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Reference-free Deflection Measurement of General-shape Bridges using Data Fusion

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ABSTRACT

Recently, an indirect displacement estimation method using data fusion of acceleration and strain (i.e., acceleration-strain-based method) has been developed. Though the method showed good performance on beam-like structures, it has inherent limitation in applying to more general types of bridges that may have complex shapes, because it uses assumed analytical (sinusoidal) mode shapes to map the measured strain into displacement. This paper proposes an improved displacement estimation method that can be applied to more general types of bridges by building the mapping using the finite element model of the structure rather than using the assumed sinusoidal mode shapes. The performance of the proposed method is evaluated by numerical simulation on a deck arch bridge model whose mode shapes are difficult to express as analytical functions. The displacements are estimated by acceleration-based method, strain-based method, acceleration-strain-based method, and the improved method. Then the results are compared with the exact displacement. The proposed method is found to provide the best estimate for dynamic displacements in the comparison, showing good agreement with the measurements as well.

KEYWORDS: *displacement, bridge, data fusion, finite element model, modal mapping*

1. INTRODUCTION

Structural health monitoring (SHM) is an essential procedure to ensure the sustainability of civil infrastructure. For the SHM, a wide variety of responses, such as acceleration, displacement, strain, and inclination, are utilized depending on structural types and features to be monitored. Despite of its intuitive feature that directly results from the external loads to the structure, displacement has been relatively less used due to two possible causes: inconvenience in its measurement and expensive cost of measurement devices. Especially, the usage of the traditional contact-type transducers, such as a linear variable differential transformer (LVDT) and a ring-type transducer, was significantly limited at the bridges due to unavailability of fixed reference point of the displacement measurement. To take the hurdle, noncontact-type devices, such as the global positioning system (GPS) and the laser Doppler vibrometer (LDV), have been emerged as alternatives (Nassif *et al*. 2005, Jo *et al*. 2013). However, high cost of such noncontact devices up to a few ten thousand dollars per sensing channel limits their real-world applications that may require a dense sensor topology.

Instead of using the noncontact-type devices, there have been research efforts to use other responses that can be converted into the displacement. Acceleration and strain are the most popular responses used to the end. Especially, Park *et al*. (2013) proposed a displacement estimation method using data fusion of acceleration and strain by extending the acceleration-based method proposed by Lee *et al*. (2010). In the regularization term, the displacement converted from strain data by the modal mapping is used to prevent the signal drift. For the modal mapping, the assumed analytical (sinusoidal) mode shapes proposed by Shin *et al*. (2012) are employed. The method by Park et al. (2013), however, inherits a limitation in application to more general types of bridges with complex shapes, such as arch and truss bridges, since the method uses assumed sinusoidal mode shapes which may be reasonably obtained only for the girder bridges.

This study proposes a reference-free method to measure the displacement using data fusion of acceleration and strain for general bridge structures. The proposed method is to extend the method by Park *et al*. (2013) to general types of bridges without use of the assumed mode shapes. To the end, the mode shapes for the modal mapping are obtained from the FE model of a structure instead of assumed sinusoidal mode shapes. The performance of the proposed method is evaluated by numerical simulations on a deck arch bridge model whose mode shapes are hard to be assumed as sinusoidal functions. The displacements are estimated by acceleration-based method, strain-based method, fusion-based method, and the improved method, and the results are compared with exact displacements to demonstrate the performance of the proposed method. From the comparison of displacements estimated by the four methods to the reference values measured by laser displacement meters, the accuracy of the proposed method has been investigated.

2. REFERENCE-FREE DEFLECTION MEASUREMENT METHOD

This section describes the principles of the displacement estimation method proposed by Park *et al*. (2013) and the modification made in the proposed method.

2.1 Acceleration-strain-based displacement estimation method

Park *et al*. (2013) have proposed the displacement estimation method by fusing the acceleration and strain. The method uses the basic form of the acceleration-based method proposed by Lee *et al*. (2010), while the regularization term is replaced by the difference between estimated displacements and displacement estimated from the strain by modal mapping method. The method can be formulated for displacement u_i at the location of x_i as:

$$\underset{u_i}{\mathrm{Min}}\,\Pi = \frac{1}{2}\left\| L_a (L_c u_i - (\Delta t)^2 \overline{a}_i) \right\|_2^2 + \frac{\lambda^2}{2}\left\| u_i - D_i \overline{\varepsilon} \right\|_2^2 \tag{1}$$

where u_i and $\overline{a}_i$ are the estimated displacement and measured acceleration at the location x_i; $\overline{\varepsilon}$ is the measured strain; Δt is the time step; L_a is a diagonal weighting matrix having the first and last entries as $1/\sqrt{2}$ and the other entries as 1; L_c is the second-order differential operator matrix of the discretized trapezoidal rule (Atkinson 2008); $\|\cdot\|_2$ is 2-norm of a vector; λ is a regularization factor; and D_i is the i th row of modal mapping matrix D that converts strain into displacement as:

$$u = D\varepsilon \tag{2}$$

The modal mapping matrix can be calculated as:

$$D = \Phi\Psi^{\dagger} \tag{3}$$

where Φ and Ψ denote mode shapes and strain mode shapes, respectively; the superscript $\dagger$ denotes the pseudo-inverse. λ is defined by Lee *et al*. (2010) as

$$\lambda = 46.81 N_d^{-1.95} \tag{4}$$

where N_d is the number of acceleration data to be converted into displacements. The solution of Eq. 1 can be expressed as

$$\begin{aligned} u_i &= (L^T L + \lambda^2 I)^{-1} (L^T L_a \overline{a}_i \Delta t^2 + \lambda^2 D_i \overline{\varepsilon}) \\ &= \begin{pmatrix} C_a \Delta t^2 & C_\varepsilon \end{pmatrix} \begin{pmatrix} \overline{a}_i \\ \overline{\varepsilon} \end{pmatrix} \end{aligned} \tag{5}$$

where $C_\varepsilon = (L^T L + \lambda^2 I)^{-1} \lambda^2 D_i$.

The mode shapes and strain mode shapes may be directly estimated from measurements, which would be very expensive. Instead, Park *et al*. (2013) employed assumed sinusoidal mode shapes and corresponding strain mode shapes, proposed by Shin *et al*. (2012).

In this paper, an improved method is proposed by employing a modal mapping matrix derived from an FE model of the structure as (Foss and Hauge 1995):

$$D = \Phi^{FE}(\Psi^{FE})^{\dagger} \tag{6}$$

where Φ^{FE} and Ψ^{FE} are the mode shapes and the strain mode shapes obtained from the FE model. Note that, the number of used modes must be smaller than the number of strain measurements to avoid the under-determined modal mapping matrix.

The accuracy of the estimated displacement can be quantified by employing a percentage RMSD (root mean square deviation) as:

$$RMSD\ (\%) = \sqrt{\sum_{i=1}^{N}(u_i^{est} - u_i^{ref})^2} \Bigg/ \sqrt{\sum_{i=1}^{N}(u_i^{ref})^2} \times 100 \tag{7}$$

where u_i^{est} and u_i^{ref} are the estimated and reference displacements, respectively; and N denotes the number of data samples.

3. DECK-ARCH BRIDGE MODEL

The example used in this study is a 2D open-spandrel deck arch bridge model shown in Figure 1. The model has a deck which locates above the arch and the deck is supported by a number of vertical columns rising from the arch. The Rainbow Bridge at Niagara Falls and the Cold Spring Canyon Arch Bridge are the famous examples of the deck arch bridges.

The model is composed of 34 members: 12 deck members, 12 arch members, and 10 vertical columns. All members are modelled as frame elements. N# and A# denote the nodes and supports on the deck, respectively. The span length of the bridge is 120 m, and the height of the arch is 20 m. The sectional properties of members for deck, arch, and vertical columns are shown in Table 1.

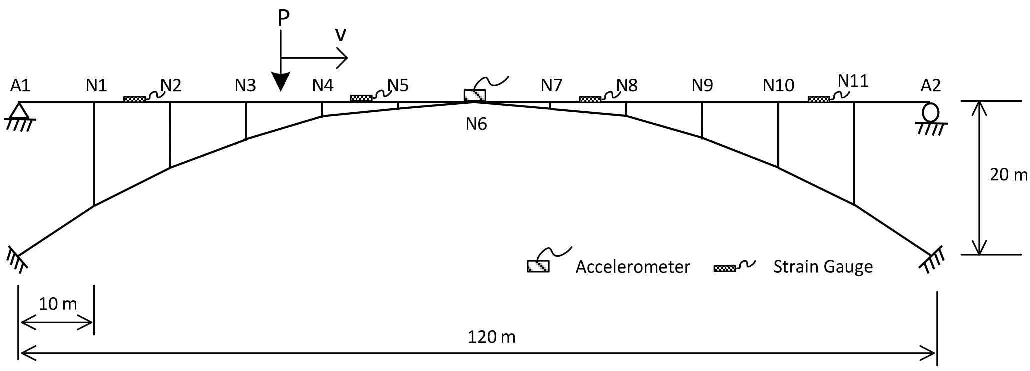


Figure 1 Deck arch bridge model with sensor topology.

Table 1. Structural properties of deck arch bridge model.

Members	Deck	Arch	Vertical Column
Sectional area	0.656 m^2	0.280 m^2	0.167 m^2
2^{nd} moment of inertia	1.453×10^{-1} m^4	3.087×10^{-1} m^4	6.535×10^{-2} m^4
Elastic modulus	200 GPa		
Mass density	7850 kg/m^3		

The displacement, acceleration, and strain of the beam are simulated using MATLAB Simulink. A vertical load moving from left to right of the deck with a constant speed (v=10 m/s), shown in Figure 2, is employed to generate non-zero mean displacements. The load is the combination of a moving static load of 43.2 ton (DB24 truck load specified in Korean highway bridge design code) and zero-mean Gaussian random load with a standard deviation of 13 ton simulating dynamic loading effect. Acceleration is assumed to be measured at N6, while strains on the deck are obtained at the mid spans of four deck members between N1-N2, N4-N5, N7-N8, and N10-N11. The simulated acceleration and strains are made artificially contaminated by adding 5 % noise in RMS (root mean square) to emulate the practical measurement. The displacement simulated at N6 is used as the reference to evaluate the accuracy of estimated displacements. Note that acceleration and displacement are obtained in the vertical direction, while the strains are obtained on the bottom surfaces of the deck in the longitudinal direction to capture the bending strain.

Since four strain data are available in this example, the first four modes are employed to build the modal mapping relationship. Figure 3 shows the first four mode shapes of the FE model, compared with the sinusoidal shapes based on the assumption of a simply supported prismatic beam. The visual comparison clearly shows the difference between the two types of mode shapes, particularly for the first and third mode shapes near the supports. Their MAC (modal assurance criterion) values are 0.718, 0.936, 0.651, and 0.988, respectively. Thus, it can be expected that the displacement estimated near the supports may have considerable error when the assumed modes are used.

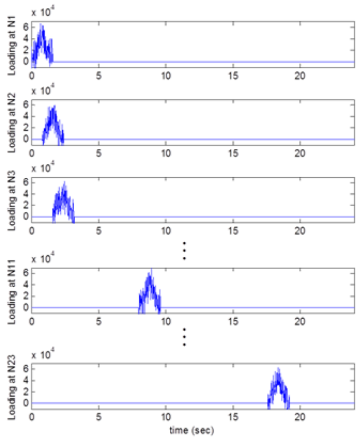


Figure 2 Vertical moving load.

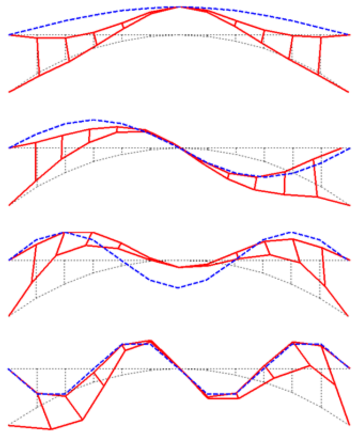


Figure 3 First four mode shapes of FE model (solid lines) compared with assumed ones (dashed lines).

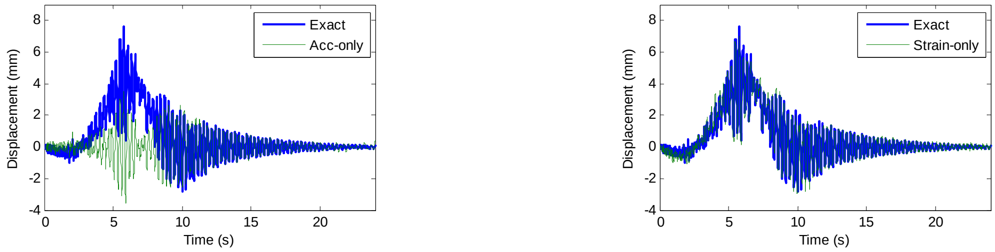
4. COMPARISON OF ESTIMATED DISPLACEMENT

The improved method is validated from numerical simulations carried out on the example bridge model. The displacements excited by a moving load are estimated by four methods: i.e., acceleration-based method (Lee *et al*. 2010), strain-based method (Kang *et al*. 2007), acceleration-strain-based method (Park *et al*. 2013), and the improved acceleration-strain-based method, and the results are compared for the validation.

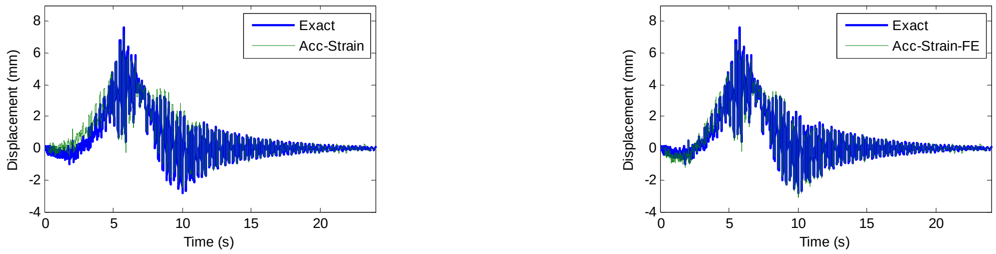
4.1.1 Comparison of displacements at N6

Figure 4 shows the comparison of the displacements estimated by four methods with exact one simulated from the MATLAB Simulink. The acceleration-based method cannot estimate the nonzero-mean pseudo-static displacement component as shown in Figure 4(a). The strain-based method can somewhat estimate the static component as shown in Figure 4(b), while the dynamic component cannot be estimated accurately. The acceleration-strain-based method gives an incorrect the displacement due to the incorrect modal mapping as in Figure 4(c). Meanwhile, the improved acceleration-strain-based method estimates very accurate displacement overlapped with the exact one as in Figure 4(d), despite of the complexity of the deck arch model. This clarifies the performance of the improved method for a complex structure whose mode shapes may not be easily assumed as analytical functions.

The accuracy of the estimated displacements can be investigated in the different aspects by looking at the frequency domain. Figure 5 shows the power spectral density (PSD) of the estimated displacements compared with that of exact one. Figs. 5(a) and 5(b) show errors of the acceleration-based and the strain-based methods in low and high frequency range, respectively. The acceleration-strain-based method shows slightly larger error in estimating the pseudo-static components near 0Hz than the improved method (Figs. 5(c) and 5(d)).

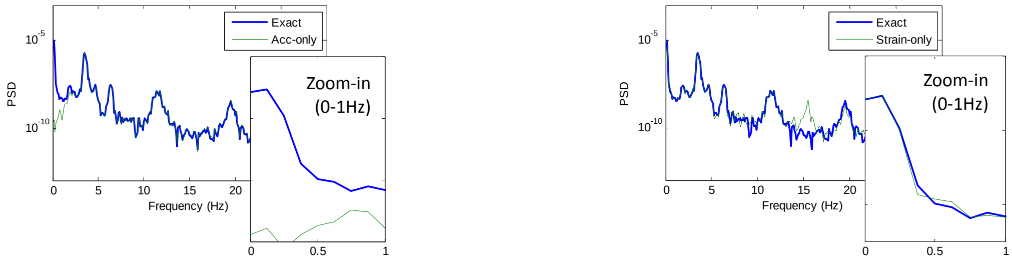


(a) Acceleration-based method.
(b) Strain-based method.



(c) Acceleration-strain-based method.
(d) Improved acceleration-strain-based method.

Figure 4 Comparison of displacements estimated at N6 for deck arch bridge model by four different estimation methods with exact one.



(a) Acceleration-based method.
(b) Strain-based method.

(c) Acceleration-strain-based method.
(d) Improved acceleration-strain-based method.

Figure 5 Comparison of PSD of displacements estimated at N6 for deck arch model by four different estimation methods with exact one.

The accuracy of the estimated displacements by the four methods is quantified using the percentage RMSD described in Eq. 7. Figure 6 shows that the acceleration-based method provides the largest RMSD of 85.9 % at N6. The strain-based method yields small error of 17.7 %, since the method uses accurate modal mapping using the FE mode shapes. The acceleration-strain-based method gives a larger RMSD value of 20.5 % than the strain-based method, which means the error in the assumed mode shapes may bring significant error in the estimation due to incorrect modal mapping. The improved method has the smallest RMSD value of 10.6 % owing to accurate modal mapping using the FE model. The estimated RMSD values quantitatively show the performance of the improved method compared with the other existing methods for the general types of bridge structures.

4.1.2 Comparison of displacements at other locations

Figure 6 shows the RMSD values of the displacements estimated by four methods at the left half of the deck: N1-N6. All methods except the acceleration-based method show increasing RMSD values as the location of the estimation gets closer to the left support due to smaller amplitude of displacement. In addition, the incorrect modal mapping near the support significantly increases the error of the acceleration-strain-based method. The improved method has the smallest RMSD values for all points.

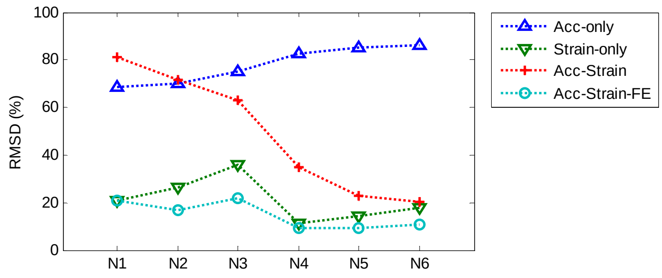


Figure 6 RMSD of displacements at N1-N6.

4.1.3 Effect of FE model inaccuracy

The FE model may be subjected to modelling errors, which may cause estimation errors in the modal properties. To see the effect of the inaccurate FE model to the estimation, the FE model is perturbed by introducing element-level errors in the elastic modulus. Three types of perturbations with 11.5, 23.1, and 34.6 % in RMS (i.e., uniform perturbation in the range of ±20%, ±40%, and ±60% of the initial value) are considered. Figure 7 shows the first four mode shapes of a perturbed FE model with 34.6 % errors in RMS in comparison with those from the original FE model. Figure 7 shows that the perturbation caused considerable discrepancy particularly on the higher mode shapes to be used for the modal mapping.

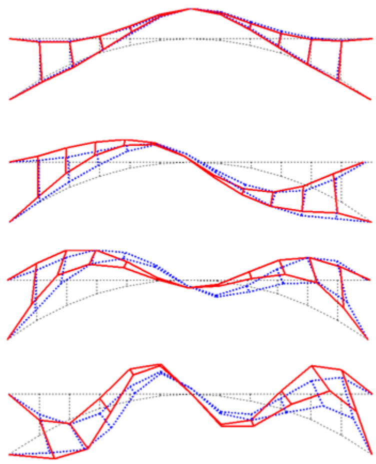


Figure 7 First four mode shapes of perturbed FE model with 11.0 % errors in RMS (dotted lines) compared with those from original model (solid lines).

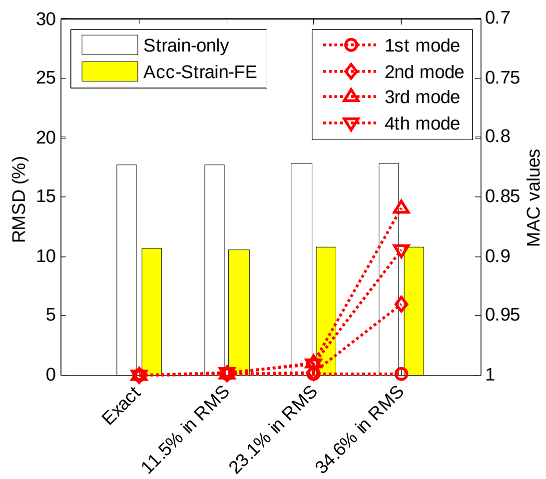


Figure 8 RMSD of displacements estimated at N5 for perturbed deck arch FE models.

Figure 8 shows the RMSDs of the estimated displacements at N6 by the strain-based and the improved methods when the perturbed FE models are used for the modal mapping. The MAC values of the perturbed models are plotted together to show the effect of perturbation to the mode shapes. By the incremental perturbation, the mode shapes are found to change incrementally. However, even with the significant perturbation, the RMSD of the estimated displacement rarely changes for both the strain-based and the improved methods. In the case of with perturbation of 34.6 % in RMS, the RMSD by the improved method is 10.8 %, which is much smaller than 20.5 % by the acceleration-strain-based method in Section 4.1.1. When the exact FE model was used, the RMSDs are 10.6 and 17.7 % for the stain-based and the improved methods, respectively. The slight increase by the large perturbation is because the RMS is significantly affected by accuracy of the low frequency components, as shown in the result of the acceleration-based method. The MAC values of Figure 8 shows that the large perturbation makes bigger change as the mode order increases, and the first mode which significantly affects the RMSDs is barely changed. The inaccuracy in higher modes resulted in inaccurate estimation of dynamic displacements that derives the slight increase of the RMSDs. This illustrates the improved method using the mode shapes of the FE model is very effective when the structure is complex, even with a somewhat inaccurate FE model.

4.1.4 Measurement of strains at load carrying members

In the case of the deck arch bridge, the arch and vertical columns are the major load carrying members to resist the dead loads that comprise majority of the total load applied to the structure, while the deck is designed to transfer the live load to the arch system. Considering the case with strain gauges on the members other than deck, another sensor topology shown in Figure 9 is considered. Figure 10 shows the displacement estimated at N6 using the improved method. The result is very close to the exact one with 9.69 % in RMSD, which is smaller than the value of 10.6 % by the strain measurements on the deck described in Section 4.1.1.

The mode shapes cannot be assumed in an analytical form for the whole structure when the structure has a complex shape. Therefore, the acceleration-strain-based method using the assumed sinusoidal mode shapes is applicable only when the strain gauges are on the deck. This example shows that the proposed improved method based on the FE model has big advantage when the strain sensors are placed on non-deck members.

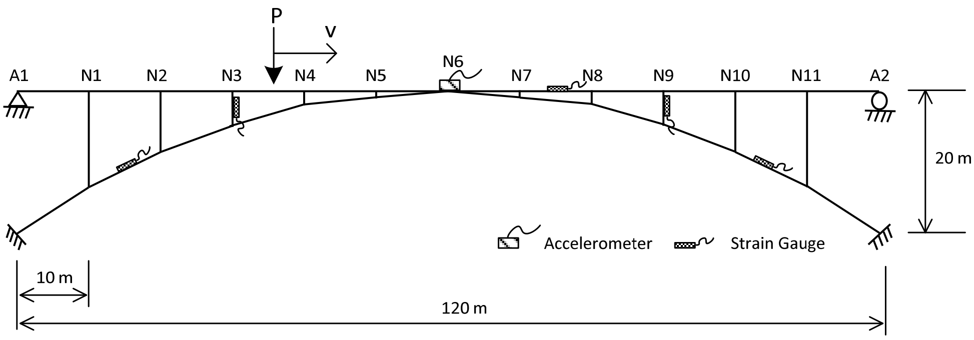


Figure 9 Topology for strain measurement at load carrying members.

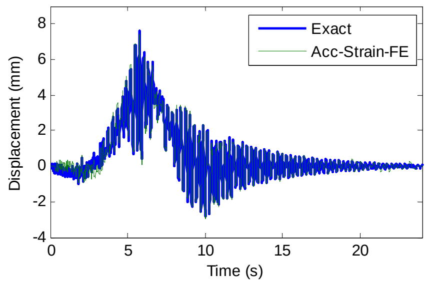


Figure 10 Displacement estimated at N6 using strain measurement on load carrying members.

5. CONCLUSIONS

In this paper, an improved displacement estimation method based on data fusion of acceleration and strain has been proposed for the application to general types of bridge structures whose mode shapes may not be assumed in analytical (e.g., sinusoidal) function. The improvement has been made by employing the mode shapes from an FE model in the modal mapping of strain to displacement. The performance of the improved method has been verified by numerical simulation on a deck arch bridge model with complex shape. The estimated displacements by four methods, acceleration-based method, strain-based method, acceleration-strain-based method, and improved method, have been compared. The result of this study can be summarized as:

(1) In the numerical simulations on the deck arch model, the proposed method estimated displacements with better accuracy than the other methods at all locations of the structure owing to the accurate modal mapping using the FE model.

(2) The perturbation of the FE model has increased the inaccuracy of the improved method. However, at the center locations, large perturbation (34.6 % in RMS) resulted in the RMSD errors of 10.8 %, which were less than 20.5 % by the acceleration-strain-based method using assumed sinusoidal mode shapes for the deck, which shows the effectiveness of the proposed method with somewhat inaccurate FE model.

(3) The proposed method estimated the displacement equivalently well using the strain data on non-beam type members such as truss and arch, which shows its good compatibility of the improved method to more general types of structures.

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