% and 9.1%; and (iii) 15.4% and 6.9%. Future work will consider the development of robust and reliable DHR identification method with higher accuracy and execution of a reliability assessment of the approach.

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