

# Vibration Serviceability Analysis of Aluminum Pedestrian Bridges Subjected to Crowd Loading

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# ABSTRACT

Use of aluminum as material for pedestrian bridges is increasingly becoming popular due to its high strength-to-weight ratio and reduced susceptibility to corrosion during the service life of the bridge. However, these structures have low intrinsic damping and mass. As a result, they tend to be lively under operational loads and often exhibit large amplitude vibrations. Controlling the excessive vibration response and assessing the serviceability are the main design criteria for pedestrian bridges. Different codes of practice have been developed by different countries to assess the vibration serviceability of pedestrian bridges based on simplified models of pedestrian induced walking load. Two general approaches are mainly followed in different guidelines to design pedestrian bridges for vibration serviceability. The first approach focuses on avoiding natural frequencies of the structure that coincide with the normal walking frequency range. The second approach is to limit the vibration response in terms of the predicted acceleration within the desired comfort limit. This paper presents serviceability assessment of two full scale aluminium pedestrian bridges with different vibration characteristics. The natural frequencies of the bridges create near resonant conditions with the first as well as the higher harmonics of walking frequencies. Crowd testing was performed on the two bridges with different crowd densities and measurements of accelerations were recorded at three different locations along the bridge span in both vertical and lateral directions. The comfort-based evaluations of the two bridges were performed by comparing the measurements with the predicted and the acceptable limits of vibrations as recommended in codes and several guidelines. The present study demonstrates that the current codes and guidelines are not fully applicable to all kinds of pedestrian bridges, specifically for bridges with resonance in higher harmonics of walking. Also the study has shown extensive inconsistency in predicted responses by the guidelines. It is recommended that the current guidelines should account for resonance in higher harmonics of walking. Further experimental study is recommended on full scale pedestrian bridges with different dynamic characteristics to investigate the applicability of these guidelines for different kinds of footbridges.

KEYWORDS: Pedestrian bridges, serviceability, design guidelines, crowd load

#### **1. INTRODUCTION**

In recent years, serviceability of lightweight and slender pedestrian bridges has been the focus of many researchers. Due to their low mass and stiffness, these structures have natural modes, which often coincide with the human walking frequencies. As a result, these footbridges tend to suffer high amplitude vibrations leading to serviceability issues. Some of the famous incidents of serviceability failures of pedestrian bridges include the London Millennium footbridge (LMF) [1], the Pont du Solferino in Paris [2] and the T-Bridge, Japan [3]. The primary cause of these high profile incidents was excessive vibration under resonance with walking frequencies of the pedestrians. In design codes, the serviceability problems are considered by giving limits to the structural vibration modes to avoid resonance. These frequency limits are known as critical frequencies. If the natural vibration under the human induced excitations within acceptable values. The acceptable limits of vibrations are decided based on experiments and experiences on human perception to vibration level. While pedestrians are relatively insensitive to low amplitude vibrations in the vertical direction of bridge oscillation, a small level of lateral oscillation can affect walking behaviour. Various codes and guidelines have proposed limits related to lateral vibrations, but they are not consistent with each other [4].

It is expected that bridge vibrations are often enhanced by groups of pedestrians or continuous crowds as compared to single pedestrian loading. Hence it is important to consider design loads from multiple traffic scenarios in assessing serviceability of pedestrian bridges. After the high profile incident of LMF, a few guidelines [5-8] focused on crowd loading and corresponding synchronous vibration of pedestrian bridges. Although multiple simplified design methodologies have been developed to incorporate crowd effects, there is still a need to validate these guidelines for different bridge types. To the authors' knowledge, these guidelines have not been applied to aluminum pedestrian bridges. The main objective of this study is to assess several design guidelines [5-8] for assessing vibration serviceability of aluminum pedestrian bridges.

In the current study, experiments have been performed to investigate the dynamic behaviour of two aluminum bridges under crowd loading. The bridges have the same cross sectional and material properties but different spans. Due to its high strength-to-weight ratio, and the benefits over other metals in terms of life cycle cost, aluminum is increasingly being considered for use in bridge applications. However, no attention has been given towards the serviceability study of aluminum pedestrian bridge while these structures may experience severe serviceability issues under certain loading scenarios. The current study focuses in verifying the applicability of the current design guidelines for aluminum pedestrian bridges under crowd excitation.

# 2. DESCRIPTION OF EXPERIMENTS

### 2.1. The Testing Platform

Two aluminum pedestrian bridges of spans 12.2 m and 22.9 m, respectively, were studied. They were constructed solely for research purposes in the Structural Laboratory of University of Waterloo, Canada. The bridges were assembled with bolted connections from a patented modular product called Make-A-Bridge<sup>®</sup> by MAADI Group. The extruded members were T-6061 aluminum. Both specimens were 1.35 m in width and 1.140 m in height with identical cross sectional properties. The bridge can be constructed with or without lateral cross-bracing under the deck to vary the lateral stiffness. In this paper, results are presented for the case of no lateral cross-bracing modelling a bridge that is relatively soft, laterally. More details of the modular bridge specimens are presented elsewhere (Dey et al. [9]). The bridge specimens and one of the typical bolted joints are shown in Fig. 2.1. The 12.2 m and 22.9 m bridges weighed respectively 982 kg and 1,735 kg.



Figure 2.1 (a) Bridge specimen of span 12.2 m (b) Bridge specimen of span 22.9 m [9] (c) Typical bolted joint [9]

In order to measure the accelerations of the structures, the pedestrian bridges were instrumented with twelve low-frequency, high-sensitivity accelerometers, which have operable frequency range of 0.1 Hz to 200 Hz. The accelerometers were placed on the bottom chords at quarter and mid-points along the length, both laterally and vertically using aluminum mounting blocks. The acceleration data were recorded using three 4-channel A/D data acquisition modules (daisy-chained); model DT9837A manufactured by Data Translation.

Prior to conducting walking tests on the pedestrian bridges, set of modal tests were performed to estimate the

dynamic properties of the structures. A description of the tests and methodology to estimate the corresponding modal properties, which are used in this paper for the crowd loading analysis, is provided in a recent work by Dey et al. [9], The estimated modal natural frequencies and damping are listed in Table 2.1

	Late	eral	Vertical	
Bridge specimens	Natural frequency(Hz)	Damping Ratio	Natural Frequency (Hz)	Damping Ratio
12.2 m	2.3	0.250	11.8	0.012
22.9 m	1.2	0.012	4.5	0.008

Table 2.1 First modal frequencies and damping of the pedestrian bridges

# 2.2 Crowd Test

A set of pedestrian walking tests were performed on the two bridges including single as well as groups of pedestrians. The current study focuses only on the tests under multiple pedestrians. The tests conducted on the two bridges involved two people walking synchronously and asynchronously, and walking of groups of pedestrians with varying densities. Table 2.2 reports the number of pedestrians involved in different tests and corresponding average mass of the crowd crossing the bridge. For statistical significance, the crowd tests on the 12.2 m bridge specimen were repeated 30 times while 10 trials were conducted for each set of tests for the 22.9 m specimen. During the tests, the pedestrians were asked to walk at a normal pace to generate random crowd loading on the bridges. Fig. 2.2 shows several pictures of the crowd tests on the bridges.

	12.2 m pedestrian bridge			22.9 m pedestrian bridge			
No		Average mass (kg)	Standard			Standard	
	Description of tests		deviation	Description of tests	Average	deviation	
			of mass	Description of tests	mass (kg)	of mass	
			(kg)			(kg)	
1	2 persons walking	66 5	21	2 persons walking	67.5	11.6	
	synchronously	00.5	2.1	synchronously	07.5	11.0	
2	2 persons walking	66 5	2.1	2 persons walking	67.5	11.6	
	asynchronously	00.5	2.1	asynchronously	07.5	11.0	
3	4 persons or 0.2 $p/m^2$	69	14	4 persons or 0.1 $p/m^2$	65	4.1	
4	8 persons or 0.5 $p/m^2$	67	9	6 persons or 0.2 $p/m^2$	72	7.5	
5	17 persons or $1.0 \text{ p/m}^2$	68	14	10 persons or 0.3 $p/m^2$	70	11.8	
6				15 persons or 0.5 $p/m^2$	71	10.8	
7	7		$22 \text{ persons or } 0.7 \text{ p/m}^2 \qquad 68$		12.4		

Table 2.2 Test matrix for the two bridges



Figure 2.2 Crowd tests on the 12.2 m (left) and 22.9 m (right) bridge specimens

#### **3. OVERVIEW OF DESIGN GUIDELINES ON PEDESTRIAN BRIDGES**

There are only few guidelines [5-8], which have incorporated the effect of crowd on the serviceability of pedestrian bridges. Although these guidelines are based on different assumptions and approaches to consider the crowd effect in the bridge and corresponding response predictions, they evaluate the serviceability of the pedestrian bridges through two main steps. The first step involves limiting the structural frequencies to those outside a critical frequency range (Table 3.1). When the natural frequencies of the structures are outside the range, they automatically satisfy the maximum comfort level for the occupants. On the other hand, if the frequencies fall within the critical range, the second step of the design methodology is followed. In this step, a detailed dynamic analysis has to be performed and the predicted vibration level should be within the acceptable limit of vibration in accordance to the guideline. These acceptable limits are given in the design guidelines and there is generally no consensus on these limits. As these limits are based on human perception, which may vary between individuals, these limits are by themselves highly uncertain. However, current guidelines have proposed deterministic values of these limits based on past experiences and experiments. Table 3.1 describes the critical frequency ranges and the acceptable limits according to different guidelines.

Codes		Critical Free	quencies (Hz)	Limit Acceleration (m/s <sup>2</sup> )	
		Vertical	Lateral	Vertical	Lateral
	Eurocode 5	<5	<2.5	0.70	0.4
	British National Annex to Eurocode 1	<8	<1.5	2.0 (upper bound)	
	SÉTRA	1-5	0.3-2.5	1.0 (mean)	0.3 (mean)
	HIVOSS	1.25-4.6	0.5-1.2	1.0 (mean)	0.3 (mean)

Table 3.1 Critical frequencies and acceptable limits of vibration by current guidelines

#### 3.1. Methodologies for Dynamic Analysis

In order to evaluate the vibration level under crowd excitations, dynamic response analysis has to be performed. The guidelines, mentioned in Table 3.1, have considered various modeling approaches of human induced loads and corresponding calculation methodologies for dynamic response. While, Eurocode 5 [5] has proposed direct equations for response predictions, the British National Annex [6], the French guideline SÉTRA [7] and the European guideline HIVOSS [8] have characterized the pedestrian loading under crowd conditions and predicted the response either through the SDOF approach or finite element analysis.

The Eurocode 5 has been developed to estimate acceleration response under several persons walking on a timber pedestrian bridge. As the methodology is not material dependent, it has been applied for aluminum bridges in the current study. For n persons crossing the bridge, the vertical and lateral accelerations of the bridge can be estimated respectively through the following two equations:

$$a_{\nu,n} = 0.23 a_{\nu,1} n k_{vert} \tag{3.1}$$

$$a_{h,n} = 0.18 \, a_{h,1} n k_{hor} \tag{3.2}$$

where,  $a_{v,l}$  is the vertical response under one individual walking, which is given by:

$$a_{\nu,1} = \begin{cases} \frac{200}{M\xi}, & \text{for } 0 < f_{\nu} \le 2.5\\ \frac{100}{M\xi}, & \text{for } 2.5 < f_{\nu} \le 5.0 \end{cases}$$
(3.3)

 $a_{h,l}$  is the lateral response under one individual walking, which is given by:

$$a_{h,1} = \frac{50}{M\xi}, 0.5 \le f_h \le 2.5 \tag{3.4}$$

In the above equations, M and  $\xi$  are the mass and damping of the structure and the factors,  $k_{vert}$  and  $k_{hor}$ , depend on the vertical and lateral structural frequencies,  $k_v$  and  $k_h$ .

The British National Annex to Eurocode 1 has proposed different load models in the vertical direction based on either pedestrians walking in a group together or continuous traffic uniformly distributed over the bridge deck area. However, this guideline has not proposed any dynamic response analysis in the lateral direction. It just requires checking of the stability of the structure in the lateral direction through a damping parameter depending only on the pedestrian and the structure mass along with the structure damping. The vertical moving load, applied on the bridge under n pedestrians crossing the bridge together, is given by:

$$F = (0.4G)k(f_v)\sqrt{1 + \gamma(n-1)\sin(2\pi f_v t)}$$
(3.5)

where, G is the pedestrian weight,  $k(f_{\nu})$  is combined factor as a function of vertical natural frequency of  $f_{\nu}$ , and  $\gamma$  is a factor to allow for the unsynchronised combination of pedestrian actions depending on the damping of the structure. For the continuous crowd scenario, where steady state is achieved, the applied load is distributed over the area, which is given by:

$$w = 1.8 \frac{(0.4G)}{A} k(f_v) \sqrt{\frac{\gamma n}{\lambda}} \sin(2\pi f_v t)$$
(3.6)

In the above equation,  $\lambda$  is the factor that reduces the effective number of pedestrian in proportion to the enclosed area of the mode of interest and equals to 0.634 for the first mode shape of a simply supported beam.

The French SÉTRA and the European HIVOSS guidelines have a similar approach to predict pedestrian bridge responses. They only differ in considering the effect of additional mass from crowd in the response calculations. While HIVOSS incorporates the additional mass effect if it crosses more than 5% the modal mass of the pedestrian bridge, the SÉTRA guideline does not have any limitations on this parameter. The SÉTRA guideline estimates two different bounds of responses considering structural frequencies of the empty structure and the structure with the occupied pedestrians. For the current study, the HIVOSS guideline has not been considered for response predictions due to its similarity with the SÉTRA guideline. The SÉTRA guideline specifies four different classes of bridges with five different traffic classes. The random load due to a stream of *n* pedestrians corresponding to a specific crowd density ( $d p/m^2$ ) is simplified to an equivalent number of pedestrians ( $n_{eq}$ ), which are uniformly distributed over the bridge deck, i.e.:

$$n_{eq} = \begin{cases} 10.8\sqrt{n\xi}, for \ d \le 1.0\\ 1.85\sqrt{n}, for \ d > 1.0 \end{cases}$$
(3.7)

The magnitude of distributed load per area (A) in any direction (vertical or lateral) is defined as:

$$p = \frac{\alpha G \psi n_{eq}}{A} \tag{3.8}$$

In the above equation,  $\alpha$  is the dynamic load factor and is 0.4 and 0.1 for the first and second harmonics in the vertical direction. It has values of 0.05 and 0.01 for, respectively, the first and second harmonics in the lateral direction.  $\Psi$  is the reduction factor and is a function of structural frequency.

In the current study, as both the bridges were simply supported and can be assumed to behave as simply supported beams, the resonant maximum acceleration under the load models from the British National Annex and the SÉTRA guideline can be estimated through the SDOF approach, which is given by:

$$a_m = \frac{P_m}{2\xi_m M_m} \tag{3.9}$$

Here,  $P_m$ ,  $M_m$  and  $\xi_m$  are the generalized (modal) load, mass and damping ratio of the structure. For a distributed load p per unit length, the generalized load is given by  $2pL/\pi$  for first mode of vibration and is half for the second mode of vibration. Similarly, the modal load for moving load F for any mode is  $2F/\pi$ .

# 4. EVALUATION OF SEVICEABILITY

In this section, serviceability of the two bridges under different crowd scenarios is evaluated according to the design guidelines listed in Table 3.1 (except HIVOSS). Firstly, the fundamental frequencies of the bridges in both vertical and lateral direction are compared with the critical ranges of frequencies. If the natural frequencies fall

within this range, the second step of the assessment is performed, otherwise it is assumed that the structures automatically satisfy the comfort limits as proposed by the corresponding guideline.

### 4.1. Evaluation through Frequency Criteria

The 12.2 m bridge specimen has its fundamental lateral frequency at 2.3 Hz while the first vertical frequency is 11.81 Hz. According to Table 3.1, the pedestrian bridge automatically satisfies the maximum comfort level as the vertical frequency is outside the critical range. Thus, further analysis is not required in this direction. However, the lateral frequency lies within the critical range according to Eurocode 5 and the SÉTRA guideline and thus, dynamic analysis needs to be performed in lateral direction to evaluate its serviceability. A similar evaluation has also been conducted for the 22.9 m bridge specimen with lateral and vertical frequencies being 1.2 Hz and 4.5 Hz, respectively. All of the codes recommend dynamic evaluation of vibration to assess the serviceability in both the directions for this structure. In the following section, the dynamic analysis of the pedestrian bridges has been performed under crowd excitation and the maximum predicted and measured responses are compared with the limits to assess the serviceability.

#### 4.2. Evaluation of Vibration level to Crowd Excitation

Eurocode 5 has suggested predicting the peak acceleration of the pedestrian bridges under crowd excitation through the application of the direct equations (Eq. 3.1 and Eq. 3.2). The British National Annex and SÉTRA guidelines apply the load models as proposed on the pedestrian bridges and estimate peak acceleration through the SDOF approach (Eq. 3.9). The maximum acceleration occurred at the mid-point of the bridge spans in all cases. Hence only the peak measurements at the centre of the bridges have been considered for this study. The results of the serviceability assessment under different pedestrian loading scenarios are plotted in Figure 4.1 for both the pedestrian bridges in the vertical as well as the lateral direction.



Figure 4.1 (a) Comparison of measured and predicted peak acceleration with acceptable limits for (a) the 12.2 m pedestrian bridge in vertical direction, (b) the 12.2 m pedestrian bridge in lateral direction, (c) the 22.9 m pedestrian bridge in vertical direction, and (d) the 22.9 m pedestrian bridge in lateral direction (here 'P' stands for pedestrian)

As discussed in the previous section, the 12.2 m bridge specimen is safe in terms of serviceability and no dynamic analysis is required according to all of the codes. Despite this fact, the measured maximum acceleration of the pedestrian bridges under different traffic scenario are compared with the acceptable vibration limits in Fig. 4.1 (a) and it is observed that the measurements have crossed the limits specified by Eurocode 5 and the SÉTRA guideline. During the two pedestrian walking case, the pedestrians were walking in their normal walking pace,

which is near 2 Hz. It is expected that the higher level of vibration might result from a near resonant condition with the sixth harmonics of the walking frequency. Similarly, for a denser crowd  $(1.0 \text{ p/m}^2)$ , the speed of walking become slow and sometimes, the structural frequency of 11.8 Hz may have near resonant condition with the seventh harmonic of the slow walking frequency (1.67 Hz), and thus generates a higher level of vibration.

In the lateral direction, the design guidelines have underestimated the measurements (Fig. 4.1 (b)). The predictions have not indicated any serviceability issue although measurements have crossed the limit values during all loading scenarios. The significant lower predictions might be due to the measured high damping values (20%) in the structures. None of the guidelines specify any damping values for aluminum alloys, although the codes suggest a damping of as low as 0.4% -1% for metals like steel. As the damping of a structure depends on the fixity of connections, support condition and friction between structural and non-structural components, the high value of damping is not surprising. However, in reality the actual damping of structures is not known at the design stage and it is common practice to assume the damping values suggested in the codes. It is obvious that a lower value of damping will increase the predictions. It will be interesting to see in future study the sensitivity of these predictions to the effect of damping uncertainty, which is not in the scope of the current work.

Through the evaluation of the frequency criteria, dynamic analysis has to be performed for the 22.9 m bridge specimen in both the directions. Fig. 4.1(c) and Fig. 4.1(d) show the serviceability assessment of the bridge specimen in the vertical and lateral directions respectively. It is observed that the predictions are overestimating the measurements in the vertical direction. The peak measurements under any loading scenario has crossed the limits defined by the SÉTRA guideline and the Eurocode 5, leading to a serviceability issue, although the measurements are safe according to the upper limit of the British National Annex. Only denser traffic leads to a serviceability issue according the British National Annex. However, all the predictions are over the limits and lead to a very conservative design scenario. It is worth mentioning that the vertical mode of the bridge (4.5 Hz) falls within the second harmonic of the walking frequency range and thus has a chance of resonating with the faster walking speed. As all the codes consider resonance up to the second harmonic, and overestimate the response in resonant condition [12], the predictions are very high as compared to the measurements.

Similar to the 12.2 m bridge specimen, the predictions are underestimated in lateral direction for the 22.9 m bridges specimen, although the SÉTRA guideline reports a serviceability issue under denser crowd loading like the measurements. In any case of crowd loading, the Eurocode 5 predicts a very low vibration level below the limits. As mentioned earlier, the estimated high damping might be one of the reasons behind these low predictions for the 22.9 m bridge specimen. Further study is recommended, however, to confirm this.

### 5. CONCLUSIONS

In this study, serviceability assessment of two full scale aluminum pedestrian bridges has been performed in accordance with three design guidelines, namely: Eurocode 5, the British national Annex to Eurocode 1 and the French SÉTRA guideline, under different crowd loading cases. A suite of tests was conducted on these two bridges with different groups of pedestrians crossing each structure. The oscillations of the bridges were measured in terms of the acceleration parameter at different locations on the bridges. Firstly, the structural frequencies in the vertical as well as the lateral directions were verified with the critical ones to decide upon the requirement of dynamic analysis of the bridges. Except for the 12.2 m bridge specimens in the vertical direction, peak responses to crowd loading had to be compared with the acceptable limits to assess the corresponding serviceability. An important conclusion of the experimental investigation is that resonance with higher than second harmonics can cause serviceability issues. It is recommended to investigate in the future the maximum numbers of harmonics that should be incorporated in the design for serviceability. The peak responses in the vertical direction for the 22.9 m bridge specimen are also in agreement with the previously found fact that the codes of practices overestimate the response in the resonant scenario. As damping is one of the important parameters, along with structural vibration frequency, in designing for serviceability, it is suggested that the effect of damping on the predictions by different guidelines should be further explored.

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