6th International Conference on Advances in Experimental Structural Engineering
11th International Workshop on Advanced Smart Materials and Smart Structures Technology
August 1-2, 2015, University of Illinois, Urbana-Champaign, United States

Development of Adaptive Multi-Rate Interface for Real-time Hybrid Simulation

A. **Maghareh**[1], **J.P. Waldbjoern**[2], **S.J. Dyke**[3], **A. Prakash**[4], **A.I. Ozdagli**[5]

1 PhD Student, Lyles School of Civil Engineering, Purdue University, West Lafayette, IN, United States.
E-mail: amaghare@purdue.edu
2 PhD Candidate, Department of Civil Engineering, Technical University of Denmark, Copenhagen, Denmark.
E-mail: jpwa@byg.dtu.dk
3 Professor, School of Mechanical Engineering, Purdue University, West Lafayette, IN, United States.
E-mail: sdyke@purdue.edu
4 Assistant Professor, Lyles School of Civil Engineering, Purdue University, West Lafayette, IN, United States.
E-mail: aprakas@purdue.edu
5 Research Fellow, Lyles School of Civil Engineering, Purdue University, West Lafayette, IN, United States.
E-mail: aozdagli@purdue.edu

ABSTRACT
Real-time hybrid simulation (*RTHS*) is a powerful cyber-physical technique which provides an efficient method for global/local system evaluation of civil structures subject to extreme dynamic loading. Due to the need for meeting hard real-time constraints, one of the major factors which determines the ability of the RTHS to represent the true system level behavior is the fidelity of the numerical substructure. Higher-order finite element models entail additional demand for computational resources, which in turn may limit achievable sampling frequencies and/or introduce delays that can degrade stability and performance in RTHS. The goal of this research is to develop a new multi-rate compensation interface to effectively enable the use of more complex numerical models, running at a slower sampling rate, coupled with an experimental substructure, running at a higher sampling rate. The effectiveness of the proposed method is experimentally verified. Furthermore, a set of simulated studies are implemented to systematically compare the performance of the proposed method to existing methods. Compared to the existing methods, the proposed technique incorporates a built-in delay compensation feature, leads to smaller error, especially, at higher sampling frequency ratios and input signals with high-frequency content, and does not induce chattering at the coupling frequency.

KEYWORDS: *Real-time hybrid simulation, Multi-rate real-time hybrid simulation, Adaptive multi-rate interface, AMRI.*

1. INTRODUCTION

As civil engineering structures evolve to meet the needs of future generations, there is an increasing demand to address ongoing challenges such as demonstrating the effectiveness of performance-based design, considering soil-structure interaction, and utilizing new materials capable of reducing earthquake impact [1]. These challenges justify the need for extending and evolving our experimental capabilities for evaluating structural response and performance in a suitable and cost-effective manner. The necessity to assess the dynamic performance of rate-dependent structural components, and recent advances in systems with *hard real-time computing* capabilities, have led researchers to conduct real-time hybrid simulation. With the introduction of new seismic mitigation techniques and devices, such as rubber bearings, viscous dampers, friction dampers, sloshing dampers, magneto-rheological dampers and electro-rheological dampers, earthquake engineers have developed new techniques to evaluate structural dynamic performance using *hybrid simulation* (*HS*), which imposes hard real-time constraints on the digital components. In RTHS, the interface interaction between the substructures is enforced by servo-hydraulic actuators or a shake table which acts as the transfer system. This transfer system must be controlled to ensure that all interface boundary conditions are satisfied in real time.

The power of hybrid simulation and real-time hybrid simulation lies in its promise to accelerate the rate at which research in earthquake engineering is conducted. In the past decade, an increasing number of researchers have utilized HS methods as an alternative to quasi-static or shake table testing. The capability of HS and RTHS to capture both, the local response of individual structural components and the global system behavior, under realistic loading allows great flexibility. Unlike shake table tests that are constrained by size and shape of the structure, HS and RTHS can be conducted on different types of structures in different loading configurations [2].

Many projects have used HS/RTHS to investigate a variety of topics related to seismic and wind engineering. Recently, researchers have begun to rely on HS or RTHS to assess local and global responses, to compare various aspects related to design guidelines, particularly design codes [3].

In classical RTHS, global stability and performance dictate the sampling frequency [4], and it is usually chosen to be an integer multiple of the digital servo-controller's sampling frequency, such as 1024Hz [5]. Due to stringent real time constraints and the fact that the time required to solve a high-order numerical model is usually much greater than the RTHS time-step, low-order numerical models that usually do not require a significant amount of time to solve are chosen by researchers, see Fig. 1.1. These simplified low-order models are limited in their ability to represent the underlying dynamics of the numerical substructure, especially for complex and/or non-linear systems. To overcome this challenge, two approaches are available: *parallel real-time computing* and *multi-rate RTHS* (*mrRTHS*).

Parallel real-time computing has the potential to enable execution of computationally intensive numerical models in RTHS. However, currently there are very few openly available platforms that are suitable for writing and executing parallel computations in real time [6]. Most current real-time systems only support sequential processing, in which a real-time workload may use only one processor core at a time, or multi-processing, in which a real-time workload may use multiple independent processor cores. Recently, a computational platform based on a federated scheduling model that exploits both intra-task and inter-task parallelism, was developed to enable execution of high fidelity numerical models within the real-time constraints of RTHS [7].

In multi-rate RTHS, the numerical substructure (or a portion of it) is executed at a slower rate than the experimental substructure. Running the numerical substructure at a slower rate provides more time to complete the task of time integration for more complex *finite element* (*FE*) models, see Fig. 1.2. However, in order to implement mrRTHS successfully from stability and performance perspectives, an effective rate-transitioning method is needed to compute the command signal properly. In this study, we propose a new rate-transitioning scheme, posted in *the NEEShub*. Also, a new evaluation procedure for the use of rate-transitioning techniques in mrRTHS is proposed and compared to some of the existing techniques.

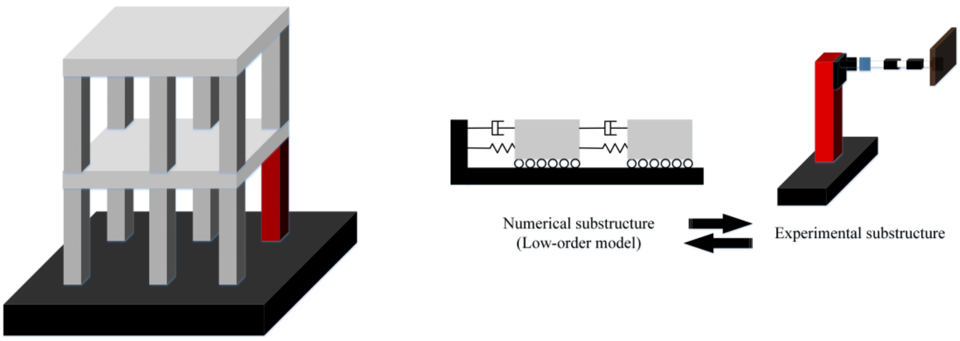


Figure 1.1 A conventional RTHS with computationally-inexpensive numerical substructure

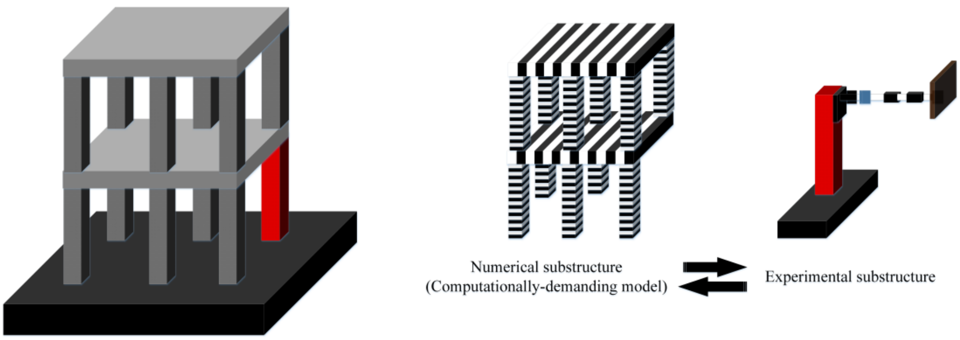


Figure 1.2 RTHS with computationally-expensive numerical substructure

The concept of multi-rate RTHS was first developed by Nakashima and Masaoka due to the computing/processing limitations in the late 90's [8]. In their RTHS setup, a novel sub-stepping technique was developed so that the experimental substructure is continuously loaded. The computation of the numerical substructure was executed at each main integration time step, Δt, while a smooth command signal generation task is executed at each sub-step, δt. The two tasks are computationally independent and separated. The sub-stepping technique, combined with the use of priority based multi-tasking, produced a smooth command signal for the experimental substructure [8]. This work was later pursued by Bonnet [9].

In this paper, we develop new rate-transitioning and compensation techniques that enable researchers to simulate the numerical substructure (or a part of it) at a slower execution rate compared to the experimental substructure. This approach facilitates the use of complex, high-fidelity numerical models and thus reduces modeling error in RTHS. To evaluate the performance of different rate-transitioning techniques, a metric is used, and the performance of the proposed technique is compared with the three existing rate-transitioning techniques in a set of multi-rate numerical simulations. Finally, the newly-developed rate-transitioning and compensation technique is experimentally validated for multi-degree-of-freedom mrRTHS.

2. ADAPTIVE MULTI-RATE INTERFACE (AMRI)

Currently, three methods are available that allow the numerical and experimental substructures to run at two different rates in an RTHS. The first two methods are based on polynomial extrapolation and the third method is based on linearly predicted acceleration. The first method was developed by Bonnet, see [9]. In this method, the control signal is extrapolated through a compensation method developed by Horiuchi *et al.* in [10, 11] using current and previous data points from the numerical substructure. The second method was established by Wallace *et al.* using least-square polynomial fitting, see [12]. The third method by Horiuchi and Konno is based on the assumption that there is a linear acceleration as a function of time, see [13].

Adaptive multi-rate interface (*AMRI*) is developed, allowing the numerical and experimental substructures to run at two rates, enabling the use of computationally-demanding numerical models while maintaining a good actuator tracking control. Here, the numerical substructure is executed with a coarse integration time, referred to as Δt. After selecting a set of bases, such as polynomial or exponential, sampling frequency ratio between the numerical and experimental substructures, and compensation time, a finer control signal is generated using the proposed method with a fine integration time, referred as δt. Herein, we define some of AMRI's parameters:

SFR: sampling frequency ratio (Δt /δt)
X: input signal w/ coarse integration increment Δt
Y: output signal w/ fine integration increment δt
M: compensation order
R: number of orthogonal bases used for interpolation
r: interpolation order
p: compensation coefficient, *p*Δt is time to be compensated

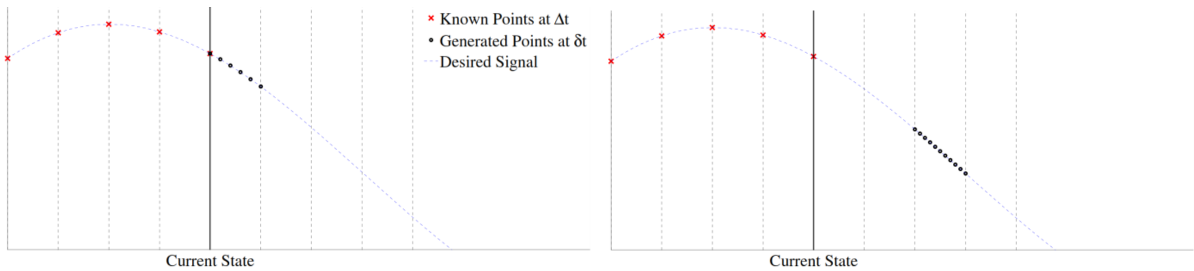


Figure 2.1 Compensation and rate transitioning: (a) *p* = 1, *SFR* = 5; (b) *p* = 3, *SFR* = 10

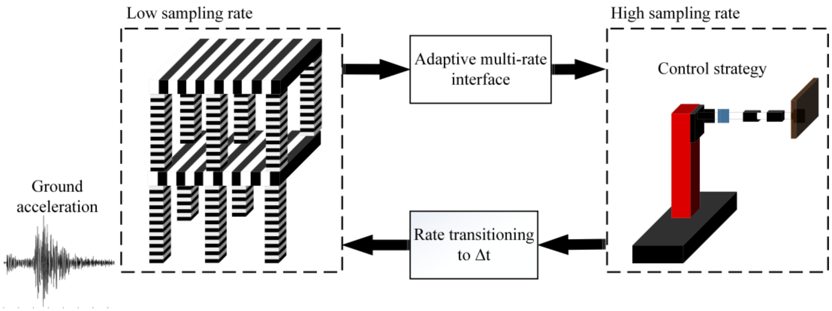


Figure 2.2 Use of adaptive multi-rate interface in multi-rate RTHS

In AMRI, *N+M+p-1* displacement points (current and previous) with the integration increment of Δt are used to generate *SFR*+1 points of displacement commands with the integration increment of δt. As examples, two cases with 1-step and 3-step compensation and sampling frequency ratios of 5 and 10 are provided in Fig. 2.1. Fig. 2.2 shows a simplified framework in which the adaptive multi-rate interface can be used in the implementation of mrRTHS. For a better understanding of how the proposed rate-transitioning scheme functions, AMRI computations are divided into three sequential steps: compensation, extrapolation, and interpolation.

Compensation: In this step, Eq. 2.1 is used to compensate and predict the command signal,

$$C(z) = \alpha_1 z^{-p} + \alpha_2 z^{-p-1} + \cdots + \alpha_M z^{-p-M+1} \tag{2.1}$$

where $p\Delta t$ is the compensated time, $p \in \{1, 2, 3 \ldots\}$ and z is the complex variable in the Z-domain. Eq. 2.1 is a time-varying, discrete transfer function and the coefficients $\{\alpha_1 \ldots \alpha_M\}$ are obtained at each time step by solving Eq. 2.2.

$$\begin{pmatrix} X_{n-p-N+1} & X_{n-p-N} & \cdots & X_{n-p-N-M+2} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n-p} & X_{n-p-1} & \cdots & X_{n-p-M+1} \end{pmatrix}_{N\times M} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_M \end{pmatrix} = \begin{pmatrix} X_{n-N+1} \\ X_{n-N+2} \\ \vdots \\ X_n \end{pmatrix} \tag{2.2}$$

Extrapolation: After obtaining the α values, the next p points $\{X_{n+1} \ldots X_{n+p}\}$ are extrapolated using Eq. 2.3.

$$\begin{pmatrix} X_{n-p+1} & X_{n-p} & \cdots & X_{n-p-M+2} \\ \vdots & \vdots & \ddots & \vdots \\ X_n & X_{n-1} & \cdots & X_{n-M+1} \end{pmatrix}_{p\times M} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_M \end{pmatrix} = \begin{pmatrix} X_{n+1} \\ X_{n+2} \\ \vdots \\ X_{n+p} \end{pmatrix} \tag{2.3}$$

Interpolation: In this step, Chebyshev polynomials of the first kind are used as a set of orthonormal bases for interpolation and rate transitioning from Δt to δt. The Chebyshev polynomials of the first kind are defined by the recurrence relation in Eq. 2.4.

$$T_1(s) = 1;\ T_2(s) = s;\ T_{(n+1)}(s) = 2sT_n(s) - T_{(n-1)}(s) \tag{2.4}$$

$$\begin{pmatrix} T_1[(p+1-r)\Delta t] & \cdots & T_R[(p+1-r)\Delta t] \\ \vdots & \ddots & \vdots \\ T_1[p\Delta t] & \cdots & T_R[p\Delta t] \end{pmatrix}_{r\times R} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{pmatrix} = \begin{pmatrix} X_{n-r+p+1} \\ \vdots \\ X_n \\ \vdots \\ X_{n+p} \end{pmatrix} \tag{2.5}$$

These polynomials must be adjusted to be within a general range of $[a, b]$, where, $a = (p + r - 1)\Delta t$ and $b = p\Delta t$. Next, Eq. 2.5 is solved to obtain $\{\beta_1, \beta_2 \ldots \beta_R\}$. Using the β coefficients, the command signal at the higher sampling time δt can be computed in Eq. 2.6.

$$Y(h) = \beta_1 T_1(h) + \beta_2 T_2(h) \ldots \beta_R T_R(h) \tag{2.6}$$

where $h \in \{(n+p-1)\Delta t, (n+p-1)\Delta t + \delta t, (n+p-1)\Delta t + 2\delta t \cdots (n+p)\Delta t\}$.

3. VERIFICATION OF ADAPTIVE MULTI-RATE INTERFACE

Tracking performance evaluation of the transfer system is a necessary preliminary step in RTHS. To evaluate the performance of AMRI, two case studies of transfer system tracking dynamics are simulated, see Fig. 3.1. The servo-hydraulic actuator, identified in *the Intelligent Infrastructure Systems Laboratory* at Purdue University, is used in this study where transfer function of the plant from command to measured displacement is modeled by:

$$G(s) = \frac{4.52 \times 10^9}{s^4 + 577s^3 + 3.68 \times 10^5 s^2 + 6.28 \times 10^7 s + 4.93 \times 10^9} \tag{3.1}$$

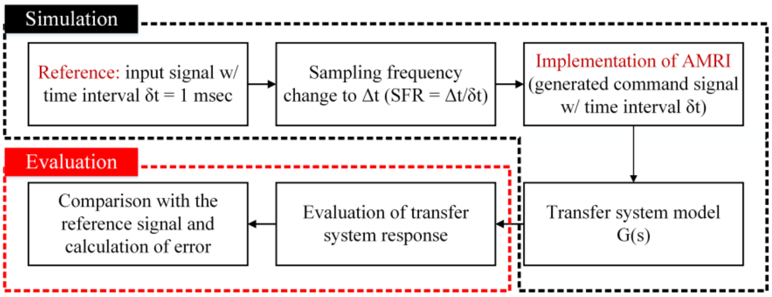


Figure 3.1 Procedure used to evaluate the AMRI performance for various reference signals

3.1. Simulated Case Study I

Two significant strengths of the adaptive multi-rate interface are its effective performance for input signals with high-frequency content and large sampling frequency ratios. To evaluate the performance of the proposed interface, a series of simulated case studies are implemented in which the input is a sinusoidal signal with various frequencies (1-49Hz) and sampling frequency ratios (2, 4, 5, 8, and 10). The corresponding normalized tracking errors using Eq. 3.2 are shown in Fig. 3.2a.

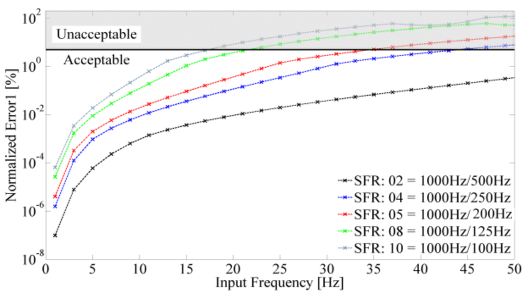
$$NE\% = \frac{\max(|X_i^{sim} - X_i^{ref}|)}{\max(|X_i^{ref}|)} \tag{3.2}$$

The simulation results shown in Fig. 3.2a allow the user to have a better understanding of the error stemming from the multi-rate implementation of a real-time hybrid simulation using AMRI. In this analysis, the frequency spectrum of the command signal is assumed to be known. For instance, the shaded region in Fig. 3.2a results in less than 5% transfer system tracking error using AMRI rate-transitioning scheme. Moreover, Fig. 3.2a shows that the majority of cases leads to less than 1% error.

3.2. Simulated Case Study II

To systematically compare the performance of the three existing methods and AMRI, a set of actuator tracking simulations are conducted with one time step (Δt) compensation and various sampling frequency ratios of (2, 5, 8, and 10). The displacement is a chirp signal (0-15Hz). A normalized tracking error is computed as:

$$NRMSE\% = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i^{sim} - X_i^{ref})}}{\max(|X_i^{ref}|)} \times 100 \tag{3.3}$$



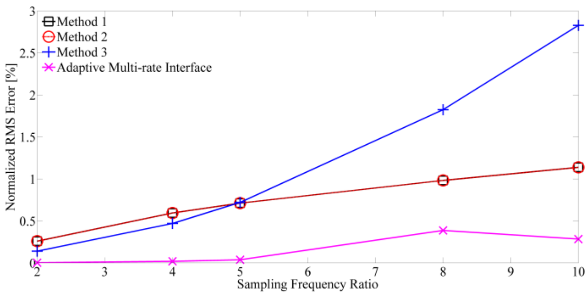


Figure 3.2 (a) Case study I: determining acceptable/unacceptable ranges for a specific multi-rate implementation error (b) Case study II: tracking performance of different rate-transitioning methods

where *NRMSE* stands for normalized root mean square error. The errors are presented in Fig. 3.2b and show that: (i) the proposed method exhibits significantly smaller error due to the sampling frequency rate transition for all sampling frequency ratios when compared to methods 1, 2, and 3; (ii) Method 1 and 2 exhibit identical performance in the simulated cases; (iii) Method 3 performs better than method 1 and 2 for smaller sampling frequency ratios but is not effective for larger sampling frequency ratios.

3.3. RTHS Case Studies

To evaluate the impact of modeling error and implement the proposed adaptive multi-rate technique, three real-time hybrid simulations are conducted. In these experiments, the numerical substructure is the 9-story structure [14] designed by Brandow and Johnston Associates (1996) for the SAC phase II steel project. The 9-story structure is well-studied as one of the benchmark control structures for seismically-excited nonlinear buildings in [15]. Two models with different levels of refinement are used for the numerical substructure: a 184 degree-of-freedom finite element model constructed using *RT-Frame2D* open-source software available at [16] and a 9 degree-of-freedom shear model with similar dynamic characteristic at [17]. In this study, the excitation is the N-S component of the *El Centro earthquake* recorded in May 1940. The seismic responses of the two models are provided in Fig. 3.3. It shows that the simple 9-story shear building model is able to capture the dominant dynamics of the more refined finite element model.

For the purpose of comparing the performance of the proposed AMRI technique, we designate the response from the real-time hybrid simulation of the 9-story structure where the numerical model is chosen to be the detailed finite element model and run at 1024Hz as *the reference response*. In the first RTHS (reference), the experimental substructure is an MR-damper placed on the first floor, see Fig. 3.5a. It should be noted that due to the computational demands of the FE model, the numerical substructure cannot be implemented on a regular xPC real-time target machine and high-performance xPC real-time target system is used instead.

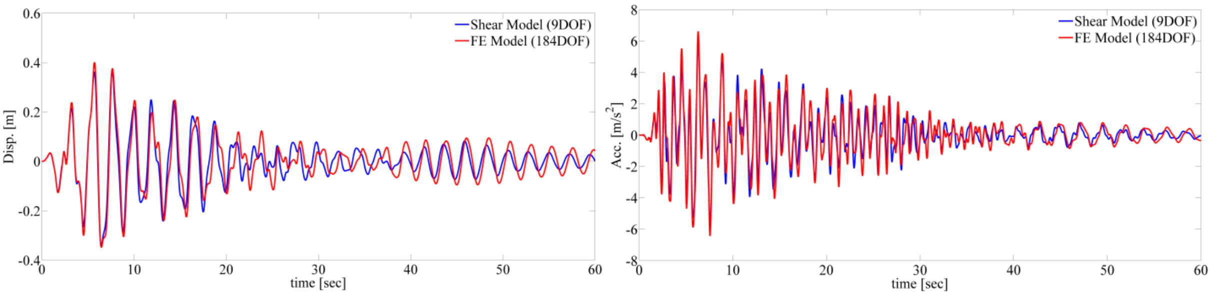


Figure 3.3 Comparison of responses obtained from two numerical models

Due to the numerical substructure being computationally demanding, three approaches may be considered: (i) obtaining the best simplified model using model reduction techniques, and using it for RTHS; (ii) using different techniques such as real-time parallel computing to enhance the available computational power; and (iii) using a multi-rate RTHS strategy to run the numerical substructure at a slower rate while the experimental substructure is run at a higher rate. However, currently there are very few openly available platforms that are suitable for writing and executing parallel computations in real-time [18] and [6] and none are yet integrated for RTHS. In this study, approaches (i) and (iii) are considered and the corresponding responses are compared.

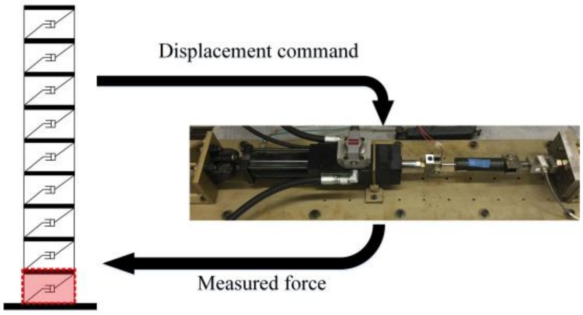


Figure 3.4 Typical RTHS with 1024Hz sampling rate and shear model numerical substructure

In the second experiment, a simplified numerical model is adopted and RTHS is conducted at 1024Hz sampling rate on a regular xPC real-time target machine, see Fig. 3.4. Finally, a multi-rate RTHS using AMRI with sampling ratio of 4 (*SFR* = 1024/256) was implemented on an xPC real-time target machine, see Fig. 3.5b. The results of these experiments are provided in Fig. 3.6.

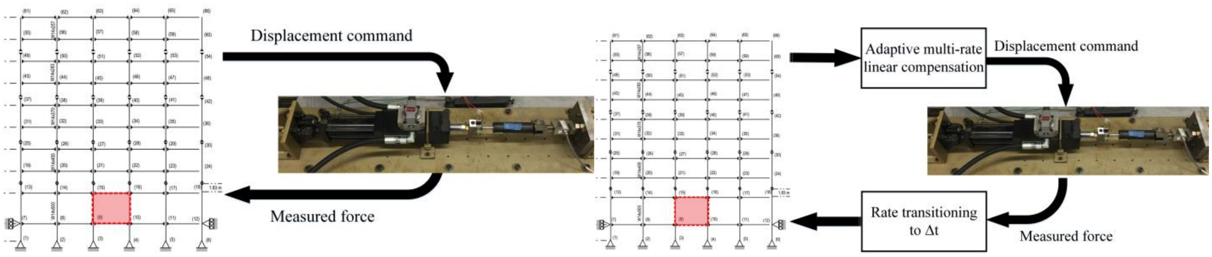


Figure 3.5 (a) Reference, conventional RTHS with 1024Hz sampling rate and FE numerical substructure
(b) Multi-rate RTHS with sampling ratio 4 (= 1024/256)

The results show that the proposed technique enables users to implement complicated experiments using a commonly available real-time target system. By comparing the responses, we observe that modeling idealization error in the numerical substructure can considerably degrade the global RTHS response. As is also evident from Fig. 3.3, the shear model captures the dominant dynamics of the finite element model. However, this insignificant modeling mismatch leads to considerable errors in the global response.

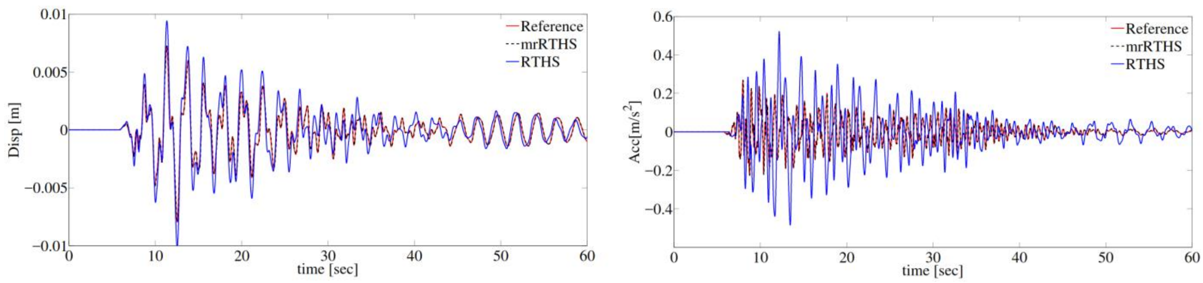


Figure 3.6 Comparison of RTHS responses at top floor

4. CONCLUSIONS

In real-time hybrid simulation, due to stringent real time constraints, high-fidelity FE models which require a significant time to solve are unsuitable. Thus, researchers established a multi-rate approach in which the computationally-demanding part of the numerical substructure is implemented at a slower rate, while the rest of the structure is executed at a higher rate to achieve a smooth, stable tracking performance. In this study, an adaptive multi-rate interface was developed to effectively enable the use of more complex numerical models, running at a slower sampling rate, coupled with an experimental substructure, running at a higher sampling rate. The effectiveness of AMRI is experimentally verified. In this experiment, we show that mrRTHS technique leads to a smaller global error compared to reducing the numerical substructure to a more simplified model. Also, we demonstrated that modeling error in the numerical substructure can considerably degrade the quality of the global RTHS response. An apparently insignificant modeling mismatch may lead to considerable errors in the global response. To mitigate this error, a user can integrate AMRI in RTHS to implement a high-fidelity FE model as numerical substructure.

ACKNOWLEDGEMENT

This material is based upon work supported by the NSF under CCF-1136075. The authors would also like to acknowledge the support from, the Danish Centre for Composite Structures and Materials funded by the Danish Council for Strategic Research within Sustainable Energy and Environment (09-067212), COWI foundation, and Otto Moensted Foundation.

REFERENCES

[1] S. J. Dyke, B. Stojadinovic, P. Arduino, M. Garlock, N. Luco, J. A. Ramirez, and S. Yim, "2020 Vision for Earthquake Engineering Research: Report on an OpenSpace Technology Workshop on the Future of Earthquake Engineering," vol. 20, no. 5, 2010.

[2] S. J. Dyke, N. Nakata, and G. Marshall, "Report on the discussions during the hybrid simulation workshop," 2013.

[3] D. Gomez, S. J. Dyke, and A. Maghareh, "On the role of hybrid and real-time hybrid simulations in advancing the practice of earthquake engineering," *Smart Struct. Syst.*, vol. 15, no. 3, pp. 913–929, 2014.

[4] A. Maghareh, S. J. Dyke, A. Prakash, and J. F. Rhoads, "Establishing a stability switch criterion for effective implementation of real-time hybrid simulation," *Smart Struct. Syst.*, vol. 14, no. 6, pp. 1221–1245, 2014.

[5] C. Chen and J. M. Ricles, "Improving the Inverse Compensation Method for Real-time Hybrid Simulation through a Dual Compensation Scheme," *Earthq. Eng. Struct. Dyn.*, vol. 38, no. 10, pp. 1237–1255, Aug. 2009.

[6] D. Ferry, J. Li, M. Mahadevan, K. Agrawal, C. D. Gill, and C. Lu, "A Real-Time Scheduling Service for Parallel Tasks," *IEEE Real-Time Embed. Technol. Appl. Symp.*, 2013.

[7] D. Ferry, A. Maghareh, G. Bunting, A. Prakash, K. Agrawal, C. Gill, C. Lu, and S. J. Dyke, "On the performance of a highly parallelizable concurrency platform for real-time hybrid simulation," *6WCSCM*, 2014.

[8] M. Nakashima and N. Masaoka, "Real-time on-line Test for MDOF Systems," *Earthq. Eng. Struct. Dyn.*, vol. 28, no. 4, pp. 393–420, Apr. 1999.

[9] P. A. Bonnet, "The Development of Multi-axis Real-time Substructure Testing," 2006.

[10] T. Horiuchi, M. Nakagawa, M. Sugano, and T. Konno, "Development of a Real-time Hybrid Experimental System with Actuator Delay Compensation," *Proc. 11th World Conf. Earthq. Eng.*, 1996.

[11] T. Horiuchi, M. Inoue, T. Konno, and W. Yamagishi, "Development of a Real-time Hybrid Experimental System Using a Shaking Table (Proposal of Experimental Concept and Feasibility Study with Rigid Secondary System)," *JSME Int.*, vol. 42, no. 2, pp. 255–264, 1999.

[12] M. I. Wallace, D. J. Wagg, and S. A. Neild, "An adaptive polynomial based forward prediction algorithm for multi-actuator real-time dynamic substructuring," *Proc. R. Soc. A Math. Phys. Eng. Sci.*, vol. 461, pp. 3807–3826, 2005.

[13] T. Horiuchi and T. Konno, "A new method for compensating actuator delay in real-time hybrid experiments," *Philos. Trans. R. Soc. A Math. Phys. Eng. Sci.*, vol. 359, pp. 1893–1909, 2001.

[14] Y. Ohtori, R. E. Christenson, B. F. Spencer, and S. J. Dyke, "Benchmark Control Problems for Seismically Excited Nonlinear Buildings," *J. Eng. Mech.*, vol. 130, pp. 366–385, 2004.

[15] S. J. Dyke, A. K. Agrawal, J. M. Caicedo, R. Christenson, H. Gavin, E. Johnson, S. Nagarajaiah, S. Narasimhan, and B. Spencer, "Database for Structural Control and Monitoring Benchmark Problems," 2010.

[16] N. Castaneda-Aguilar, X. Gao, and S. J. Dyke, "RT-Frame2D: A Computational Platform for the Real-Time Hybrid Simulation of Dynamically-excited Steel Frame Structures," 2012.

[17] A. Maghareh, M. Barnes, and Z. Sun, "Evaluation of the 9-Story Benchmark Building Shear Model," *NEEShub*, 2012. [Online]. Available: https://nees.org/resources/4103.

[18] Q. Wang and G. Parmer, "Practical, predictable, and efficient system support for fork/join parallelism," *20th Real-Time Embed. Technol. Appl. Symp.*, 2014.