Evaluation of Stiffness Reduction of RC Columns during Earthquakes Based on Acceleration Measurements

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ABSTRACT

This research aims at evaluating local structural conditions of an RC viaduct during earthquakes through the identification of loading stiffness and unloading stiffness using acceleration data. RC columns under seismic excitation have the characteristics that the stiffness when the load is being applied reduces more severely than the stiffness when the load is being removed. The stiffness reduction between the unloading and the loading stiffness is proposed for the evaluation of the nonlinear characteristics of the columns under excitation. In order to identify the loading and unloading stiffness separately from the whole time history data, an existing method for the physical model identification and its improved version are first discussed. Then, the loading and the unloading time intervals are defined by estimated displacement and restoring force time histories. The loading and the unloading stiffness is identified by the data during the loading and the unloading time intervals and then the stiffness reduction ratio is calculated. Simulation of a nonlinear SDOF system shows that the proposed method is capable of estimating stiffness reduction in the hysteresis loops. The method is also applied to a nonlinear viaduct model and the potential of localizing columns with severe nonlinearity is demonstrated.

KEYWORDS: Localized structural assessment, RC viaduct, earthquakes, loading stiffness, unloading stiffness, stiffness reduction ratio

1. INTRODUCTION

RC viaducts are one of the most typical and important structure among the transport infrastructure. They play an important role in our society ranging from human mobility to physical distribution and its shutdown would cause severe social and economic impacts. Therefore, Viaducts should be monitored, maintained and operated appropriately even under the extreme conditions caused by natural hazards. On the other hand, Japan is widely recognized as a seismically active country and viaducts have sometimes suffered from damage due to strong earthquakes. In 2011, the Great East Japan Earthquake occurred and JR Yamanote line suffered from 17 hours of suspension, in which 3 hours, more than 15 hours, and 8.5 hours were spent on the evacuation guidance, inspection/maintenance, and operation planning phases, respectively [1]. Apart from catastrophic earthquakes like the Great East Japan Earthquake, railroad operations under the management of JR-EAST have been regulated for 171 times in 14 years from 1987 to 2000 because of earthquakes [2]. These cases indicate that the inspection/maintenance phase takes much of the time necessary for the resumption of railroad operations.

Though the importance of quick inspections is widely recognized, there are several difficulties in achieving it [1]. The first problem is about inspector arrangement. After earthquakes, communication failure often occurs and it prevents railway and highway companies from gathering inspectors and making contacts with them. Traffic congestion after earthquakes also makes the access to the facilities difficult. The other problem is about decision making in inspection planning. Generally, decisions on whether an area is inspected or not and whether the area is inspected on foot or inspected from trains are made based on information such as Spectral Intensity [2] [3], seismic intensity [4] [5] and maximum ground acceleration [2]. However, these kinds of information tend to have low resolution and do not directly indicate damage to the facilities. This often leads to overly safe decisions on the inspection planning and unnecessary inspections are often conducted. In the case of JR-EAST
[2], the railway facilities were damaged only during 2 earthquakes out of 171 earthquakes which caused the regulation of railway operation. These investigations show that a detailed structural assessment method based on measurement data from sensors is effective in decreasing unnecessary time-consuming human inspections.

Structural assessment methods during earthquakes based on acceleration measurement data have been actively pursued because of the ease of acceleration measurement. A group of works based on acceleration measurement evaluate RC structures during earthquakes through modal parameters such as natural frequency [6] and equivalent stiffness derived from natural frequency [7]. A problem of this approach is that modal parameters basically represents the system’s global characteristics and their change cannot be directly associated with the local change in the system’s properties, i.e. damage. On the other hand, there is a research [8] which evaluates restoring force characteristics of each story of MDOF models by acceleration measurement data as shown in Figure 1.1. However, the applicability of this method to viaducts is limited because the applicability of the MDOF model in Figure 1.1 to viaducts is limited.

Based on the discussion above, this research aims at localized structural assessment of RC viaducts under seismic excitation by identifying the stiffness reduction of each columns based on acceleration measurement data. In Section 2, stiffness variation of RC columns in their hysteresis loops is explained and parameters examined for the localized structural assessment are defined. In Section 3, an existing method for the identification of physical model, i.e. mass-normalized damping, stiffness and input matrices, is introduced and the method is improved to identify the varying stiffness in the hysteresis loops. In Section 4, the overall procedure for the evaluation of stiffness reduction in the hysteretic loops is presented. The proposed method is first applied to a nonlinear SDOF model in Section 5 and stiffness reductions of the model during various excitations are examined. The method is then applied to the excitation and response time histories of a nonlinear viaduct model under seismic excitation in Section 6. Section 7 presents the conclusions.

2. NONLINEAR CHARACTERISTICS OF RC COLUMNS UNDER SEISMIC EXCITATION

A typical shape of the load-displacement plot of an RC column is illustrated in Figure 2.1. The hysteresis loop of the column can be characterized by two different stiffness values, namely loading stiffness and unloading stiffness. The loading stiffness, $K_{\text{loading}}$ in Figure 2.1, is a gradient of the hysteresis loop when the load is being applied and it reduces sharply as the column experiences large deformation. On the other hand, the unloading stiffness, $K_{\text{unloading}}$ in Figure 2.1, is a stiffness value when the load is being removed. This stiffness is less dependent on the system’s deformation and maintains relatively large values during the whole excitation process. The loading and the unloading stiffness provide relevant information of the shape of the hysteresis loops and are often used in the modeling of nonlinear characteristics of RC columns in practical situations [9].

In this paper, the stiffness reduction defined as

$$R_{\text{stiff}} = \frac{K_{\text{unloading}} - K_{\text{loading}}}{K_{\text{unloading}}} \quad (2.1)$$

is proposed as a damage indicator of the column. When the stiffness reduction ratio is large, the loading stiffness takes much smaller value than the unloading stiffness and the severe degradation of the column is indicated. On the other hand, when the stiffness reduction ratio takes a small value, the column can be regarded as close to linear. This stiffness reduction ratio can be a relevant indicator of the column’s nonlinear characteristics.
3. DEVELOPMENT OF A PHYSICAL MODEL IDENTIFICATION METHOD FOR THE ESTIMATION OF THE LOADING AND THE UNLOADING STIFFNESS

This section first introduces an existing method for the identification of a physical model, i.e. mass-normalized damping, stiffness and input matrices. The method is not directly applicable to the identification of the loading and the unloading stiffness because of the linear assumption employed in the identification process. Therefore, an improved version of the physical model identification method is developed to identify the loading and unloading stiffness separately.

3.1. A Least Square Physical Model Identification method (LSID)

This method [10] identifies mass-normalized damping, stiffness and input matrices from the ground acceleration and the response acceleration, velocity and displacement time histories. The equations of motion at time \( t = t_1, t_2, ..., t_N \) is expressed as

\[
C \dot{x}(t_1) + K x(t_1) + B \ddot{x}_g(t_1) = -\ddot{x}(t_1)
\]

\[
C \dot{x}(t_2) + K x(t_2) + B \ddot{x}_g(t_2) = -\ddot{x}(t_2)
\]

\[
\vdots
\]

\[
C \dot{x}(N_{\text{data}}) + K x(N_{\text{data}}) + B \ddot{x}_g(N_{\text{data}}) = -\ddot{x}(t_N)\]  

in which \( C, K, B \) are mass-normalized damping, stiffness and input matrices, respectively. \( \ddot{x}_g \) is measured ground acceleration and \( \dot{x}, \ddot{x} \) and \( x \) are acceleration response, velocity and displacement response, respectively. The velocity and the displacement response are estimated by the integral of the measured acceleration time history by trapezoidal rule and the following high-pass filtering. The matrices \( C, K, B \) can be calculated by

\[
[C \ K \ B]^T = R^+ b
\]  

in which

\[
R = [\begin{array}{cccc}
\dot{x}(t_1) & \dot{x}(t_2) & \cdots & \dot{x}(N_{\text{data}}) \\
 x(t_1) & x(t_2) & \cdots & x(N_{\text{data}}) \\
 \ddot{x}_g(t_1) & \ddot{x}_g(t_2) & \cdots & \ddot{x}_g(N_{\text{data}})
\end{array}]^T
\]  

\[
b = -[\ddot{x}(t_1) \ \ddot{x}(t_2) \ \cdots \ \ddot{x}(t_N)]^T
\]

and \( R^+ \) is the pseudo-inverse of the matrix \( R \).

A problem of this method is that a linear model shown in Eq. 3.1 is fitted to the excitation and the response time history data. In particular, the restoring force always takes the same value for the same displacement and it is impossible to identify two different stiffness values, i.e. the loading and the unloading stiffness, from a set of time history data.

3.2. The Improved Version of the Least Square Identification Method (Improved LSID)

A general form of equation of motion with hysteretic damping and stiffness can be expressed as

\[
\ddot{x}(t) + \int_{\ddot{x}(0)}^{\ddot{x}(t)} C(\tau) d\dot{x}(\tau) + \int_{\ddot{x}(0)}^{\ddot{x}(t)} K(\tau) d\dot{x}(\tau) = -B \ddot{x}_g(t)
\]  

in which the time-varying hysteretic characteristics of the mass-normalized damping and stiffness matrices are
represented by the time index. By taking the difference of the generalized equation of motion between time \( t \) and \( t + 1 \), the incremental form of the equation of motion in Eq. 3.5 is derived as follows;

\[
d\ddot{x}(t) + \int_{x(t)}^{x(t+1)} C(\tau) \, d\dot{x}(\tau) + \int_{x(t)}^{x(t+1)} K(\tau) \, dx(\tau) = -Bd\ddot{x}_g(t)
\]

When the sampling period is sufficiently small and the mass-normalized damping and stiffness matrices can be regarded as constant during the time step, Eq.3.6 can be approximated by

\[
d\ddot{x}(t) + C(t)d\dot{x}(t) + K(t)dx(t) = -Bd\ddot{x}_g(t)
\]

The improved LSID solves the incremental form of the equations of motion in Eq.3.7 at time \( t = t_1, t_2, ..., t_N \) simultaneously instead of solving linear equations of motion simultaneously as shown in Eq.3.1. By this approach, averaged mass-normalized damping and stiffness matrices only during the selected time intervals is calculated. An advantage of this improved method is that Eq.3.7 in each time interval approximates the system’s nonlinear characteristics and the loading/unloading stiffness can be identified separately using time intervals when the load is being applied/removed, respectively.

4. OVERALL FLOW OF THE EVALUATION OF STIFFNESS REDUCTION

The evaluation of stiffness reduction of an RC column in its hysteresis loop starts with the determination of loading and unloading time intervals. The loading and the unloading intervals are then used to identify the loading and the unloading stiffness of the column. This section explains these two steps in detail.

4.1. Determination of the loading and the unloading time intervals

The determination of the loading and the unloading time intervals in this paper is based on the displacement and mass-normalized restoring force time histories estimated by the measured acceleration data. The displacement time history can be estimated by the integral of the acceleration time history data by the trapezoidal rule followed by high-pass filtering. The rough estimation of the mass-normalized force can be made by identifying mass-normalized damping and input matrix by the improved LSID using the whole time history of interest and computing

\[
-\ddot{x}(t) - C\dot{x}(t) - B\ddot{x}_g(t)
\]

The determination of the loading and the unloading time intervals by the estimated displacement and force time history data proceeds twofold as shown in Figure 4.1. First, the turning points in the hysteresis loops are determined. Each turning point is defined as the point where the sign of the displacement increment changes and the signs of the force at the adjacent turning points are the opposite (Figure 4.1(a)). Then, the loading time intervals are defined as the intervals before the turning points when the displacement is between 50% and 90% of the value at the turning points (Figure 4.1(b)). The unloading time intervals are defined in a similar manner by picking up time intervals after the turning points when the displacement is between 75% and 95% of the value at the turning points. Time history data during the loading and the unloading time intervals thus determined are separately used for the identification of the stiffness of each column when the load is being applied and removed, respectively.

4.2. Evaluation of the loading and the unloading stiffness of each column

The mass-normalized stiffness matrix when the load is being applied and removed, \( K_{\text{loading}} \) and \( K_{\text{unloading}} \) can be identified by applying the improved LSID described in Section 3.2 to the time history data during the loading and the unloading time intervals defined in Section 4.1. When the measurement data of a viaduct presented in Figure 4.2(a) are obtained at 5 measurement points indicated by the red points, the identified model is a simplified 5DOF model shown in Figure 4.2(b). The loading and the unloading stiffness of the spring placed between the \( i^{th} \) measurement point and the ground, \( K_{\text{loading}}^i, K_{\text{unloading}}^i \) is calculated by applying the same
displacement $1$ at each measurement point, i.e.

$$K_{\text{loading}}^{i} = K_{\text{loading}}^{i_{th \text{row}}} \cdot 1 \tag{4.2}$$

$$K_{\text{unloading}}^{i} = K_{\text{unloading}}^{i_{th \text{row}}} \cdot 1 \tag{4.3}$$

When the system’s displacement is $1$, there is no relative displacement between any of the two measurement points and the restoring force by the springs between two measurement points is 0. Therefore, the restoring force calculated by Eq.4.2 and Eq.4.3 is the restoring force only by the spring connecting the ground and the $i^{th}$ measurement point when the displacement $1$ is applied, namely stiffness of the $i^{th}$ spring. The loading and the unloading stiffness thus calculated is used to compute the stiffness reduction ratio defined by Eq.2.1.

5. EVALUATION OF STIFFNESS REDUCTION RATIO OF A SDOF MODEL

In this section, the proposed method is validated by a nonlinear SDOF model presented in Figure 5.1. A nonlinear hysteretic model of an RC column designated in the railway design standard [9] is implemented in this model. A high-pass filter with the cutoff frequency of 0.3Hz is used to estimate the displacement response. The end portions of excitation and response time history with the length of 20s (2000 points) are used for the identification of the loading and the unloading stiffness. In this case, the original model for data generation shown in Figure 5.1 and the model identified by the improved LSID method is exactly the same and the stiffness reduction ratio of the identified model can be compared with the ratio of the original model.

The comparison of the stiffness reduction ratios are made for various earthquakes and the results are shown in Figure 5.2. The difference between the stiffness reduction ratio obtained by the improved LSID and its analytical value is 3.30% at maximum, while the identification of stiffness reduction in the hysteresis loop is shown to be impossible by the LSID method.

6. EVALUATION OF STIFFNESS REDUCTION RATIO OF THE 3D VIADUCT MODEL
In this section, the proposed method is applied to the nonlinear model of a viaduct shown in Figure 4.2(a). The nonlinearity of the viaduct is implemented by inserting nonlinear rotational springs with RC nonlinear characteristics [9] into the upper and the lower part of each column. High-pass filter with the cutoff frequency of 0.3Hz are applied after the numerical integration.

In this case, a comparison of the stiffness reduction ratio with its analytical value is impossible because the original model shown in Figure 4.2(a) is different from the identified model in Figure 4.2(b) and the analytical value of the stiffness reduction ratio which rigorously corresponds to the ratio of each column of the identified model in Figure 4.2(b) does not exist. Therefore, for the purpose of the comparison with the analytical values, stiffness against inertia force \( K_{\text{inertia}} \) is defined as

\[
\mathbf{x}_{\text{inertia}} = \mathbf{K}^{-1}(-\mathbf{1})
\]  

\[
(K_{\text{inertia}})^{i\text{th spring}} = \frac{-1}{(x_{\text{inertia}})^{i\text{th element}}}
\]

in which \( \mathbf{x}_{\text{inertia}} \) and \( \mathbf{K} \) in Eq.6.1 are the displacement of the system against inertia force with the acceleration \( \mathbf{1} \) and mass-normalized stiffness matrix, respectively. The stiffness against inertia force for each spring in the identified model is calculated by dividing the inertia force \( (-\mathbf{1}) \) by the displacement against the inertia force at the \( i\text{th} \) measurement point.

The viaduct model is first excited by the NS component of the Great East Japan earthquake measured at Sendai and the identified and the analytical values of the stiffness against inertia force are shown in Figure 6.1. In this analysis, the loading and the unloading time intervals are picked up from 5.0s after the maximum displacement point because the nonlinear characteristics of RC columns depend on the maximum experienced displacement and the accuracy of data around the maximum displacement point is not high because of the high-pass filtering applied to estimate the velocity and the displacement time histories. In this case, the error between the identified and the analytical values of the stiffness against inertia is less than 3%. The stiffness reduction ratio for each spring numbered 1 to 5 in Figure 4.2(b) is presented in Figure 6.2 and the figure shows that the all columns
Then, damage is introduced to the viaduct model by changing the nonlinear characteristics of the nonlinear spring 9 and 19 in Figure 4.2(a). The load-displacement plots of the nonlinear springs when the damaged viaduct model is excited by the same earthquake as the case above are shown in Figure 6.3. The figures show that the RC columns structurally associated with the 5th measurement point shows severe loss of restoring force due to the stiffness reduction during the excitation.

The portion of data from 5.0s after the maximum displacement point is used for the identification of the loading and the unloading stiffness and the stiffness reduction ratio is calculated. The identified and the analytical values of the stiffness against inertia force of each column numbered 1 to 5 in Figure 4.2(b) during the loading and the unloading time intervals are shown in Figure 6.4 and the stiffness reduction ratios for the springs 1 to 5 are presented in Figure 6.5. It should be noted that the unloading stiffness structurally relevant to the damaged columns also decreases as shown in Figure 6.4. The values of the stiffness reduction ratios of the columns presented in Figure 6.5 are 55%, 50%, 48%, 55% and 66% for the spring 1 to 5 respectively and the ratio at the damaged column is more than 10% higher than the ratios at the other columns. The results are obtained only from the data of the damaged viaduct under strong excitation. If the measurement data during other previous...
earthquakes with smaller amplitudes are available, the identified unloading stiffness is closer to the linear stiffness of the columns and the stiffness reduction ratio can be a more sensitive damage indicator.

7. CONCLUSIONS

This paper proposes a localized structural assessment method for the columns of an RC viaduct under seismic excitation based on acceleration measurement. The proposed method identifies the loading stiffness and the unloading stiffness in the hysteresis loops separately and examines the stiffness reduction ratio defined by the loading and the unloading stiffness. This method enables the localized evaluation of the shape of the hysteresis loops of each columns of the viaduct.

An improved system identification method for the identification of mass-normalized stiffness matrix was first developed to identify the loading and the unloading stiffness separately from measured acceleration time history data. By employing the improved system identification method, the stiffness reduction ratio was successfully calculated for the nonlinear SDOF model in Section 5. The proposed method is applied to the 3-dimensional viaduct model and the columns with exceptionally low restoring force were localized by examining the stiffness reduction ratio.

In the validation by the viaduct model, the excitation had a long duration and sufficient number of the loading and the unloading time intervals were defined. However, it is an ideal case and in most cases the number of reliable data points is not sufficient for the identification of the loading and the unloading stiffness. Furthermore, nonlinear characteristics of real structures are generally more complex than the nonlinear model used in this paper. To address these problems, estimation of the loading and the unloading stiffness by combining data from multiple seismic excitations can be effective. Further validation of this method using multiple excitation data can lead to a reliable localized structural assessment method for RC viaducts applicable to real conditions.

REFERENCES