



Alternative Acceleration Performance Assessment Method for Seismic Shaking Tables

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ABSTRACT

Shaking table testing has been regarded as one of the most effective experimental methods to evaluate the seismic response of structural systems subjected to earthquake ground motions. Test specimens are mounted on the table and shaken to failure by driving servo-hydraulic actuators to follow historical or artificial earthquake accelerations. However, precisely reproducing a prescribed acceleration time history over the frequency of interest is challenging because shaking tables are eventually nonlinear by nature. In addition, the interaction between table and specimen affects the control accuracy of shaking tables significantly. Therefore, various control algorithms has been developed to accommodate the control issue in the past decades. This paper reviews current acceleration performance assessment methods for shaking tables first. The pros and cons of each method are indicated and discussed thoroughly. Finally, an alternative assessment method is proposed considering both time-domain and frequency-domain analyses. The efficacy and feasibility of the proposed method is investigated by using a set of experimental data of shaking table tests.

KEYWORDS: *Seismic shaking table, acceleration performance, time-domain analysis, frequency-domain analysis*

1. INTRODUCTION

Seismic shaking tables are generally driven by servo-hydraulic actuators controlled by servo-controllers. The most challenging part of shaking table control is to overcome the coupling dynamics between the shaking table and the specimen. This coupling effect is well-known as the control-structure interaction (Dyke et al., 1995). Conventionally, the hydraulic actuator is displacement-controlled with a proportional-integral-differential (PID) controller which takes a combination of proportional, integral and derivative action on the difference between the target and measured displacements to generate the command of the controller. While in other approaches, velocity and acceleration feedback are added to the displacement control loop, leading to a wider system frequency bandwidth and improved system stability (Tagawa and Kajiwara, 2007; Yao et al., 2011). One of the most recognized application is the three-variable controller (TVC) which has been widely implemented by MTS Systems Corporation (Nowak et al., 2000). The three variables in the TVC are displacement, velocity, and acceleration. Both feedforward and feedback loops adopt the three variables with corresponding control gains; therefore, a total number of six gains can be tuned to meet the requirements of a shaking table test. In addition to these conventional control approaches, various control schemes have been developed in the past decade to enhance the performance and robustness of seismic shaking tables. Spencer and Yang (1998) proposed the transfer function iteration method in which the errors between the desired and achieved accelerations need to be iteratively modified offline. Stoten and Gomez (2001) developed the minimal control synthesis algorithm which allows online tuning the controller without prior knowledge of the system dynamics. Nakata (2010) presented a combined control scheme that contains acceleration feedforward, displacement feedback, command shaping, and a Kalman filter for measured displacements. Phillips et al. (2014) proposed a model-based multi-metric control strategy that takes both displacement and acceleration measurements for control calculation. Those above-mentioned methods aim to improve control accuracy of shaking table acceleration tracking over a wide range of frequencies.

Assessment methods for shaking table performance are crucial in order to investigate the control efficiency of conventional and/or new developed controllers. In this paper, the existing methods used to evaluate the performance of shaking tables are reviewed and discussed first. These methods can be mainly separated into two categories: time-domain and frequency-domain analyses. Among these methods, some of them can be quantized as a performance index while the rest of them barely provides a schematic concept of shaking table performance.

Then, an alternative assessment method for shaking table performance is proposed by combining the specific advantage of each existing method. Through penalty factors, this new method allows users to modify the quantifiable procedure which depends on the performance requirements for individual shaking table test. Finally, the efficacy and practicality of the proposed method is investigated by using real experimental data of shaking table tests.

2. TIME-DOMAIN PERFORMANCE ASSESSMENT METHODS

The purpose of a shaking table test is to duplicate a predetermined acceleration time history so that the corresponding dynamic response of the structural model fixed on the table can be investigated. In other words, shaking table control essentially deals with a trajectory tracking problem up to a maximum of six degrees-of-freedom. Therefore, time-domain analysis is a straightforward assessment method because it compares the time histories of reference acceleration and achieved acceleration.

2.1. Root-mean square error

The tracking performance of a seismic shaking table test considers the difference between the reference acceleration and the achieved acceleration. The root-mean square (RMS) error is commonly used as an index of the tracking performance for its simplicity and straightforwardness. The RMS error is defined as:

$$\text{RMS}_{\text{error}}(\%) = \sqrt{\frac{\sum_{k=1}^N (a_r[k] - a_m[k])^2}{\sum_{k=1}^N a_r[k]^2}} \times 100\% \quad (2.1)$$

where N represents the number of data point; $a_r[k]$ and $a_m[k]$ are the reference and measured acceleration at the step k , respectively. Less difference between the reference and measured accelerations leads to a smaller RMS error; therefore, a low RMS error indicates good tracking performance. RMS error is also a normalized index since the square of error is divided by the square of reference, indicating that RMS error is not affected by the intensity of ground motion. The purpose of a shaking table test is to reproduce a predetermined acceleration time history; therefore, time lag and delay between the reference and measured accelerations is not vital. The tested specimen is subjected to an identical ground motion even though time lag and delay exists between the reference and measured accelerations. Accordingly, the dynamic response of the tested specimen is not affected by the time lag and delay.

In order to support the statement above, a simple numerical simulation was conducted. The 1940 El Centro earthquake with a normalized peak ground acceleration (PGA) of 100gal was adopted as the reference acceleration. The time step and period of the reference acceleration are 1/200 second and 40 seconds, correspondingly. The dynamics of seismic shaking table was assumed as a pure-delay system with a delay time of 2, 4, 6, 8 and 10 time steps, respectively. Table 2.1 shows the corresponding RMS errors of the five cases. It is evident that a small amount of time delay results in a significant RMS error. Consequently, RMS error of acceleration time histories may not be a proper method for shaking table performance assessment. Time-shift correction must be completed before RMS error can be used as an index for shaking table performance assessment.

Table 1.1 Acceleration tracking performance of a time-delay system in terms of RMS error

Time delay (steps)	RMS _{error} (%)
2	24.35
4	46.11
6	64.33
8	79.86
10	93.90

2.2. Integral square error

An integral square error (ISE) index is calculated by taking the integral of the square of tracking error in order to investigate the overall performance (Gizatullin and Edge, 2007). The slope level of the graph is a measure of the tracking accuracy because the ISE increases rapidly when the error of each time step is significant. The ISE

index is defined as:

$$ISE[m] = \sum_{k=1}^m (a_r[k] - a_m[k])^2 \quad (2.2)$$

where m is the current integral step of ISE index. Identical 1940 El Centro time history and delay time were adopted for the numerical analysis on the ISE index. Figure 2.1 shows the ISE index of the five cases. ISE index can be treated as the denominator part of RMS error inside the square root. As a result, it is found that a larger time delay results in a steeper slope in the ISE index. However, ISE merely provides an illustration without a specific quantized index. Consequently, ISE index may not be a good indicator for shaking table acceleration performance.

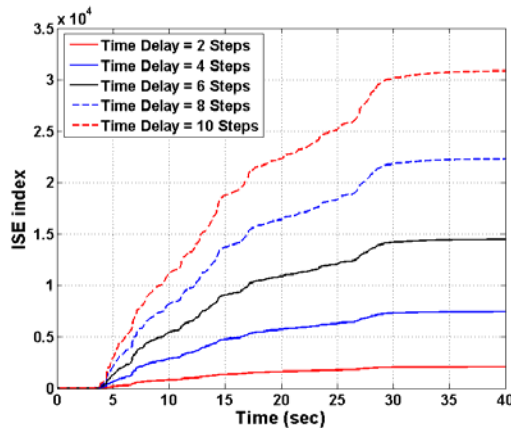


Figure 2.1 Acceleration tracking performance a time-delay system in terms of ISE index

3. FREQUENCY-DOMAIN PERFORMANCE ASSESSMENT METHODS

The reproduced acceleration can be analyzed with respect to frequency rather than time for shaking table testing when the frequency range of interest becomes a test requirement. A frequency-domain representation includes the information on magnitude and phase; therefore, it gives a clear vision of the achieved acceleration. Consequently, frequency analyses have been widely adopted to investigate the shaking table performance.

3.1. Fourier amplitude spectrum

Fourier amplitude spectrum provides the information of an acceleration time history decomposing into harmonic components of given frequencies. It illustrates the signal in terms of the strength of the various underlying frequency components. These amplitude coefficients are then used to assess the performance of a shaking table test. Generally speaking, the Fourier amplitude spectrum of the achieved acceleration is consistent with that of the reference acceleration if they are exactly identical to each other in time domain. In addition, Fourier amplitude spectrum of a specific signal is not affected by any time lag or delay. Furthermore, RMS error of Fourier amplitude spectra between the reference and achieved accelerations can be also adopted as a quantized index. Therefore, it has been considered as a good assessment method for shaking table testing.

Despite the fact that two signals share the same Fourier amplitude spectrum, it may not be absolutely stated that they are identical in time domain. In order to prove the statement, an artificial acceleration time history was generated by reversing the time series of the 1940 El Centro earthquake. Figure 3.1 shows the 1940 El Centro earthquake acceleration time history and its reversed one with a normalized PGA of 100 gal. It is evident that these two time histories are significantly different. However, identical Fourier amplitude spectrum can be obtained by taking discrete Fourier transform of the two signals as shown in Fig. 3.2. Consequently, Fourier amplitude spectrum may not be a perfect tool for shaking table acceleration performance.

3.2. Power spectral density

Power Spectral Density (PSD) investigates a signal's power intensity in the frequency domain by computing from the Fourier spectrum of this signal, providing a useful approach to characterize the amplitude versus

frequency content of a given signal. Figure 3.3 shows the PSD of the 1940 El Centro earthquake acceleration and its reversed one with a normalized PGA of 100gal. Since the PSD is correlated with Fourier spectrum, its imperfect application for evaluating the performance of the special case is alike as mentioned in section 3.1.

3.3. Spectral acceleration

Spectral acceleration (SA) is used to investigate the seismic responses of structures and equipment in earthquake engineering; therefore, it provides an indirect approach to evaluate the performance of shaking table testing. SA illustrates the peak acceleration response of a single-degree-of-freedom structure with a constant damping ratio subjected to an earthquake ground motion. Typically, the SA of the achieved acceleration is identical to that of the reference acceleration as long as the shaking table is able to duplicate the reference acceleration. In particular, for nonstructural components and systems, test response spectrum (SA of the achieved acceleration) needs to meet the required response spectrum (SA of the reference acceleration) with a tolerance range of 90% to 130% (AC156, 2007). Therefore, it is considered that SA is a great assessment method for seismic shaking tables.

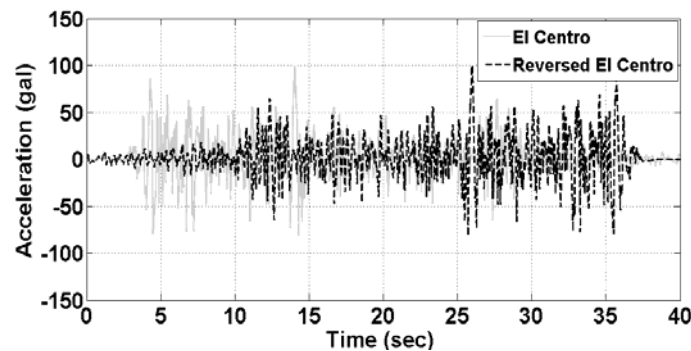


Figure 3.1 The 1940 El Centro earthquake acceleration time history and its reversed one with a PGA of 100 gal

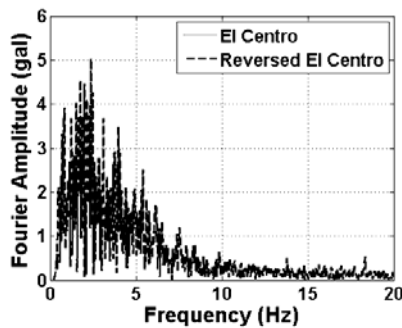


Figure 3.2 Fourier amplitude spectrum of El Centro earthquake and its reversed time histories

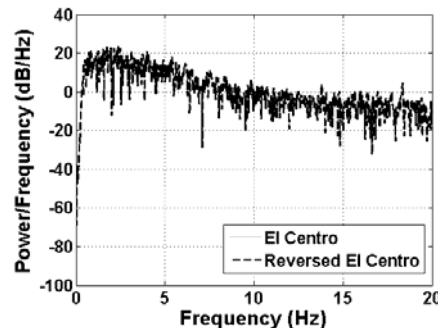


Figure 3.3 Power spectral density of El Centro earthquake and its reversed time histories

4. PROPOSED PERFORMANCE ASSESSMENT METHOD

In this study, an alternative assessment method for shaking table testing which combines the advantages of the RMS error and spectral acceleration methods. Penalty factors are added in the method to categorize the shaking table tests into several performance levels.

4.1. Minimization of root-mean square error

The RMS error, as defined in Eq. 2.1, is affected by magnitude error as well as time lag and delay between the reference and measured accelerations. A continuous reference earthquake acceleration can be represented as Fourier series:

$$a_r(t) = a_0 + \lim_{L \rightarrow \infty} \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right) \quad (4.1)$$

where $a_r(t)$ is the reference acceleration; a_n and b_n are Fourier coefficients; and L is the half period of the signal. Similarly, the measured acceleration $a_m(t)$ considered magnitude error and time delay can be formed as:

$$a_m(t) = \tilde{a}_0 + \lim_{L \rightarrow \infty} \sum_{n=1}^{\infty} \left(\tilde{a}_n \cos \frac{n\pi(t - \Delta t)}{L} + \tilde{b}_n \sin \frac{n\pi(t - \Delta t)}{L} \right) \quad (4.2)$$

where \tilde{a}_n and \tilde{b}_n are Fourier coefficients; and Δt is the averaged time delay over the frequency of interest of the earthquake ground motion and can be obtained by curve-fitting of the phase plot between the reference and measured accelerations.

As shown in Table 2.1, the RMS error is particularly sensitive to time lag or delay. Assuming the RMS error between the reference and measured accelerations can be minimized by a time shifting of τ , the shifted measured acceleration can be represented as:

$$\tilde{a}_m(t) = \tilde{a}_0 + \lim_{L \rightarrow \infty} \sum_{n=1}^{\infty} \left(\tilde{a}_n \cos \frac{n\pi(t - \Delta t + \tau)}{L} + \tilde{b}_n \sin \frac{n\pi(t - \Delta t + \tau)}{L} \right) \quad (4.3)$$

where $\tilde{a}_m(t)$ is the shifted measured acceleration. Let the summation term denoted as $F(\tau)$, the squared error in the denominator of Eq. 2.1 in continuous time can be then calculated as:

$$\lim_{L \rightarrow \infty} \int_{-L}^L (a_r(t) - \tilde{a}_m(t))^2 dt = \lim_{L \rightarrow \infty} \int_{-L}^L (a_0 - \tilde{a}_0)^2 dt + \lim_{L \rightarrow \infty} 2 \int_{-L}^L (a_0 - \tilde{a}_0) F(\tau) dt + \lim_{L \rightarrow \infty} \int_{-L}^L F(\tau)^2 dt \quad (4.4)$$

By differentiating Eq. 4.4 with respect to τ and letting the differentiated equation equals to zero, the minimum of the squared error in the denominator of Eq. 2.1 can be found by using the orthogonality relationships as well as the product-to-sum identities between sine and cosine. Finally, it is confirmed that $\tau = \Delta t$ results in a minimum of RMS error as long as the frequency component satisfies the following condition:

$$\frac{a_n}{b_n} = \frac{\tilde{a}_n}{\tilde{b}_n} \quad n = 1, 2, 3, \dots \quad (4.5)$$

Conclusively, RMS error of acceleration time histories could be a proper index for shaking table performance assessment as long as time-shift correction is completed. The τ value that leads to a minimum RMS error can be obtained by trying incremental τ values.

4.2. Penalty factor on PGA error

PGA is an important index for the earthquake intensity and used to establish building design codes. For shaking table testing, PGA has become a critical input parameter for the test structures. A shaking table test can be controversial if the PGA of the reproduced ground motion is different (either less or larger) from the reference acceleration. However, the difference of PGAs would not affect the RMS error significantly especially when the rest of the time histories match with each other well. As a result, a penalty factor on PGA difference must be considered for the performance assessment of shaking table testing. In this study, a general penalty factor on PGA error can be represented as:

$$P_{\text{PGA}} = e^{\alpha |R_{\text{PGA}} - 1|} \quad (4.5)$$

where α is the penalty coefficient that controls the growing rate of the penalty factor, and R_{PGA} is the ratio of the achieved PGA to the desired PGA. For example, assuming that a 20% of PGA error forms the acceptable margin and the penalty factor is set 2 on the margin. The corresponding $\alpha = 3.47$ can be obtained and the relationship between P_{PGA} and R is shown in Fig. 4.1. Once the achieved PGA is out of the margin, the penalty factor is increasing rapidly.

4.3. Penalty factor on SA error

SA can be used to interpret the seismic response by a value related to the natural frequency of the structural vibration, providing a more agreeable approximation to the motion of a structure than the PGA value. A tolerance range in percentage of the SA within the earthquake frequency range can be determined depending on the test requirements. A penalty factor on SA is necessary for any specific SA value that exceeds the determined tolerance range of a shaking table testing. Similar with the concept of the penalty factor on PGA error, the penalty factor on SA error can be determined based on an acceptable region. However, SA is not a single value for a specified ground motion. It forms a one-to-one mapping from varying frequencies that within the interest of research. As a result, the penalty factor on SA error should not exist as long as the SA is within the acceptable margin. On the contrary, the penalty factor on SA error grows more rapidly than that on PGA error once the SA crosses the margin. Consequently, a general penalty factor on SA error is proposed as:

$$\begin{aligned} P_{SA} &= 1 && \text{within the margin} \\ P_{SA} &= e^{\beta|R_{SA}-M|} && \text{otherwise} \end{aligned} \quad (4.6)$$

where β is the penalty coefficient that controls the growing rate of the penalty factor; R_{SA} is the ratio of the achieved SA to the desired SA in the frequency range of interest that deviates the margin the most severely; and M is the mean value of the upper bound and lower bound of the acceptable margin. Take the requirements for testing nonstructural components for example (AC156, 2007), M is 1.1 when the SA of the achieved acceleration is required to meet the required response spectrum with a tolerance range of 90% to 130%. On the other hand, the value of β can be determined once the corresponding value of penalty factor is decided when the maximum SA is on the margin. For example, $\beta=3.47$ when the penalty factor on SA error is set 2 on the margin. The exemplated penalty factor on SA error is illustrated in Fig. 4.2.

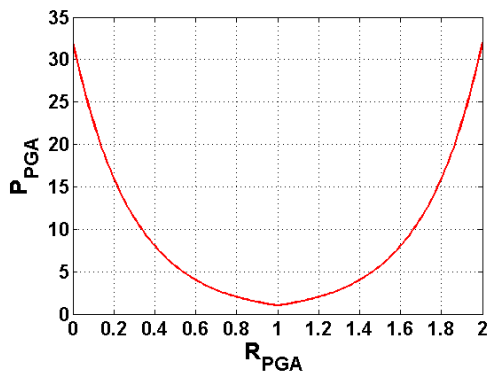


Figure 4.1 Penalty factor on PGA error with respect to the ratio of the achieved PGA to desired PGA

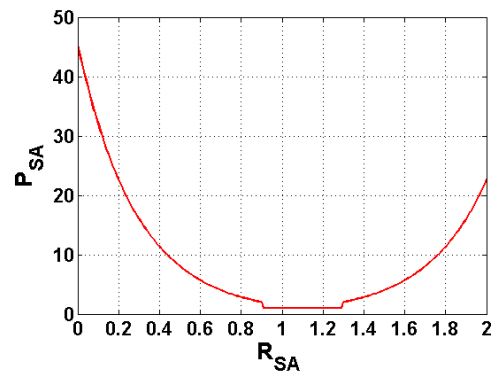


Figure 4.2 Penalty factor on SA error with respect to the maximum ratio of the achieved SA to desired SA

4.4. Performance Index

Considering the special cases that have been discussed in the previous section, a performance index (PI) for shaking table testing assessment is proposed by taking the advantages of both time-domain and frequency analyses. The performance index can be represented as:

$$PI(\%) = RMS_{error} \cdot P_{PGA} \cdot P_{SA} \quad (4.7)$$

For a generally shaking table test, a RMS error less than 5% of an acceleration time history after optimal time-shifting without adding penalty factors is considered excellent. In addition, taking the shaking table acceptance criteria of the MTS Systems Corporation for reference, the assessment performance index that can be applied to most of the shaking table testing is suggested as Table 4.1. It is noted that the penalty factors as well as the performance levels suggested in this study are preliminary and can be refined after conducting statistical analyses on a large number of shaking table test data.

Table 4.1 Performance index and the suggested performance levels

PI (%)	Performance
0-10	Excellent
10-20	Good
20-30	Fair
30-40	Marginal
≥ 40	Unacceptable

5. APPLICATION EXAMPLES

Shaking table tests using a uniaxial shaking table of National Center for Research on Earthquake Engineering (NCREE) in Taiwan are selected as the application examples of the proposed performance assessment method (Chen et al., 2014). This shaking table was made of steel with a dimension of 2500 mm x 1700 mm. The linear guide way system was adopted as the sliding mechanism between the platen and a reaction steel frame. The PID controller was provided by a domestic manufacturer in Taiwan which allows controlling three servo-hydraulic actuator with a maximum update rate of 10 kHz. An existing MTS servo-hydraulic actuator with ± 250 kN force and ± 250 mm stroke capacities was used. The maximum flow rate for the servo valve was 340 liters per minute. Three earthquake records were normalized to 100 gal and used to evaluate the performance of the shaking table: (a) the 1940 El Centro Earthquake, (b) the 1995 Kobe Earthquake, and (c) the 1999 Chi-chi Earthquake records. It is noted that the Chi-chi ground motion was recorded at the TCU129 seismic station. Two control algorithms were used including the existing PID controller as well as the PID controller with an additional feedforward (FF) controller proposed by Phillips and Spencer (2011).

The assessment method is separated into three steps: (a) to obtain the optimal RMS error by time shifting method; (b) to calculate the penalty factor based on PGA ratio; and (c) to calculate the penalty factor based SA ratio. Take the 1940 El Centro testing for example, Fig. 5.1 shows the time shifting versus RMS error relationship of the PID control case and PID with FF controller case. Accordingly, the optimal RMS error for each test is shown in the third column of Table 5.1. In the application example, the acceleration spectra within 0-15 Hz are used to assess the performance of shaking table testing. Figure 5.2 shows the SA of the PID control case and PID with FF controller case under the 100-gal 1940 El Centro ground motion. It is evident that the SA of the PID with FF controller case is always inside the predefined acceptable margin (90% to 130% of the desired response spectrum). However, the SA of the PID control case lies out of the margin when the structural frequency is higher than 3 Hz. The most severe deviated SA from the margin occurs at 8.9 Hz and the corresponding R_{SA} is 0.6. Table 5.1 lists all the required values in order to calculate the PIs for the six shaking table tests. All the three test cases driven by the PID controller are not acceptable in terms of performance levels after applying the penalty factors. On the contrary, two of the three shaking table tests activated by PID and FF controllers are considered good. It is evident that the performance of the shaking table was significantly improved after applying the additional FF controller to the control loop.

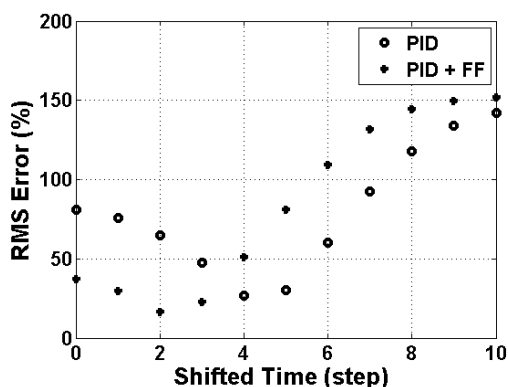


Figure 5.1 RMS error with respect to a variety of shifted time steps

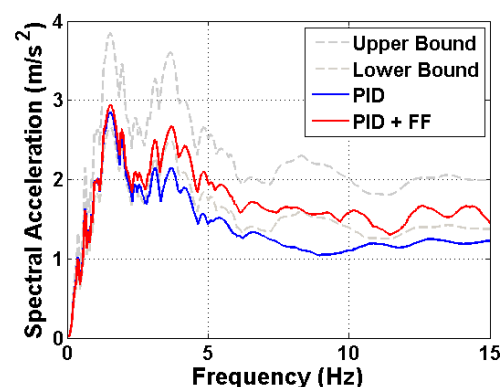


Figure 5.2 Spectral acceleration of shaking table testing of the 100-gal El Centro ground motion

Table 5.1 Performance index of the proposed assessment method for the application examples

Ground motion (1)	Controller (2)	RMS <i>error</i> (%) (3)	R_{PGA} (4)	P_{PGA} (5)	R_{SA} (6)	P_{SA} (7)	PI (%) (8)
El Centro	PID	26.55	0.97	1.11	0.60	5.66	166.80
	PID + FF	16.49	0.97	1.11	0.91	1.00	18.30
Kobe	PID	19.25	1.05	1.19	0.75	3.36	76.97
	PID + FF	12.22	1.04	1.15	0.90	1.00	14.05
Chi-chi	PID	43.05	0.72	2.64	0.54	6.97	792.15
	PID + FF	27.14	0.91	1.37	1.30	1.00	37.18

6. CONCLUSIONS AND FUTURE WORKS

An alternative performance assessment method for shaking table testing has been proposed considering both time-domain and frequency-domain analyses in this study. The efficacy and feasibility of the proposed method is investigated by using a set of experimental data of shaking table tests. User-defined penalty factors on peak ground acceleration and spectral acceleration are suggested so that the performance assessment method can be adaptive to different testing requirements. In the exemplified applications, the proposed assessment method distinguishes the compensated shaking table tests from the conventional PID-driven tests and demonstrates the efficiency of the feedforward controller for shaking table testing. In the future, statistical analyses will be conducted to derive the appropriate penalty functions and performance levels for most of the seismic shaking table testing.

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