6th International Conference on Advances in Experimental Structural Engineering
11th International Workshop on Advanced Smart Materials and Smart Structures Technology
August 1-2, 2015, University of Illinois, Urbana-Champaign, United States

Simplified Flexural Design of Bolted Side-Plated Beams with Partial Interaction

L.Z. Li [1], C. J. Jiang [2], R.K.L. Su [3]

1 *Assistant Professor, Research Institute of Structural Engineering and Disaster Reduction, College of Civil Engineering, Tongji University, Shanghai 200092, China.*
E-mail: lilingzhi@tongji.edu.cn

2 *PhD Candidate, Research Institute of Structural Engineering and Disaster Reduction, College of Civil Engineering, Tongji University, Shanghai 200092, China.*
E-mail: 99jchj@tongji.edu.cn

3 *Associate Professor, Dept. of Civil Engineering, University of Hong Kong, Pokfulam Road, Hong Kong, China.*
E-mail: klsu@hkucc.hku.hk

ABSTRACT

Reinforced concrete (RC) beams can be strengthened by bolting steel plates to the side faces, which is known as the bolted side-plating (BSP) technique. Previous experimental studies have found that the performance of BSP beams is primarily controlled by the degree of partial interaction at the steel-RC interface. In this paper, a new simplified flexural design procedure for BSP beams taking into account the partial interaction between the steel plates and the RC beam is developed, based on the outcomes of the previous experimental and numerical studies conducted by the authors. Only minor modification to the conventional design formulas of RC beams is needed to cover the flexural design of the BSP beams, by adopting some optimum strain and curvature factors. The experimental results are then employed to verify the validation of the proposed model.

KEYWORDS: *Reinforced concrete beam; Strengthening design; Partial interaction; Bolted side-plating.*

1. INTRODUCTION

Bolting steel plates to the sides faces of reinforced concrete (RC) beams has become a widely accepted retrofitting technique due to its minimal space requirement and convenience of installation [1]. It can also avoid serious debonding and peeling failures at the ends of the steel plates [2, 3], which is more competitive compared to bonding steel plates or fibre reinforced polymers (FRPs) by adhesive mortar [4, 5]. Unlike the RC beams strengthened with steel plates on the beam soffit, which usually lead to an undesirable decrease in ductility, the BSP beams have proven to possess both increased flexural strength and ductility [6-8].

However, in the BSP beams, there is partial interaction caused by a combination of both longitudinal and transverse slips on the plate–RC interface [9-11]. The degree of partial interaction, which controls the behaviour of BSP beams, is affected by both the longitudinal and transverse slips. Su and Siu proposed an analytical model to solve the longitudinal partial interaction of the BSP beams under several symmetrical loading conditions [6, 12]. Su et al. developed this analytical model to utilise it for more complicated asymmetrical loading conditions [13]. Li and Su [8, 14] conducted experimental and numerical studies to investigate the longitudinal and transverse slips. Oehlers et al. [11] established the relationship between the degree of transverse partial interaction and the stiffness as well as plastic deformability of the anchoring bolts utilised. Nguyen et al. [15] derived the relationship between longitudinal and transverse partial interactions. Li [1] proposed an analytical model for the transverse slip, thus provided an approach to consider the transverse partial interaction in BSP beams. Siu and Su [12] developed a two-alpha approach to analyse the sectional behaviour of BSP beams, in which the degree of partial interaction was measured in the longitudinal and the transverse directions by two separate indicators.

The behaviour of BSP beams is very different from that of RC beams and strengthened beams using other strengthening techniques. The existing analysis and design methods would not be applicable to the BSP beams. Therefore, this paper will develop a simplified flexural strengthening design procedure for the BSP beams. The formula used to compute the flexural strength of normal RC beams is modified to take the partial interactions in both longitudinal and transverse directions into account. Then the test result of a previous experimental study will be extracted to validate the proposed theoretical model.

2. THEORETICAL MODEL

2.1. Theoretical Base

In the computation of the ultimate moment resistance of a BSP beam section, the following assumptions are employed:

(1) The bond–slip effect of both tensile and compressive reinforcement is ignored, i.e., the strain in the rebars is the same as that in the surrounding concrete.

(2) The effects of both longitudinal and transverse slips between the bolted steel plates and the RC beam are considered.

(3) The cross-sections of both the steel plates and the RC beam remain plane respectively after deformation.

(4) The tensile strength of concrete is ignored; the compressive stress of concrete, the tensile and compressive stresses in reinforcing steel and plate steel are derived from the design stress–strain relations given in the Eurocodes [16].

(5) The shear strength of anchor bolts is computed according to the Eurocodes [17].

2.2. Material Models

The stress–strain relation for the design of concrete material in the Eurocodes [16] is adopted as shown in Figure 2.1:

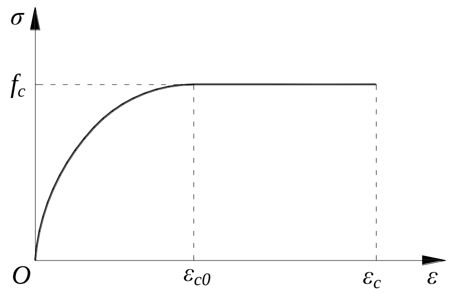

Figure 2.1 Stress–strain curve of concrete in compression condition

$$\sigma_c = \begin{cases} f_c \cdot \left[1-\left(1-\dfrac{\varepsilon_c}{\varepsilon_{c0}}\right)^2\right] & 0 \le \varepsilon_c \le \varepsilon_{c0} \\ f_c & \varepsilon_{c0} \le \varepsilon_c \le \varepsilon_{cu} \end{cases} \tag{2.1}$$

Where σ_c is the stress at strain ε_c, ε_{c0} is the strain at the maximum strength f_c, ε_{cu} is the ultimate strain. Both the reinforcement and steel plates are considered as elasto-plastic materials [16] as shown in Figure 2.2.

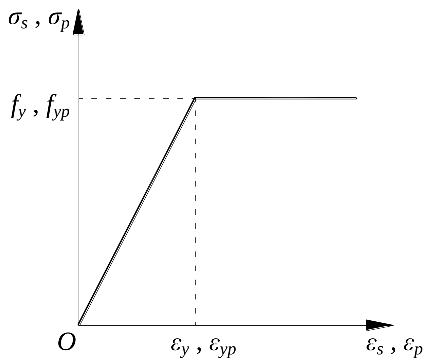

Figure 2.2 Stress–strain curve of steel reinforcement and steel plates

$$\sigma_s = \begin{cases} E_s \varepsilon_s & |\varepsilon_s| < \varepsilon_y \\ f_y & |\varepsilon_s| > \varepsilon_y \end{cases}, \qquad \text{where:} \quad E_s = f_y / \varepsilon_y \tag{2.2}$$

$$\sigma_p = \begin{cases} E_p \varepsilon_p & |\varepsilon_p| < \varepsilon_{yp} \\ f_{yp} & |\varepsilon_p| > \varepsilon_{yp} \end{cases}, \qquad \text{where:} \quad E_p = f_{yp} / \varepsilon_{yp} \tag{2.3}$$

Since the shear failure of anchor bolts is a brittle failure, the elastic shear force–slip relation is simplified for anchor bolts as shown in Figure 2.3 and whose maximum slip in BSP beams should be always less than S_{by}.

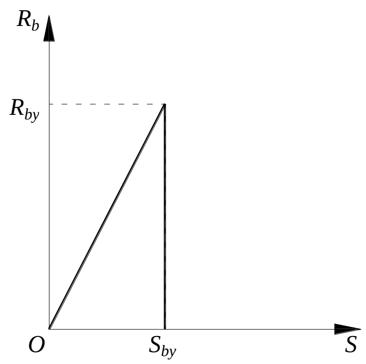

Figure 2.3 Shear force–slip curve of anchor bolts

$$R_b = K_b S \qquad S \le S_{by}, \qquad \text{where:} \quad \begin{cases} K_b = R_{by} / S_{by} \\ R_{by} = \alpha_v f_{ub} \dfrac{\pi d_b^{\,2}}{4} \end{cases} \tag{2.4}$$

Where f_{ub} and d_b are the ultimate tensile strength and the nominal diameter of anchor bolt, α_v is a modifier and a value of 0.5 or 0.6 is conventionally chosen according to the Eurocodes [17].

2.3. Sectional Analysis and Flexural Strength

The degree of partial interaction between the steel plates and the RC beam in BSP beams can be measured in the longitudinal and the transverse directions separately, by two indicators as follows (see Figure 1.4 for details):

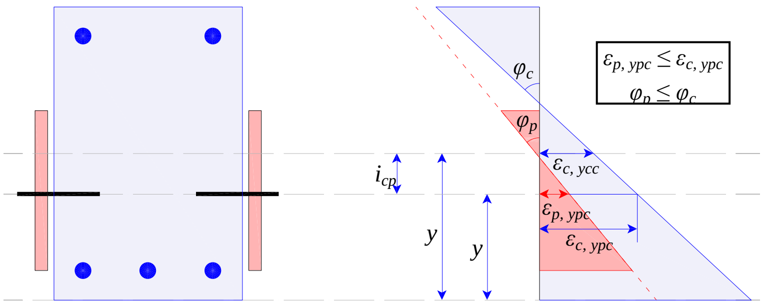

Figure 1.4 Strain profile of a BSP section with partial interaction

$$\alpha_\varepsilon = \frac{\varepsilon_{p,y_{pc}}}{\varepsilon_{c,y_{pc}}} \le 1 \tag{2.5}$$

$$\alpha_\varphi = \frac{\varphi_p}{\varphi_c} \le 1 \tag{2.6}$$

Where the strain factor (α_ε) is the ratio between the longitudinal strains of the steel plates and the RC beam ($\varepsilon_{p,\,ypc}$ and $\varepsilon_{c,\,ypc}$ respectively) at the centroidal level of the steel plates (y_{pc}), and used to indicate the degree of longitudinal partial interaction caused by the longitudinal slip (S_{lc}). The curvature factor (α_φ) is the ratio between the curvatures of the steel plates and the RC beam (φ_p and φ_c respectively), and used to indicate the degree of transverse partial interaction caused by the transverse slip (S_{tr}). According to the numerical results of a finite element analysis (FEA) by Lo et al. [18], the influence of partial interaction to the overall performance is significant, and strengthening effect of 90% could be guaranteed when the strain or the curvature factor is chosen to be no less than 0.6. This proposed value can thus be employed for both the two factors in the simplified flexural strengthening design of BSP beams as:

$$\alpha_\varepsilon = \alpha_\varphi = \alpha = 0.6 \tag{2.7}$$

Thus the cross-sectional strain profile of the BSP beam in Figure 1.4(b) can be simplified as illustrated in Figure 2.5(b). In order to obtain the flexural strength, the cross-sectional stress profile at the ultimate limit state is also illustrated in Figure 2.5(c). The concrete strain at the compressive surface reaches the ultimate strain ε_{cu}, therefore the curvature of the RC beam can be expressed by the depth of neutral axis (c) as:

$$\varphi_c = \frac{\varepsilon_{cu}}{c} \tag{2.8}$$

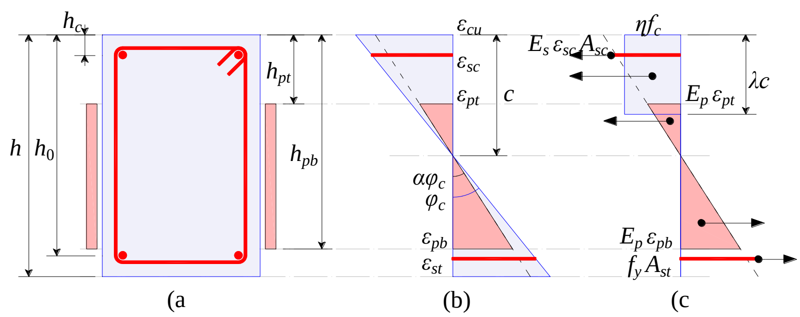

Figure 2.5 Sectional strain and stress profiles in a BSP beam:
(a) Section, (b) Strain profile, and (c) Stress profile

The strains of the compressive and tensile reinforcement can be written by their depths (h_c and h_0) as follows:

$$\varepsilon_{sc} = \varphi_c \left(c - h_c\right) \tag{2.9}$$

$$\varepsilon_{st} = \varphi_c \left(h_0 - c\right) \tag{2.10}$$

The strains of the steel plates at their top and bottom edges can also be written as following:

$$\varepsilon_{pt} = \alpha\varphi_c \left(c - h_{pt}\right) \tag{2.11}$$

$$\varepsilon_{pb} = \alpha\varphi_c \left(h_{pb} - c\right) \tag{2.12}$$

For a satisfactory strengthening design, the outmost layer of tensile reinforcement should yields before concrete crushing occurs, thus its tensile stress is the yield strength f_y at the ultimate limit state. By substituting the strains in Equations (2.9)~(2.12) into the material constitutive relations, the internal sectional axial force N_u and bending moment M_u can be obtained. Furthermore, the pure bending condition should be satisfied as:

$$N_u = \quad \eta f_c b\lambda c + E_s A_{sc} \frac{\varepsilon_{cu}}{c}\left(c - h_c\right) - f_y A_{st} + E_p t_p \frac{\alpha\varepsilon_{cu}}{c}\left(c - h_{pt}\right)^2 - E_p t_p \frac{\alpha\varepsilon_{cu}}{c}\left(h_{pb} - c\right)^2 = 0 \tag{2.13}$$

Where λ is a factor defining the effective depth of the concrete compression zone and η is a factor defining the effective strength as shown in Figure 2.5(c). It can be found that c is the only unknown in Equation (2.13) and it is convenient to solve this quadratic equation to yield the neutral axis depth c as following:

$$c = \frac{\sqrt{B^2 - 4AC} - B}{2A}$$
$$\text{where: } \begin{cases} A = \eta f_c b\lambda \\ B = E_s A_{sc}\varepsilon_{cu} - f_y A_{st} + 2E_p t_p \alpha\varepsilon_{cu}\left(h_{pb} - h_{pt}\right) \\ C = -\left[E_s A_{sc}\varepsilon_{cu} h_c + E_p t_p \alpha\varepsilon_{cu}\left(h_{pb}{}^2 - h_{pt}{}^2\right)\right] \end{cases} \tag{2.14}$$

Then the ultimate moment resistance (M_u) can be expressed as following:

$$\begin{aligned} M_u = \quad & \eta f_c b\lambda c^2\left(1 - \frac{\lambda}{2}\right) + E_s A_{sc}\frac{\varepsilon_{cu}}{c}\left(c - h_c\right)^2 + f_y A_{st}\left(h_0 - c\right) \\ & + \frac{2}{3}E_p t_p \frac{\alpha\varepsilon_{cu}}{c}\left(c - h_{pt}\right)^3 + \frac{2}{3}E_p t_p \frac{\alpha\varepsilon_{cu}}{c}\left(h_{pb} - c\right)^3 \end{aligned} \tag{2.15}$$

However, the neutral axis depth (c) solved from Equation (2.13) must be substituted into Equations (2.8)~(2.12) to check if the strains of the reinforcement and the steel plates (ε_{sc} , ε_{pt}, and ε_{pb}) surpass their corresponding yield strain (ε_y and ε_{yp}) or change their directions as following:

(1) If the yielding of the compressive reinforcement happens ($\varepsilon_{sc} > \varepsilon_y$), the stress in Equations (2.13) and (2.15) should be replaced by f_y.

(2) If the yielding of the top or the bottom edge of steel plates happens ($\varepsilon_{pt} > \varepsilon_{yp}$ or $\varepsilon_{pb} > \varepsilon_{yp}$), the corresponding triangular stress block in Figure 2.5(c) should be replaced by an echelon stress block as shown in Figure 2.6(a), and Equations (2.13) and (2.15) should be changed accordingly.

(3) If the strain of the top edge of steel plates is negative ($\varepsilon_{pt} < 0$), this means the steel plates is in tension for entire section as shown in Figure 2.6(b). Actually, this phenomenon implies the steel plates are shallow ones and attached to the tensile region of the RC beam. In this case, no modification is needed for Equations (2.13) and (2.15).

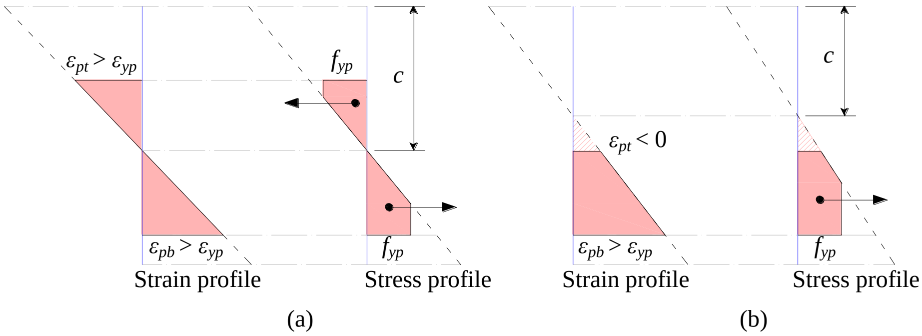

Figure 2.6 Sectional strain and stress profiles of steel plates in a BSP beam at the occurrence of (a) plate yielding and (b) plate entire-sectional tension

3. VERIFICATION BY EXPERIMENTAL RESULTS

3.1. A Brief Introduction of the Experimental Study

In a previous experimental study conducted by the authors [8], steel plates with a thickness of 6 mm and two different depths of 100 mm and 250 mm were bolted to the beam's sides by one or two rows of anchor bolts with a longitudinal spacing of 300 mm or 450 mm to yield distinct strengthening effects. To prevent the plate from buckling, which might otherwise occur in the compressive regions, buckling restraint devices were added. A four-point bending test of the BSP beam specimens, with a clear span of 3600 mm and a pure bending zone of 1200 mm, was conducted. The strengthening layout and the test set-up are shown in Figures 3.1 and 3.2.

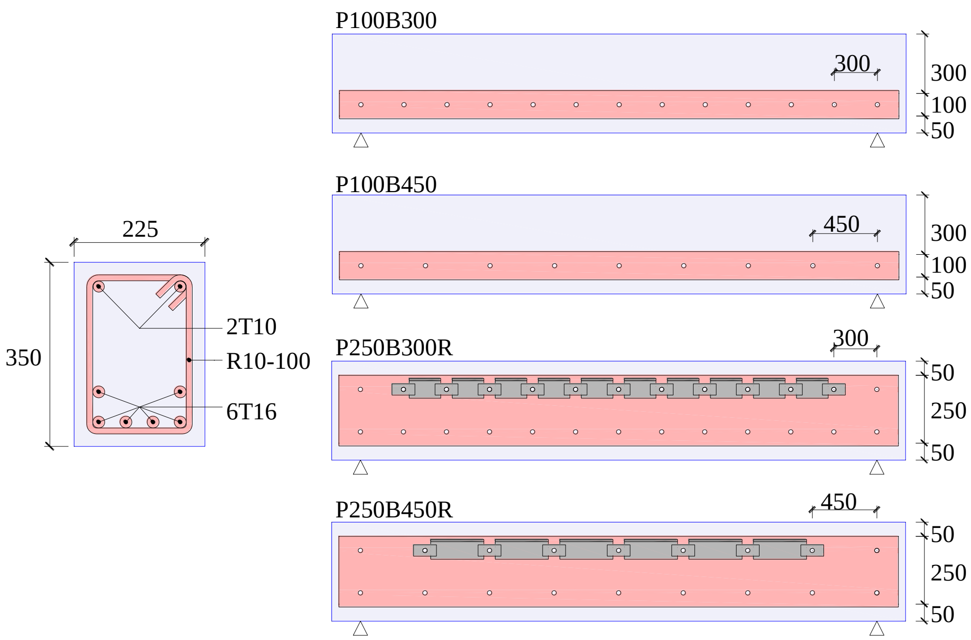

Figure 3.7 Specimen details (dimensions in mm)

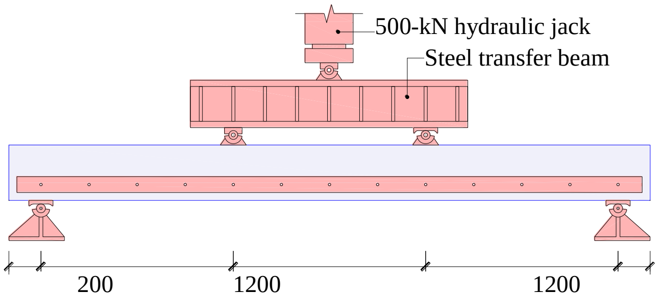

Figure 3.8 Experimental test set-up (dimensions in mm)

3.2. Comparison between Experimental and Theoretical Results

For a BSP beam subjected to four-point bending, the peak load can be expressed conveniently as $F_p = 2M_u / L_s$ (where L_s is the shear span). The results extracted from the experimental study were employed to verify the aforementioned sectional analysis method as shown in Table 3.1. It is evident that the proposed sectional analysis method can predict the peak load of the specimens with a satisfactory mean discrepancy of about 5.2%.

Table 3.1 Comparison of experimental and theoretical peak loads

Specimen	$F_{p,exp}$	$F_{p,the}$	$\lvert F_{p,\mathrm{the}} - F_{p,\mathrm{exp}} \rvert / F_{p,\mathrm{exp}}$
CONTROL	268	278	3. 7%
P100B300	317	335	5. 7%
P100B450	327	364	11. 3%
P250B300R	382	369	3. 4%
P250B450R	377	375	0. 5%
		Mean absolute error :	5. 2%

4. CONCLUSIONS

In this paper, a simplified flexural strengthening design method was proposed for the BSP beams. The modified flexural strength formula of traditional RC beams was modified by involving the influence of partial interaction. The main findings of this study are as follows:

(1) The load capacity of BSP beams would be overestimated if the assumption of full interaction is employed in the calculation. On the other hand, results that are more accurate can be obtained by taking the partial interaction on the plate–RC interface into account.

(2) The strain and curvature factors are used to quantify the longitudinal and transverse partial interaction. And only minor modification is needed for the conventional flexural strength formula of RC beams to cover the computation of the flexural strength of BSP beams, by employing the recommended strain and curvature factors.

(3) A strain or curvature factor of 0.6 can attain an optimal enhancement with a reasonable number of anchor bolts, and an excessive connection is neither economic nor necessary.

(4) The testing results of a previous experimental study show that the proposed method can predict the peak load of the specimens satisfactorily.

ACKNOWLEDGMENT
The research described here received financial support from the Research Grants Council of the Hong Kong SAR (Project No. HKU7151/10E) and National Natural Science Foundation of China (Project No. 51408436).

REFERENCES

1. Li, L.Z. (2013). New partial interaction models for bolted-side-plate reinforced concrete beams. the University of Hong Kong, Hong Kong.
2. Su, R.K.L., and Zhu, Y. (2005). Experimental and numerical studies of external steel plate strengthened reinforced concrete coupling beams. *Engineering structures***. 27:10**, 1537-1550.
3. Souici, A., Berthet, J.F., and Li, A., et al. (2013). Behaviour of both mechanically connected and bonded steel–concrete composite beams. *Engineering Structures***. 49**, 11-23.
4. Adhikary, B.B., Mutsuyoshi, H., and Sano, M. (2000). Shear strengthening of reinforced concrete beams using steel plates bonded on beam web: experiments and analysis. *Construction and Building Materials***. 14:5**, 237-244.
5. Buyukozturk, O., Gunes, O., and Karaca, E. (2004). Progress on understanding debonding problems in reinforced concrete and steel members strengthened using FRP composites. *Construction and Building Materials***. 18:1**, 9-19.
6. Siu, W.H. (2009). Flexural strengthening of reinforced concrete beams by bolted side plates. the University of Hong Kong, Hong Kong.
7. Su, R.K.L., Siu, W.H., and Smith, S.T. (2010). Effects of bolt–plate arrangements on steel plate strengthened reinforced concrete beams. *Engineering Structures***. 32:6**, 1769-1778.
8. Li, L.Z., Lo, S.H., and Su, R.K.L. (2013). Experimental study of moderately reinforced concrete beams strengthened with bolted-side steel plates. *Advances in Structural Engineering***. 16:3**, 499-516.
9. Johnson, R.P., and Molenstra, N. (1991). Partial shear connection in composite beams for buildings. *ICE Proceedings***. 91**, 679-704.
10. Siu, W.H., and Su, R.K.L. (2011). Analysis of side-plated reinforced concrete beams with partial interaction. *Computers and Concrete***. 8:1**, 71-96.
11. Oehlers, D.J., Nguyen, N.T., and Ahmed, M., et al. (1997). Transverse and longitudinal partial interaction in composite bolted side-plated reinforced-concrete beams. *Structural Engineering and Mechanics***. 5:5**, 553-563.
12. Siu, W.H., and Su, R.K.L. (2010). Effects of plastic hinges on partial interaction behaviour of bolted side-plated beams. *Journal of Constructional Steel Research***. 66:5**, 622-633.
13. Su, R.K.L., Li, L.Z., and Lo, S.H. (2014). Longitudinal partial interaction in bolted side-plated reinforced concrete beams. *Advances in Structural Engineering***. 17:7**, 921-936.
14. Su, R.K.L., Li, L.Z., and Lo, S.H. (2013). Shear transfer in bolted side-plated reinforced concrete beams. *Engineering Structures***. 56**, 1372-1383.
15. Nguyen, N.T., Oehlers, D.J., and Bradford, M.A. (2001). An analytical model for reinforced concrete beams with bolted side plates accounting for longitudinal and transverse partial interaction. *International Journal of Solids and Structures***. 38:38-39**, 6985-6996.
16. Cen. (1992). BSEN 1992-1-1-2004 Eurocode 2. Design of concrete structures - Part 1-1 General rules and rules for buildings.
17. Cen. (1993). BSEN 1993-1-8-2005 Eurocode 3. Design of steel structures - Part 1-8 Design of joints.
18. Lo, S.H., Li, L.Z., and Su, R.K.L. (2014). Optimization of partial interaction in bolted side-plated reinforced concrete beams. *Computers and Structures***. 131**, 70-80.