6th International Conference on Advances in Experimental Structural Engineering
11th International Workshop on Advanced Smart Materials and Smart Structures Technology
August 1-2, 2015, University of Illinois, Urbana-Champaign, United States

Experimental investigation of a Kriging surrogate-based finite element model updating method for bridge structures

H.Y. Jung[1], J.H. Lee[1], S.S. Jin[1], H.J. Jung[2]

1 Ph.D. Candidate, Dept. of Civil and Environmental Engineering, KAIST, Korea.
E-mail: soulpower@kaist.ac.kr
2 Associate Professor, Dept. of Civil and Environmental Engineering, KAIST, Korea.
E-mail: hjung@kaist.ac.kr

ABSTRACT
Finite element (FE) model updating methods are frequently used to evaluate the performance of a bridge. A precise initial analytical model is necessary to obtain accurate results in conventional FE model updating methods. However, a precise model consumes excessive calculation time. In this study, a new FE model updating method based on a surrogate model is presented to significantly reduce the calculation time without sacrificing accuracy. A Kriging surrogate model (or response surface model) is selected as a result of considering various kinds of surrogate models. In order to efficiently construct an accurate Kriging surrogate model, the sequential sampling strategy is introduced. The performance and efficiency of the proposed method are verified through a lab-scale experiment with a small-scale bridge model. It is demonstrated that the proposed FE model updating method has much better performance in the calculation time than the conventional one.

KEYWORDS: *Finite element model updating, Surrogate model, Kriging basis, Sequential sampling strategy*

1. INTRODUCTION

Civil infrastructure such as high-rise buildings and long-span bridges have increased gradually. These structures are generally designed to withstand for many decades. However, the performance of these structures degrades gradually due to many different causes such as fatigue, corrosion and natural hazards. In order to maintain their serviceability and prevent structural failure, continuous evaluation and reinforcement of structural performance are required. For evaluation of structural performance, a finite element (FE) model of a target structure is needed. The current design and assessment procedures do not have any quantitative linkage to actual existing structures [1]. Therefore, a process to link an FE model with the corresponding existing structure is necessary, which is called an FE model updating process.

In the FE model updating process, the accuracy and reliability of an FE model are decided by degree of precision of the model. Using a sophisticated FE model, a more accurate and reliable result can be obtained. However, the sophisticated FE model consumes excessive calculation time, and the excessive calculation time can be a burden to users.

In order to reduce the computational time of a sophisticated FE model, a surrogate model has been suggested recently. The surrogate model is a computational modeling technique based on a mathematical construction of input and output relation. It is also called a response surface model. It can replace the model updating process with the response surface [2]. By using the response surface, updating parameters for an FE model can be obtained in a relatively short calculation time.

In this study, a new FE model updating method based on a surrogate model is presented to reduce the excessive calculation time. As a result of considering various kinds of surrogate models, the Kriging surrogate model is selected. In order to minimize the calculation time, a sequential sampling method is used. The performance and efficiency of the proposed method are verified through a lab-scale experiment with a small-scale bridge model, and compared with those of the conventional FE model updating technique.

2. THEORY

At this section, the Kriging surrogate model and the sequential sampling strategy is briefly explained. A more detailed description can be found in [3-4]. The Kriging model, also known as the Gaussian process model, is one of the surrogate models originated from Geostatistics [5]. It is able to model a function as the realization of a stochastic process with a mean μ and a variance σ^2. It is based on a spatial correlation among the values of the function. The Kriging basis with k dimensions (i.e., correlation function) can be expressed as

$$\psi^{ij} = exp\left(-\sum_{p=1}^{k} \theta_p \left\|x_p^i - x_p^j\right\|\right) = \mathrm{corr}[y(x^i), y(x^j)] \tag{2.1}$$

where the subscript "*p*" denotes the dimension of a sample *x*, the superscripts "*i*" and "*j*" indicate *i*-th and *j*-th samples, respectively, and $\left\|x_p^i - x_p^j\right\|$ is the Euclidean distance between the two samples in a parameter space with 2-norm. The Kriging basis has parameters corresponding to each dimension (i.e., $\boldsymbol{\theta}_p = [\theta_1, \theta_2, \dots, \theta_p]$) which determine how fast the correlation decays in each dimension (i.e., input).

The correlation matrix of all the samples can be constructed as

$$corr[\boldsymbol{Y}, \boldsymbol{Y}] = \begin{bmatrix} corr[y(x^1), y(x^1)] & \cdots & corr[y(x^1), y(x^n)] \\ \vdots & \ddots & \vdots \\ corr[y(x^n), y(x^1)] & \dots & corr[y(x^n), y(x^n)] \end{bmatrix} \tag{2.2}$$

The covariance matrix can be derived from the correlation matrix ($\boldsymbol{\psi}$) as

$$COV(\boldsymbol{Y}, \boldsymbol{Y}) = \boldsymbol{\sigma}^2 corr(\boldsymbol{Y}, \boldsymbol{Y}) = \boldsymbol{\sigma}^2 \boldsymbol{\psi} \tag{2.3}$$

In order to construct the Gaussian process, the estimation of parameters such as μ and σ^2 is required. The parameters can be estimated by maximum likelihood estimation (MLE), and its log-likelihood function is given as

$$ln(L) = -\frac{n}{2} ln(\sigma^2) - \frac{1}{2} ln|\boldsymbol{\psi}| - \frac{(\boldsymbol{Y} - \boldsymbol{1}\mu)^T \boldsymbol{\psi}^{-1} (\boldsymbol{Y} - \boldsymbol{1}\mu)}{2\sigma^2} \tag{2.4}$$

where $\boldsymbol{1}$ is an n-by-1 unit vector and $\boldsymbol{Y}$ is the true function value of the observed samples.

Taking the derivatives of Eq. 2.4 with respect to $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}^2$, repsectively, and setting these to zeros, MLEs of $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}^2$ are expressed as

$$\hat{\mu} = \frac{1^T \boldsymbol{\psi}^{-1} \boldsymbol{Y}}{1^T \boldsymbol{\psi}^{-1} \boldsymbol{1}} \tag{2.5}$$

$$\hat{\sigma}^2 = \frac{(\boldsymbol{Y} - \boldsymbol{1}\hat{\mu})^T \boldsymbol{\psi}^{-1} (\boldsymbol{Y} - \boldsymbol{1}\hat{\mu})}{n} \tag{2.6}$$

MLEs for μ and σ^2 are sequentially computed from the correlation matrix ($\boldsymbol{\psi}$), so the only remaining parameter to be determined is θ_p in Eq. 2.1. In order to find the optimal θ_p, thus, the optimization method is applied by maximizing Eq. 2.4 under the observed samples.

After some manipulation, the augmented log-liklihood can be obtained as

$$ln(L) = \left(\frac{-1}{\hat{\sigma}^2 (1 - \hat{\boldsymbol{\psi}}^T \boldsymbol{\psi}^{-1} \hat{\boldsymbol{\psi}})}\right) (\hat{y} - \hat{\mu})^2 + \left(\frac{\hat{\boldsymbol{\psi}}^T \boldsymbol{\psi}^{-1} (\boldsymbol{Y} - \boldsymbol{1}\hat{\mu})}{\hat{\sigma}^2 (1 - \hat{\boldsymbol{\psi}}^T \boldsymbol{\psi}^{-1} \hat{\boldsymbol{\psi}})}\right) (\hat{y} - \hat{\mu}) \tag{2.7}$$

where $\hat{\mu}$ and σ^2 are the MLEs from Eqs. 2.5 and 2.6, $\hat{\boldsymbol{\psi}}$ is a correlation vector between the new (x_{new}) and observed samples (*X*), i.e., $\hat{\boldsymbol{\psi}} = corr[y(x_{new}), \boldsymbol{Y}]$. The prediction ($\hat{y}$) is obtained by maximizing the log-likelihood function in Eq. 2.7. Taking the derivative of Eq. 2.7 with respect to $\hat{y}$ and setting this to zero, the prediction ($\hat{y}$) is obtained as

$$\hat{y}(x_{new}) = \hat{\boldsymbol{\mu}} + \hat{\boldsymbol{\psi}}^T \boldsymbol{\psi}^{-1} (\boldsymbol{Y} - \boldsymbol{1}\hat{\boldsymbol{\mu}}) \tag{2.8}$$

By evaluating the prediction values, the Kriging surrogate model can be constructed in the linear combination form.

The sequential sampling strategy is an adaptive sampling approach and the sampling continues until the target accuracy of the Kriging model is achieved. Figure 2.1 describes the basic concept of the sequential sampling strategy. In the strategy, a new sample is added to improve hte accuracy of the suroogate model and reduce the variance. The new sample has high likelihood of improvement based on statistical interpretation of prediction. In this study, the sequential sampling strategy based on expected improvement (i.e., EI(x)) is introduced and the expected improvement approach is the criteria to evaluate how much improvement of current best value is expected if a new sample is obtained [2]. A more detailed explanation can be found in [3-4].

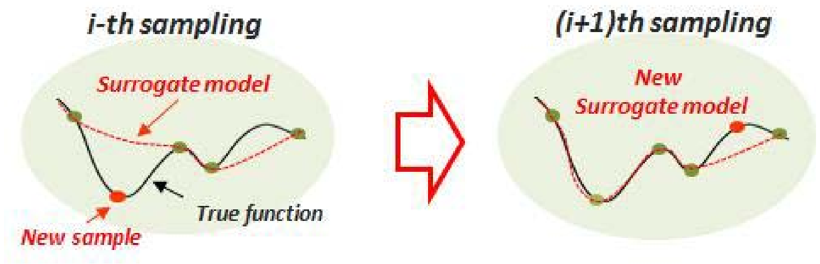


Figure 2.1 Construction of the surrogate model with the sequential sampling strategy

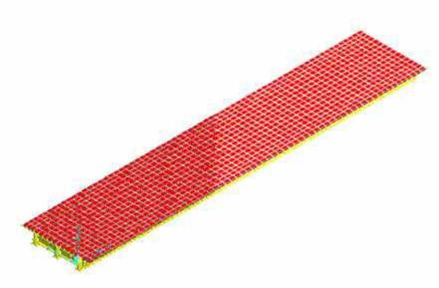
3. EXPERIMENTAL VALIDATION

In order to verify the effectiveness of the proposed method, experimental investigation is performed. First, an experimental test is conducted with a small-scale bridge model which is a simple span with three girders and nine cross beams. It is made of Polycarbonate. Table 3.1 shows the material properties of the bridge model. The model is supported by a hinge and a roller at its both ends, respectively. An FE model for the bridge model is constructed with 309 frame elements and 1056 shell elements by using SAP 2000. Figure 3.1 shows a small-scale bridge model and its FE model.

Table 3.1 Material properties

Parameter	Value
Length	2 m
Width	0.39 m
Young's modulus	2.0~3.0 GPa
Density	1.2 g/cm^2

(a) Small-scale bridge model



(b) FE model

Figure 3.1 Bridge model

An impact test is carried out to identify the dynamic characteristics of the bridge model such as the natural frequencies. The acceleration responses at the three different locations are measured with accelerometers and the deflection at mid-span is measured with a laser displacement sensors. The natural frequencies of the bridge model are identified by the peak picking method from the measured acceleration responses.

The Young's moduli of the main girders (E_{girder}), slab (E_{slab}) and cross beams (E_{cross}), which mainly affect the behavior in the longitudinal and transverse directions, are selected as the updating parameters for a surrogate model. Also, the 1st and 2nd bending natural frequencies and the 1st torsional natural frequency and the deflection at mid-span are selected as the target outputs.

To build an initial Kriging model, the 10 samples are first generated by Latin hypercube sampling (LHS). And then, a total of 100 samples are additionally generated as the validation data set for evaluating the accuracy of the initial Kriging model. Finally, a four Kriging model (i.e., 3 natural frequencies and 1 deflection) are constructed by using the sequential sampling strategy as mentioned in the previous section. Figure 3.2 shows the process of constructing the Kriging surrogate model with the sequential sampling strategy. For the stopping criteria, the thresholds of R-squared value (R^2) and root mean square error (RMSE) are set to 0.98 and 0.005, respectively. As the number of the infilled samples (i.e., x_{infill}) is increased, the accuracy of the Kriging model is improved.

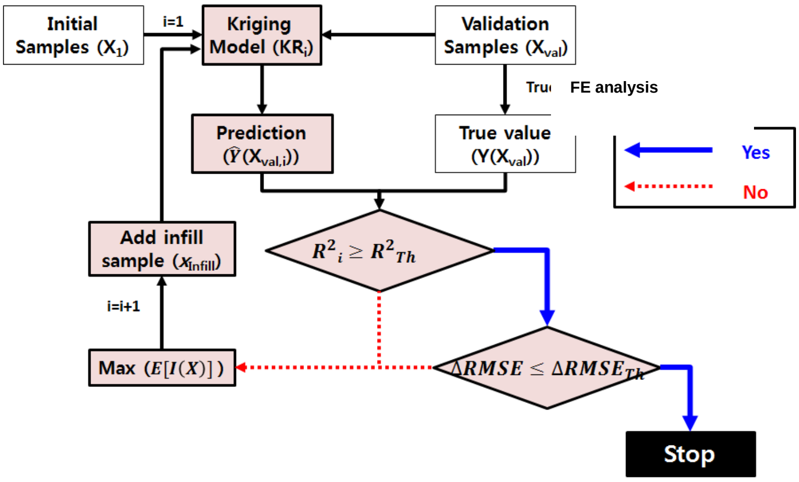


Figure 3.2 Flow chart for constructing the Kriging surrogate model

Except the samples which are used to construct the initial surrogate model, additional 300 samples are randomly selected in order to validate the performance of the surrogate model. For the 300 samples, the target outputs predicted from the surrogate model and those obtained from the FE model analysis are compared. The R^2 and RMSE values are computed as shown in Figure 3.3. The figures show that the results from the Kriging surrogate model are quite similar to those from the FE model analysis. It is, thus, expected that the Kriging model could be reproduce the target outputs accurately.

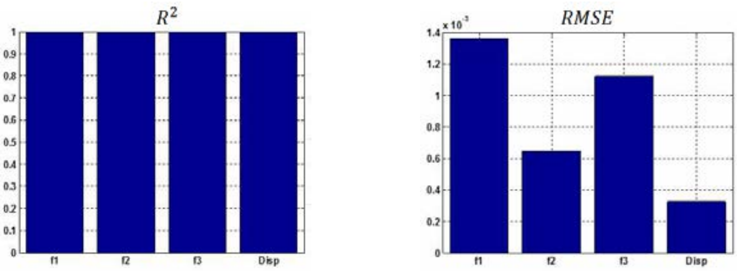


Figure 3.3 R^2 and RMSE of the 300 sample points

Based on the constructed Kriging surrogate models, the FE model updating process is carried out. The objective function for model updating is formulated in terms of the residuals between the predicted values from the Kriging model and the measured ones from the test of target outputs such as three natural frequencies and the mid-span deflection:

$$J = \sum_i \omega_i^f \left(\frac{f_i^{FEM} - f_i^{EXP}}{f_i^{EXP}}\right)^2 + \omega^u \left(\frac{u_{mid}^{FEM} - u_{mid}^{EXP}}{u_{mid}^{EXP}}\right)^2 \quad (3.1)$$

$$\text{subject to } \sum_i \omega_i^f + \omega_{mid}^u = 1, \; \omega_i^f, \omega_{mid}^u \geq 0.$$

where f_i^{FEM} and f_i^{EXP} denote the i-th natural frequencies from the prediction (i.e., the Kriging model) and the experiment, respectively; u_{mid}^{FEM} and u_{mid}^{EXP} indicate the mid-span deflection from the prediction and the experiment, respectively; and ω_i^f and ω_{mid}^u are weighting factors for natural frequencies and mid-span deflection, respectively. In FE model updating process, an optimization technique is used to minimize the objective function. In this study, the genetic algorithm (GA) is used for FE model updating, because it has a capability of global search based on the population. The GA is able to indirectly evaluate the accuracy of the Kriging model globally during the model updating process. In addition, the Nelder-Mead simplex algorithm is used to improve the solution from the GA. SAP2000 and MATLAB are linked to perform modal analysis iteratively with optimization tools of MATLAB.

Tables 3.2 and 3.3 summarize the results of FE model updating based on the Kriging surrogate model with the sequential sampling strategy. In Table 3.2, the updated values of the target outputs (i.e., 3 natural frequencies and a mid-span deflection) from the proposed method are compared with the measured values from the experiment. As seen from the table, all the updated values get much closer to the corresponding measured values. This means that the accuracy of the FE model is significantly improved by conducting the Kriging surrogate-based FE model updating. Table 3.3 shows the updated values of the updating parameters (i.e., the three Young's moduli). The total computation time for the proposed FE model updating method is less than an hour while the time for the conventional FE model updating method is longer than 8 hours. This experimental investigation clearly demonstrates that the Kriging surrogate model could be a cost-effective substitute for a time-consuming FE analysis.

Table 3.2 FE model updating results

Target outputs	*Experimental result*	*Initial FE model*		*Updated FE model (Kriging model)*	
		Value	*Relative error*	*Value*	*Relative error*
f_1	11.482 Hz	11.330 Hz	-1.32 %	11.461 Hz	-0.18 %
f_2	21.053 Hz	22.442 Hz	6.59 %	21.013 Hz	-0.19 %
f_3	41.520 Hz	40.607 Hz	-2.19 %	41.095 Hz	-1.02 %
u_{mid}	6.20 mm	6.33 mm	2.09 %	6.22 mm	0.32 %

Table 3.3 Results of updating parameters

Updating parameter	*Initial value*	*Updated value*
E_{slab}	2.5 GPa	2.080 GPa
E_{girder}	2.5 GPa	2.757 GPa
E_{cross}	2.5 GPa	2.083 GPa

4. CONCLUSION

In this study, a new FE model updating method based on the Kriging surrogate model with the sequential sampling strategy is proposed. First, a brief introduction of the theoretical background on the Kriging model and the sequential sampling strategy is addressed. Then, the experimental validation of the proposed method is conducted by using a small-scale bridge model. It is demonstrated that the proposed method has a quite similar accuracy compared with the conventional FE model analysis approach. It means that the proposed method is able to construct an accurate surrogate model. At the same time, the Kriging surrogate-based FE model updating method shows a drastic reduction of calculation time for FE model updating compared with the conventional approach. Therefore, the proposed FE model updating method could be a substitute for the time-consuming conventional FE model updating method which needs a lot of FE analysis.

AKCNOWLEDGEMENT

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2013R1A1A2011351) and a grant (13SCIPA01) from Smart Civil Infrastructure Research Program funded by Ministry of Land, Infrastructure and Transport (MOLIT) of Korea government and Korea Agency for Infrastructure Technology Advancement (KAIA).

REFERENCES

1. Catbas, F. N., Kijewski-Correa, T. L., and Aktan, A. E., eds. (2013). Structural Identification (St-Id) of Constructed Systems: Approaches, Methods and Technologies for Effective Practice of St-Id, ASCE, Structural Engineering Institute, Reston, VA.
2. Jones, D. R. (2001). A Taxonomy of Global Optimization Methods Based on Response Surfaces. *Journal of Global Optimization*. **21:4**, 345-383.
3. Jin, S. S., and Jung, H. J. (2015). Sequential Sampling Method for Surrogate Modelling of Time-consuming Finite Element Analysis. *The Seventh International Conference on Structural Health Monitoring of Intelligent Infrastructure*, Torino, Italy.
4. Jin, S. S., and Jung, H. J. (2015). Sequential Sampling Strategy with Kriging Model for Surrogate-based Finite Element Model Updating. *Computer-Aided Civil and Infrastructure Engineering*. under review.
5. Krige, D. G. (1994). A Statistical Approach to Some Basic Mine Valuation Problems on the Witwatersrand. *Journal of the South African Institute of Mining and Metallurgy*. **94:3**, 95-111.