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Dynamic Similitude Mechanism and Design Approaches of the Base-isolated Structures

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ABSTRACT

Dynamic similitude design approach for the small-scaled model via shaking table test is of importance to investigate the seismic behavior of prototype structure. To study the dynamic mechanism of base-isolated structure, a simplified dynamic analysis model with isolation layer for the structure was analyzed, the corresponding similitude relationship was put forward, and structural effect-based similitude approach for base-isolated structures was proposed. Different from the traditional similitude method, this structural effect-based similitude approach focuses on the similitude of structural properties and corresponding structural responses, which are either linear or nonlinear, rather than material properties in elastic stage. To verify the effectiveness of this approach, a quarter-scaled model was designed and tested on shaking table. The experimental results are proved to be very effective for predicting dynamic responses of the prototype base-isolated structure.

KEYWORDS: *base-isolated; model test; similitude approach; transfer layer; isolation layer*

1. INTRODUCTIONS

The seismic base isolation is becoming a more familiar technology to the protection of whether new designed or existing buildings since 1980s[1]. Different from the traditional earthquake-resistance methods, in which the whole structure are designed to be exposed directly to the earthquake ground motions, the role of the base isolator under superstructure is to isolate the superstructure from the horizontal and vertical components of the earthquake ground motions[2]. The design methods of base isolation systems have been gradually improved as more efforts have been taken ages in this field. Experimental studies, especially shaking table tests, have always been playing an important role in the improvement of the base-isolated structures.

Base-isolated structural models on shaking tables could be assorted into two types, taking consideration on whether there are prototypes or not[3]~[6]. Models without a prototype refer to the ones whose superstructure is scaled down from the prototype, whereas the isolation layer is not. A model with a prototype means that both its superstructure and its base isolation layer are strictly designed from the prototype based on the similitude law [7]. Till present, researches of shaking table tests for base-isolated structures were mainly focus on the latter one, in other words, less researches were involved the similitude design of isolation layer.

Generally, Base isolation structure consists of three parts, i.e. the superstructure, transfer layer, and the isolation layer. Being subjected to various seismic earthquake inputs, the superstructure and the transfer layer are generally in the elastic state or slightly damaged while isolation layer is in elastic or elastic-plastic state. Therefore, similitude design of the base-isolated structure should focus on the design of the isolation layer. According to the traditional similitude design method, base isolator in the model should keep geometrically similar to that in the prototype and should be place in the same position. However, small-sized base isolator is difficult to reproduce in practice, and it proved be more difficult to produce small-sized base isolator with the same material to a certain prototype. Therefore, in the present cases of base-isolated structures shaking table tests, further study are desiderated to provide practical similitude design approaches for the similarity of the prototype, and guidance for the interpretation of the testing results from the scale model to the prototype.

2. SIMILITUDE MECHANISM OF BASE-ISOLATED STRUCTURES

Physical quantities of the structure models concerned in shaking table tests are shown in Table 2.1. In the present case of base-isolated models, those physical quantities of the isolation layer are most concerned.

In the similitude design of base-isolated structures, as shown in Fig. 1.1, the prototype structure is usually treated as three parts, i.e. the isolation layer, transition layer and the superstructure, to be designed. Taking the fabrication cost of the bearings into consideration, the similitude factors of the stiffness in the base isolation layer can be satisfied in priority and other similitude factors can be finally determined through the coordination of various parts.

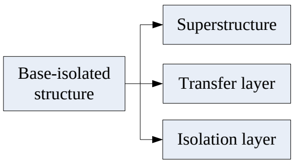


Figure 1.1 Process diagram of similitude design for base-isolated structure

2.1 Similitude Design of the Isolation Layer

Bilinear model is used to simulate the restoring force characteristics of the isolation layer of the model, because related research [8] showed that it could effectively simulate the base isolation system in the nonlinear response characteristics under different level earthquake action. Indicators such as initial stiffness, K_0, yield force, F_y and yield stiffness, K_d are included. To realize the similitude, the isolation layer restoring force curve of the model and that of the prototype should meet the similitude requirement shown in Fig. 2.1, in which the curves should keep being similar to each other during the whole loading process.

The superstructure of a base-isolated structure can be idealized as a i-degree-of-freedom (DOF) system with one horizontal DOF for each other. The isolation layer can be idealized as a 6 DOF system with three transverse DOF and three torsion DOF.

Bearings in base isolation system are easy to be in tension due to the existence of horizontal and vertical rotation of the superstructure as shown in Fig. 3, and it would be serious when the superstructure is in high aspect ratio or irregular plane layout [8]. Assuming the transfer layer above the isolation layer to be a rigid body, and six kinds of stiffness, i.e. three Transverseal stiffness, K_X,K_Y, K_v and three rotational stiffness, K_{eX}, K_{eY}, K_{ev} are included.

Table 2.1 Measuring parameters of tests for base-isolated structures

Measuring parameters	Parameters
Dynamic characteristics	Frequency,Period,Damping ratio
Seismic response	Maximum acceleration,Maximum displacement,Stress in key point
Restoring fore characteristics	Hysteretic curve

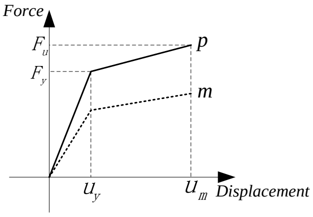


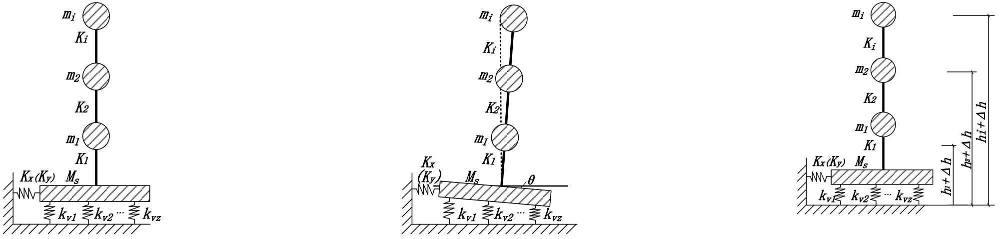
Figure 2.1 Force-displacement curves of the base isolation layer of the model and the prototype

Structural calculation model is simplified as shown in Fig. 2.2 to take the complexity of multidimensional and the practical requirement of preliminary calculation into consideration [9], in which M_s stands for equivalent mass of the transfer layer, m_1~m_i represents equivalent mass of the superstructure from first layer to i layer, K_1~K_i represents equivalent stiffness of the superstructure from first floor to ith floor.

Transverseal stiffness of the isolation layer are mainly determined by the number and properties of the bearings in it, whereas rotational stiffness are also influenced by the arrangement of the bearings besides to a large extent. As shown in Fig. 2.2(a), taking the X direction as the example, the relationship between the Transverseal stiffness and the rotational stiffness is

$$K_{\theta x} = \left\{ \sum_{j=1}^{z} (j - Z/2 - 1/2)^2 / (Z-1)^2 \right\} B^2 K_v \tag{2.1}$$

Where Z is the number of bearings rows and B is the width of the transfer layer.



a) b) c)

Figure 2.2 Diagram of the numerical model for base-isolated structures

As seen in Equation (1), the rotational stiffness of the isolation layer can be justified by changing the width of the transfer layer and the arrangement of the bearings if the characteristics and number of the bearings are unchangeable in the similitude design of the base-isolated structures.

2.2 Similitude Design of the Transfer Layer

Since the isolation layer is designed as a whole, the aspects ratios and numbers of the single bearing is not necessary to be exactly compatible to its counterpart in prototype. New problems in transfer layer are brought by this convenience in isolation layer: (1) beams in transfer layer may have insufficient stiffness; (2) overturning may happen to the model during the testing because of the increased height and decreased number of the bearings. As two present solutions to these two problems, the height of the transfer layer is increased and additional beams are designed in the model, which will apparently lead to geometrical distortion in similitude.

Actual height of the transfer beam, h_A^m, of the model is assumed to be larger than that of the theoretical height, h_T^m, by Δh, then the height of the mass point one to mass point i are

$$h_{1,A}^m = h_{1,\mathrm{T}}^m + \Delta h \tag{2.2}$$

$$h_{2,\mathrm{A}}^m = h_{2,\mathrm{T}}^m + \Delta h \tag{2.3}$$

$$h_{i,\mathrm{A}}^m = h_{i,\mathrm{T}}^m + \Delta h \tag{2.4}$$

Seismic response of the model will be amplified under the same earthquake excitation with the incensement of the height, which will lead to deviation in the testing results. At present, this deviation is either been ignored or been modified in the conversion of the model data to the prototype data. This paper argues that similitude of the structural property and dynamic response can be satisfied by adjustment of the parameters in isolation layer as the size, stiffness and mass of the transfer layer are changed.

2.3 Similitude Design of the Superstructure

Superstructure of the base-isolated structure can be scaled down by available traditional similitude design method. Natural frequency and mode shape should be guaranteed in priority to other physical quantities as traditional similitude design method is difficult to be put into application due to the small proportion and limited test conditions. Taking the masonry superstructure as an example, seismic action and shear bearing capacity of the model should be similar to that of the prototype in this case.

$$\frac{F_E^p}{R_v^p} = \frac{F_E^m}{R_v^m} \tag{2.5}$$

$$\frac{F_E^p}{R_v^p} = \frac{F_E^m}{R_v^m} \Rightarrow \frac{m^p a^p}{f_{vE}^p A^p} = \frac{m^m a^m}{f_{vE}^m A^m} \Rightarrow \frac{a^m}{a^p} = \frac{m^p f_{vE}^m A^m}{m^m f_{vE}^p A^p} \tag{2.6}$$

In which, F_E^p, F_E^m means seismic action of the prototype and the model; R_v^p, R_v^m means shear bearing capacity of the prototype and the model; m^p, m^m means mass of the prototype and the model; f_{vE}^p, f_{vE}^m means seismic shear strength of the prototype and the model; a^p, a^m means acceleration of the prototype and the model. Also, for other structure types, the similitude design should ensure the key physical parameters in priority to conform to similitude law.

2.4 Expression of Similitude Approach for Base-isolated Structures

According to the analysis above in combination with similitude relationship in structural level, geometry size, stiffness of isolation layer, yield force of isolation layer, mass of the whole structure, acceleration are taken as the five governing factors and other parameters can be obtained by dimensional analysis method as shown in Table 2.2. S_L is determined according to the test conditions such as the size and capacity of the shaking table. Stiffness of isolation layer is a combination of stiffness of all bearings in it, so the numbers and specific arrangement of the bearings in the model are not necessary to be exactly the same to that of the prototype, which lead to more flexibility in choosing bearings and reduce the difficulties of actual model design work.

S_F is been determined based on the force-displacement relationship obtained from the preliminary quasi-static test on the base bearings. S_m is obtained based on the consideration of both bearing capacity of available shaking table and the actual mass of the global model. S_a is determined based on the consideration of the resemblance constant of geometry size, testing goals and the bearing capacity of the shaking table.

Since these five governing factors are difficult be satisfy the similitude requirements, the resemblance constants of the superstructure should be obtained in priority, then the resemblance constants of geometry size, stiffness and force of the transfer layer are obtained thereafter.Also, resemblance constants of geometry size and mass of the isolation layer are finally determined after the analysis of the size, number, and arrangement of the bearings.

Table 2.2 Similitude relationship for base-isolated structures during the whole process

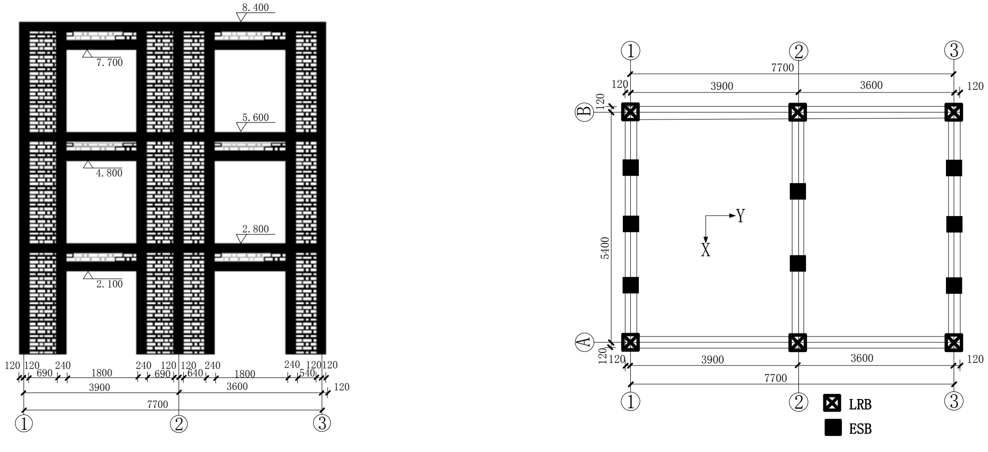
	Parameter	Formula of Similitude	Similarity Constant	Isolation Layer	Transfer Layer	Super -structure
Structural Property	Geometry size L	S_L	0.25	×	×	√
	Stiffness K	S_K	0.1159	√	√	
	Mass m_1	S_m	0.029	×	×	
	Force F	S_F	0.029	√	×	
	Deformation y	$S_y=S_F/S_k$	0.25	√	√	
Equivalent Material Property	Equivalent elastic modulus E	$S_E=S_k/S_L$	0.4636	√	√	
	Equivalent stress σ	$S_\sigma = S_F / S_L^2$	0.464	√	√	
	Equivalent mass density ρ	$S_\rho = S_m / S_L^3$	1.856	√	√	
Dynamic Property	Time, Period t	$S_t=(S_m/S_K)^{0.5}$	0.5	√	√	
	Acceleration a	$S_a=S_F/S_m$	1	×	√	
	Gravity acceleration g	$S_g=1$	1	√	√	

Remark：√ means satisfying the resemblance constant, and × means not satisfying the resemblance constant.

3. SIMILITUDE DESIGN EXAMPLE OF BASE-ISOLATED STRUCTURE

3.1 Overview of the Prototype

The dimension of the three-storey masonry prototype is given in Fig. 1. Each storey is 2.8m high. It is 7740mm by 5400mm in plane. The 240mm thick walls are made from fired perforated bricks of *Mu10*, whereas the mortar is *M10.* Plane and elevation layout of this building is shown as Fig. 3.2. The superstructure of the base-isolated case rests on 18 bearings, i.e. 6 laminated rubber bearings (LRB) and 12 elastic sliding bearings (ESB), and the arrangement and their details are shown in Fig. 3.1(b) and Table 3.1.



（a）Superstructure （b）Plan view of isolator layer

Figure 3.1 The plan view and elevation of the base-isolated prototype

3.2 Similitude design of the model

3.2.1 Superstructure

As the primary goal of the test is to verify the effectiveness of the isolation layer in horizontal and vertical direction, so acceleration distortion in both directions should be avoided, i.e. S_g=1, S_a=1. Taking the vertical carrying capacity of the shaker and specification of the masonry structure into account, resemblance constant of geometry size is obtained as 1/4. The global mass of the prototype is 690t, and taking the mass of the model to be 20t can meet the requirement of equation (6), so the resemblance of the mass is 0.029.

3.2.2 Isolation layer

Taking the equivalent horizontal stiffness, K_x, K_y, as the governing factors, and 8 bearings, i.e. 4 elastic sliding bearings (ESB) and 4 laminated rubber bearings (LRB), are designed and fabricated as shown in Fig. 3.2. Details of the bearings are shown in Table 3.2. Resemblance constants of horizontal stiffness, S_K, and sliding force S_F of the isolation layer are obtained as 0.1159 and 0.029. Since governing factors, S_L, S_a ,S_m, S_K, S_F are known, other parameters can be obtained by dimensional analysis method as shown in Table 2.2.

3.2.3 Transfer layer

Transfer layer, which is located in the middle of the isolation layer and the superstructure, is composed of transfer beams shown in Fig. 3.2, which are 1200mm×600mm in cross section. To guarantee the stiffness of the transfer layer and to cooperate with rotational stiffness adjustment, the model transfer layer is not designed strictly based on resemblance constant of geometry size and the transfer beams are designed as shown in Fig. 3.2. The final fabricated model is shown in Fig. 3.3.

Table 3.1 Parameters of the isolators of the prototype

Type	Horizontal stiffness （*kN*/mm）	Vertical static stiffness （*kN*/mm）	Horizontal sliding force （*kN*）
ESB	0.311	378.67	34.42
LRB	0.143	4.2	---

Table 3.2 Parameters of the isolators of the model

Type	Horizontal stiffness （*kN*/mm）	Vertical static stiffness （*kN*/mm）	Horizontal sliding force （*kN*）
ESB	0.108	56.00	3.675
LRB	0.025	0.087	---

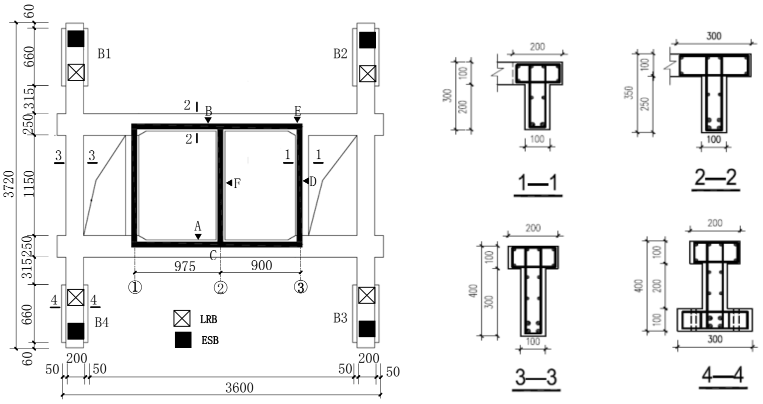


Figure 3.2 Arrangement of the isolator of the model

Figure 3.3 The model

4. SIMILITUDE EFFECTIVENESS EVALUATION

Model masonry structure was made of small bricks, which are cut from normal *MU10* bricks, and *M2.5* mortar. Concrete used in construction columns are simulated with mortar in model and reinforced bar is replace with galvanized iron wire. To carry additional mass, the thickness of the slab is designed to be 40mm instead of 27.5mm, which is the value strictly calculated by similitude law.

The model test was carried out on the 6 DOF shaking table system in the State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University. The table size is 4m×4m, and the maximum payload is 250kN. The maximum accelerations are 4.0g horizontally and 2.0g vertically. Maximum strokes of actuators are ±100mm in X direction and ±50mm in Y and Z direction, whereas piston velocities are 1000mm/s in X direction and 600mm/s in Y and Z direction. The sensors for the model structure consisted of 18 accelerometers, 10 displacement transducers and 8 3-D load cells, which were placed at the isolation layer, and different floor levels, as shown in Table4.1 and Fig. 3.2.

Table 4.1 Sensor layout

Sensor Type	No.	Floor	Direction	Position	Sensor Type	No.	Floor	Direction	Position
Accelerometer	A1	Roof	X	A	Accelerometer	A10	First floor	X	A
	A2	Roof	Y	A		A11	First floor	Y	A
	A3	Roof	Z	A		A12	First floor	Z	A
	A4	Third floor	X	A		A13	Isolation layer	Z	S3
	A5	Third floor	Y	A		A14	Isolation layer	Z	S2
	A6	Third floor	Z	A		A15	Isolation layer	Z	S4
	A7	Second floor	X	A		A16	Isolation layer	Z	S1
	A8	Second floor	Y	A		A17	Second floor	X	C

	A9	Second floor	Z	A		A18	Third floor	X	C
Sensor Type	No.	Floor	Direction	Position	Sensor Type	No.	Floor	Direction	Position
Displacement Transducer	D1	Roof	X	B	Position transducer	D10	Second floor	Y	D
	D2	Third floor	X	B	Force transducer	F1	Isolation layer	X,Y,Z	S1Left
	D3	Second floor	X	B		F2		X,Y,Z	S1Right
	D4	First floor	X	B		F3		X,Y,Z	S2Left
	D5	First floor	Y	D		F4		X,Y,Z	S2Right
	D6	First floor	X	E		F5		X,Y,Z	S3Left
	D7	Isolation layer	Z	F		F6		X,Y,Z	S3Right
	D8	Roof	Y	D		F7		X,Y,Z	S4Left

Five various seismic ground motions with different frequency contents, i.e. El-Centro waves, Wenchuan waves, Pasadena waves, Shanghai waves and Eastern Japan earthquake waves, were applied to excite the model on the table. Each wave was multiplied by time duration and amplitude constants shown in Table 2.2 to satisfy the similitude requirements for the quarter-scale model.

The effectiveness of the similitude design method was evaluated by comparing the structural responses of the testing results and numerical results. The finite element model consists of isolation layer and the superstructure, in which masonry wall and concrete slab used SHELL element, concrete beams and columns used BEAM elements and isolation layer used COMBIN elements [9].

4.1 Structural Dynamic Property

Table 4.2 shows the comparison of the natural frequencies and mode shape of model structure and prototype. The results demonstrated the significant similarity of the first two natural frequencies and mode shapes, but less simulation in the following frequencies.

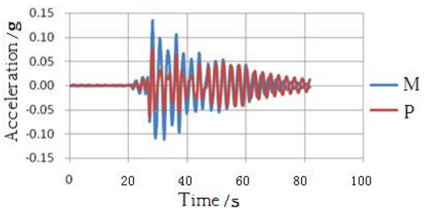
Table 4.2 Frequency and mode of the base-isolated masonry structure

Order	Natural frequency (*Hz*)		Mode shape	
	FEM Analysis	Model Tests	FEM Analysis	Model Tests
1	0.37	0.39	X	X
2	0.37	0.39	Y	Y
3	1.04	0.44	Torsion	Torsion
4	6.65	4.39	X	X
5	7.00	4.44	Z	Y
6	7.70	4.54	Y	Z

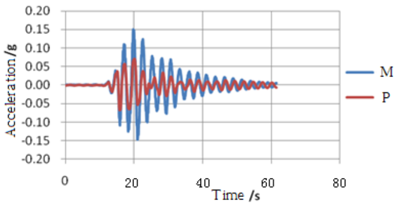
4.2 Acceleration Response

4.2.1 Acceleration response time history

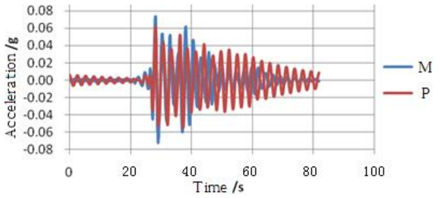
The acceleration response time history of the prototype and the model under Pasadena and SHW2 waves are shown in Figures 4.1 and 4.2, respectively. It can be seen that the acceleration response time history of the model and the prototype under basic and rare occurred seismic action are in reasonable agreement, though the accelerations of the former are some larger than that of the latter. It is mainly because that the isolation effectiveness of the numerical analysis is usually better than that of the test. From the Fig. 4.1(a), it can be clearly seen that error has been introduced in to the data. The error source consists of inertia force from the testing apparatus and the friction force from the bearings and the transfer layer.



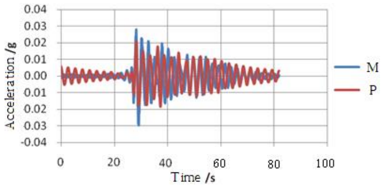
(a) Acceleration response under frequently occurred seismic action of Pasadena wave



(d)Acceleration response under frequently occurred seismic action of SHW2 wave

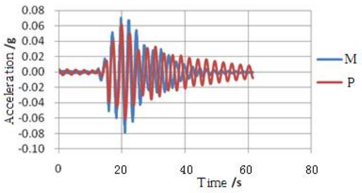


(b)Acceleration response under basic seismic action of Pasadena wave

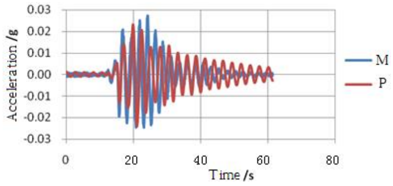


(c) Acceleration response under rare occurred seismic action of Pasadena wave

Figure 4.1 Acceleration response time history of the isolation layer under Pasadena wave



(e)Acceleration response under basic seismic action of SHW2 wave

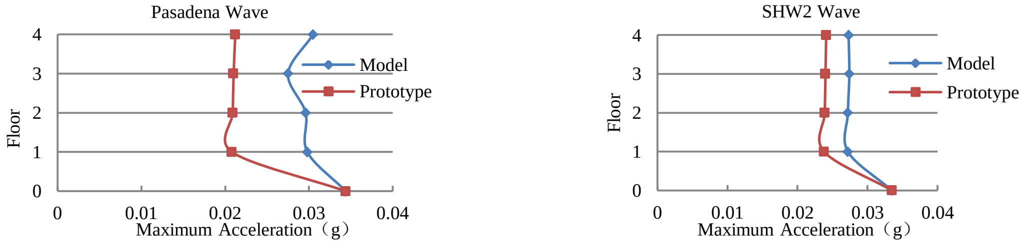


(f) Acceleration response under rare occurred seismic action of SHW2 wave

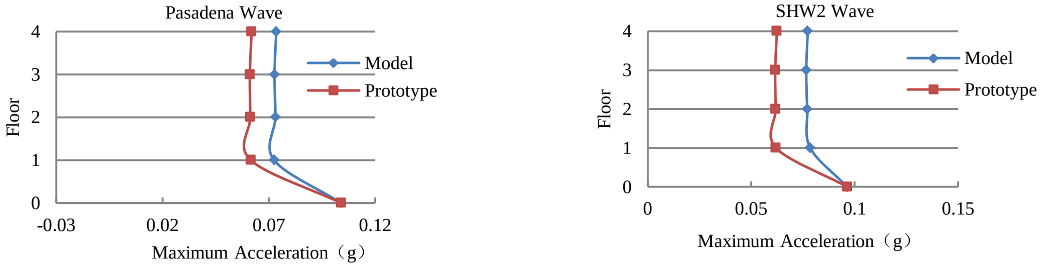
Figure 4.2 Acceleration response time history of the isolation layer under SHW2 wave

4.2.2 Acceleration response envelope

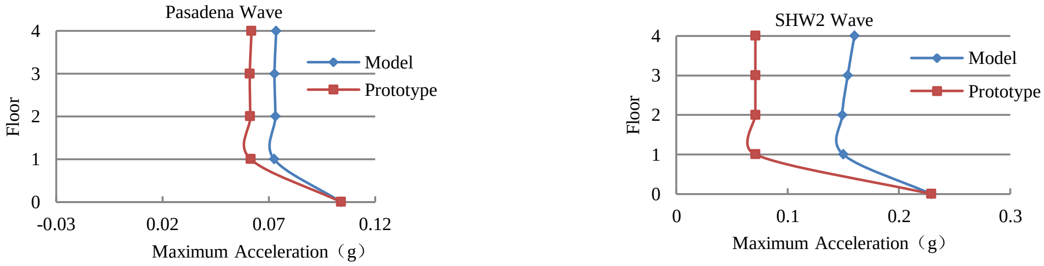
Maximum acceleration responses of the model and the prototype under Pasadena and SHW2 waves are shown in Fig. 4.3. Floor acceleration responses of both the model and the prototype are found to be very similar to the base floor acceleration, which indicates that the superstructure vibrate approximately as a rigid body, as assumed in the model. The maximum acceleration values of the model and the prototype are in reasonable agreement and the distributing characteristics of the model are similar to that of the prototype if the influence of the isolation layer is ignored.



(a) Maximum acceleration under frequently occurred seismic action



(b) Maximum acceleration under basic seismic action



(c) Maximum acceleration under rare occurred seismic action

Figure 4.3 Acceleration response of the structure

5. CONCLUSION

A structural effect-based similitude approach for base-isolated structural model on shaking table is proposed, the similitude relationships of the physical quantities and the corresponding similitude constants are illustrated, while taking a base-isolated three-storey masonry building as an example on the shaking table to be subjected to various earthquake actions. Based on the comparison results, the following conclusions can be obtained,

(1) The proposed structural effect-based similitude design approach is proved to be an effective way to establish a small-sized base-isolated structural model on shaking table test.
(2) Three parts of the base-isolated structure model, which are superstructure, transfer layer and isolation layer, can be designed to realize the similarities of significant but different physical quantities, respectively.
(3) The acceleration responses of the superstructure are slightly underestimated from the model tests due to the uncertainties of the friction forces, particularly for the responses under frequently occurred earthquake actions .

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