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RTHS WITH CONCURRENT MODEL UPDATING ON A DISTRIBUTED PLATFORM

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ABSTRACT
In this work, a novel RTHS concept with concurrent model updating is introduced to overcome the limitation of having static (non-evolving) numerical models of complex physical components present at multiple locations in the structure. In this RTHS, the numerical substructure includes a time invariant linear structural model and time variant nonlinear models of the complex physical component at all the locations within the structural model that are not modeled physically. The time variant non-linear models can accept and update parameters identified using the data collected from the actual physical component in real time during RTHS. To avoid the additional computational overhead associated with parameter identification as compared to conventional RTHS, the numerical substructure and the model updating program run on two locally distributed real time operating systems (xPC) at different rates. In this paper, global and local behavior of this novel RTHS concept with concurrent model updating is validated through a virtual distributed platform. Robustness of the model updating algorithm, accuracy, and multi-rate effects between two RT Simulink systems are discussed.

KEYWORDS: *Model Updating, Real Time, Hybrid Simulation, dRTHS, Constrained Unscented Kalman Filter*

1. INTRODUCTION

Development and application of Hybrid Simulation (HS) and Real Time Hybrid Simulation (RTHS) has been gradually expanding during recent decades. One appealing benefit of RTHS is that only the critical component must be fabricated physically (experimental substructure) and the rest of structure can be numerically modeled (numerical substructure). RTHS has shown its advances in evaluating auxiliary devices such as dampers and base isolators where the functionality of those devices (normally for vibration control purpose) is quite unique and distinguishable from the structural components. For structural components (column, bridge pier, joints and other connections, etc.) evaluation, a given component might be used in multiple instances within the structure. Modelling errors in the numerical substructure may contribute significantly to the global response which affects the fidelity of a RTHS if one takes the approach of using a limited number of the repeated component as the experimental substructure. Thus, it is critical to accurately identify and update the numerical model of those highly nonlinear components during HS to preserve the fidelity of the experiment.

The opportunity to conduct model updation on the fly has recently been recognized by researchers developing novel testing approaches. Rather than only exchanging information at the interface (displacement, acceleration or restoring force), information to improve the model of the numerical substructure can also be extracted from the response of the physical substructure. It can then be used to improve the representation of similar components in the numerical model. Kwon *et al.* [1] first introduced the concept of representing an entire structure with several key physical components, and modifying their numerical models using the physical response in real-time. The numerical model used in simulation consisted of a collection of Bouc-Wen models with predetermined parameters. During HS, a weighting factor was identified for each Bouc-Wen model until the summation of their weighted responses matched the measured physical response. Thus, the accuracy of this approach depends highly on the initial collection of Bouc-Wen models chosen. In the subsequent years, several techniques to apply model updating in HS have been developed, mostly in the unscented Kalman filter family. Those approaches include using the constrained unscented Kalman filter (CUKF) in RTHS [2] and the unscented Kalman filter (UKF) algorithm in HS [3] and RTHS [4] to identify Bouc-Wen model parameters.

Experimental results in the aforementioned work demonstrate the feasibility of model updating in HS and the associated improvement in testing accuracy.

In conventional RTHS, computational resources are dedicated to computations associated with the numerical substructure. Due to the fast sampling rate, numerical substructure sometimes needs to be simplified to meet the execution deadline. Model updating algorithms require additional CPU resources in RTHS with model updating (RTHSMU) and are normally computational intensive. Meanwhile, it is the model updating accuracy and convergence which are more critical to RTHSMU performance but not the updating speed. Thus, it is ideal to separate the model updating from the RTHS core (conventional RTHS components) to avoid overhead added by the model updating algorithms. In this paper, a distributed RTHS platform is introduced involves two real time operation system (xPC), where the master xPC runs the core RTHS components and slave xPC runs the model updating in a reduced sampling rate. The numerical substructure in the master xPC receives updated model parameter from the slave xPC.

2. REAL TIME HYBRID SIMULATION WITH MODEL UPDATING

Consider the equation of motion for a reference structural with nonlinear components in conventional simulation written as:

$$\mathbf{M\ddot{x} + C\dot{x} + Kx + R(x,\dot{x},\theta_R) = -M\Gamma\ddot{x}_g} \tag{2.1}$$

where **M**, **C**, **K** are the linear mass, damping, stiffness matrices of the reference structure, **R** is the internal restoring force provided by the critical nonlinear components within the numerical model, θ_R are the parameters of the nonlinear components, x, $\dot{x}$ and $\ddot{x}$ are structural responses (displacement, velocity, acceleration), and $\ddot{x}_g$ denotes earthquake excitation.

$$\mathbf{M^N\ddot{x}^N + C^N\dot{x}^N + K^Nx^N + F^E(x^E,\dot{x}^E) + R^N(x^N,\dot{x}^N,\theta_R) = -M\Gamma\ddot{x}_g} \tag{2.2}$$

$$\mathbf{M^E\ddot{x}^E + C^E\dot{x}^E + K^Ex^E + R^E(x^E,\dot{x}^E) = F^E(x^E,\dot{x}^E)} \tag{2.3}$$

where the superscript $(\ )^N$ and $(\ )^E$ denote the portions of the reference structure included in the numerical and experimental substructures respectively, $\mathbf{M = M^E + M^N}$, $\mathbf{C = C^E + C^N}$, $\mathbf{K = K^E + K^N}$, and $\mathbf{R = R^E + R^N}$. $\mathbf{F^E}$ denotes the measured force in the experimental substructure. The fidelity of RTHS is based on how accurately Eq. 2.2 and 2.3 represent the Eq. 2.1 when implemented.

To focus on the analysis of the impact of model updating, we assume boundary condition continuity is preserved ($\mathbf{x^E = x^N}$ and $\mathbf{\dot{x}^E = \dot{x}^N}$). Because it is relatively straightforward to identify the properties of a linear structure $\mathbf{M^E}$, $\mathbf{C^E}$, $\mathbf{K^E}$ prior to testing, the accuracy of the RTHS depends mainly on the ratio of $\mathbf{R^N / R}$ and the modeling error in $\mathbf{R^N(x^N,\dot{x}^N,\theta_R)}$. In many past RTHS studies such as those with isolated dampers as the physical components, the nonlinear restoring force is dominated by those and $\mathbf{R^E >> R^N}$. However, when the physical specimen is selected to include structural components that are used in multiple instances within a structure, a significant portion of the nonlinear behavior resides in the numerical substructure ($\mathbf{R^N >> R^E}$) and there maybe modeling errors present in $\mathbf{R^N}$. Thus, the modified formulation of RTHS which includes model updating is:

$$\mathbf{M^N\ddot{x}^N + C^N\dot{x}^N + K^Nx^N + F^E(x^E,\dot{x}^E) + R^N(x^N,\dot{x}^N,\tilde{\theta}_R) = -M\Gamma\ddot{x}_g} \tag{2.4}$$

$$\mathbf{M^E\ddot{x}^E + C^E\dot{x}^E + K^Ex^E + R^E(x^E,\dot{x}^E) = F^E(x^E,\dot{x}^E)} \tag{2.5}$$

$$\mathbf{\tilde{\theta}_R = \Psi(R^E, x^E, \dot{x}^E, \theta_\Psi)} \tag{2.6}$$

where Ψ indicates the model updating is performed in real-time, θ_Ψ is the parameter being updated through the chosen model updating algorithm and $\tilde{\theta}_R$ is the recursively identified nonlinear model parameter that minimizes the associated cost function. Thus, in this implementation, the numerical restoring force $R^N(\mathbf{x^N}, \mathbf{\dot{x}^N}, \boldsymbol{\tilde{\theta}}_\mathbf{R})$ adapts in real-time based on the physical responses.

2.1. Constrained Unscented Kalman Filter

Consider a stochastic nonlinear discrete-time dynamic system:

$$\boldsymbol{\theta}_\mathbf{k} = \mathbf{F}(\boldsymbol{\theta}_\mathbf{k-1}, \mathbf{u}_\mathbf{k-1}, \mathbf{k-1}) + \mathbf{w}_\mathbf{k-1} \tag{2.7}$$

$$\mathbf{y}_\mathbf{k} = \mathbf{H}(\boldsymbol{\theta}_\mathbf{k}, \mathbf{k}) + \mathbf{v}_\mathbf{k} \tag{2.8}$$

where F and H are process and observation models. For a parameter estimation problem, $\boldsymbol{\theta}_\mathbf{k-1}$ is the system parameter vector. Assume for all $k \geq 1$, input u_k, measurement y_k, and the PDFs of $\boldsymbol{\rho}(\boldsymbol{\theta}_\mathbf{0})$, $\boldsymbol{\rho}(\mathbf{w})$, $\boldsymbol{\rho}(\mathbf{v})$ are known. Also, $\mathbf{w}$ and $\mathbf{v}$ are the process noise and measurement noise, with zero mean and known variances, represented by $\mathbf{Q}$ and $\mathbf{R}$. $\boldsymbol{\theta}_\mathbf{0}$ is the initial condition (guess) of the parameter estimation vector.

Consider parameter set to be estimated θ_k has interval limit $d_k \leq \theta_k \leq e_k$, the interval constrained unscented transformation (IUCT) is defined as $[\boldsymbol{\gamma}_\mathbf{k}, \boldsymbol{\Theta}_\mathbf{k}] = \boldsymbol{\Phi}_\mathbf{ICUT}(\boldsymbol{\hat{\theta}}_\mathbf{k}, \mathbf{P}^{\boldsymbol{\theta\theta}}_\mathbf{k}, \mathbf{d}_\mathbf{k}, \mathbf{e}_\mathbf{k}, L, \lambda_C)$ [5], where the current estimation parameter $\boldsymbol{\theta}_\mathbf{k}$ is projected to additional 2L sigma points (L is the number of parameter to be identified) $\boldsymbol{\Theta}_\mathbf{k}$, the mean $\boldsymbol{\hat{\Theta}}_\mathbf{k} = \sum_{0}^{j=2L} \gamma_{j,k} \boldsymbol{\Theta}_\mathbf{j,k}$ with weighting factors γ_j ($j = 1..2L$), and $\sum_{j=0}^{2L} \gamma_j = 1$.

Once we have defined the ICUT, the forecast step is given as:

$$[\boldsymbol{\gamma}_\mathbf{k-1}, \boldsymbol{\Theta}_\mathbf{k-1|k-1}] = \boldsymbol{\Phi}_\mathbf{ICUT}(\boldsymbol{\hat{\theta}}_\mathbf{k}, \mathbf{P}^{\boldsymbol{\theta\theta}}_\mathbf{k}, \mathbf{d}_\mathbf{k}, \mathbf{e}_\mathbf{k}, L, \lambda_C) \tag{2.9}$$

$$\boldsymbol{\Theta}_\mathbf{j,k|k-1} = \mathbf{F}(\boldsymbol{\Theta}_\mathbf{k-1|k-1}, \mathbf{u}_\mathbf{k-1}, k-1) + \mathbf{w}_\mathbf{k-1} \tag{2.10}$$

$$\boldsymbol{\hat{\theta}}_\mathbf{k|k-1} = \sum_{j=0}^{2L} \gamma_{j,k-1} \boldsymbol{\Theta}_\mathbf{j,k|k-1} \tag{2.11}$$

$$\mathbf{P}^{\boldsymbol{\theta\theta}}_\mathbf{k|k-1} = \sum_{j=0}^{2L} \gamma_{j,k-1} [\boldsymbol{\Theta}_\mathbf{j,k|k-1} - \boldsymbol{\hat{\theta}}_\mathbf{k|k-1}][\boldsymbol{\Theta}_\mathbf{j,k|k-1} - \boldsymbol{\hat{\theta}}_\mathbf{k|k-1}]^\mathbf{T} + \mathbf{Q}_\mathbf{k-1} \tag{2.12}$$

$$[\boldsymbol{\gamma}_\mathbf{k}, \boldsymbol{\Theta}_\mathbf{k|k-1}] = \boldsymbol{\Phi}_\mathbf{ICUT}(\boldsymbol{\hat{\theta}}_\mathbf{k|k-1}, \mathbf{P}^{\boldsymbol{\theta\theta}}_\mathbf{k|k-1}, \mathbf{d}_\mathbf{k}, \mathbf{e}_\mathbf{k}, L, \lambda_C) \tag{2.13}$$

$$\mathbf{Y}_\mathbf{j,k|k-1} = \mathbf{H}(\boldsymbol{\Theta}_\mathbf{j,k|k-1}, k) \tag{2.14}$$

$$\mathbf{\hat{y}}_\mathbf{k|k-1} = \sum_{j=0}^{2L} \gamma_{j,k} \mathbf{Y}_\mathbf{j,k|k-1} \tag{2.15}$$

$$\mathbf{P}^\mathbf{yy}_\mathbf{k|k-1} = \sum_{j=0}^{2L} \gamma_{j,k} [\mathbf{Y}_\mathbf{j,k|k-1} - \mathbf{\hat{y}}_\mathbf{k|k-1}][\mathbf{Y}_\mathbf{j,k|k-1} - \mathbf{\hat{y}}_\mathbf{k|k-1}]^\mathbf{T} + \mathbf{R}_\mathbf{k} \tag{2.16}$$

$$\mathbf{P}^{\boldsymbol{\theta}\mathbf{y}}_\mathbf{k|k-1} = \sum_{j=0}^{2L} \gamma_{j,k} [\boldsymbol{\Theta}_\mathbf{j,k|k-1} - \boldsymbol{\hat{\theta}}_\mathbf{k|k-1}][\mathbf{Y}_\mathbf{j,k|k-1} - \mathbf{\hat{y}}_\mathbf{k|k-1}]^\mathbf{T} \tag{2.17}$$

$\mathbf{P}^{\boldsymbol{\theta\theta}}_\mathbf{k|k-1}$ is the forecast error covariance, $\mathbf{P}^\mathbf{yy}_\mathbf{k|k-1}$ is the innovation covariance, $\mathbf{P}^{\boldsymbol{\theta}\mathbf{y}}_\mathbf{k|k-1}$ is the cross covariance. Similar to classic KF update, the estimate step (also known as data assimilation step) is defined:

$$\mathbf{K}_{\mathbf{k}} = \mathbf{P}^{\theta\mathbf{y}}_{\mathbf{k|k-1}}(\mathbf{P}^{\mathbf{yy}}_{\mathbf{k|k-1}})^{-1} \tag{2.18}$$

$$\hat{\boldsymbol{\theta}}_{\mathbf{k|k}} = \mathbf{K}_{\mathbf{k}}(\mathbf{y}_{\mathbf{k}} - \hat{\mathbf{y}}_{\mathbf{k|k-1}}) \tag{2.19}$$

$$\boldsymbol{P}^{\theta\theta}_{\boldsymbol{k|k}} = \boldsymbol{P}^{\theta\theta}_{\boldsymbol{k|k-1}} - \boldsymbol{K}_{\boldsymbol{k}}\boldsymbol{P}^{\boldsymbol{yy}}_{\boldsymbol{k|k-1}}\boldsymbol{K}^{\boldsymbol{T}}_{\boldsymbol{k}} \tag{2.20}$$

where $\mathbf{K}^{\mathbf{k}}$ is the Kalman gain matrix and $\mathbf{P}^{\theta\theta}_{\mathbf{k|k}}$ is the data-assimilation error covariance.

3. VIRTUAL DISTRIBUTED RTHS IMPLEMENTATION

RTHSMU on the distributed platform is validated through a numerical example, where the behavior of a 3-story-frame with identical nonlinear braces is studied, shown in Figure. 3.1. In the simulation, only the first story brace serves as experimental substructure which response is considered to be accurate, and the other braces are numerically modeled as a Bouc-Wen system with initial error. During RTHSMU, the model updating algorithm receives measured input (displacement D_m) and output (measured restoring force R_m) from the experimental substructure and real time estimated parameter sets updates the numerical Bouc-Wen [6] model to improve the RTHS fidelity. Two real time xPCs were executed concurrently in the virtual distributed RTHS; the master xPC governs the core RTHS including numerical substructure (linear frame and nonlinear braces), experimental substructure (1st floor nonlinear brace) with master computer sampling rate f_m of 2000Hz, and the slave xPC ran the model updating with different sampling rate f_s from 200Hz to 2000Hz.

3.1. Simulation Case Study

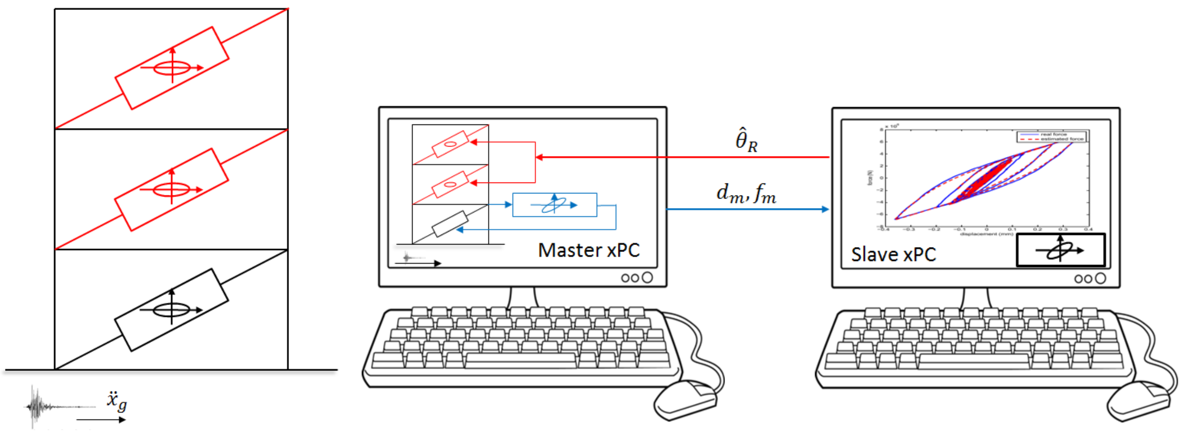


Figure 3.1 Numerical example and implementation scheme

Consider a 3-story frame with natural frequency at 1.5 Hz, 3.1 Hz and 4.8 Hz, with mass m of 3.5×10^3 kg, stiffness k of 1.8×10^3 kN/m and 5% damping at each floor. A scaled El-Centro earthquake is used as the ground motion excitation. The hysteretic behavior of the brace is modeled with a Bouc-Wen equation as:

$$R_m(D^m, z) = \alpha_B k_B D^m + (1-\alpha_B)k_B z \tag{3.1}$$

$$\dot{z} = A_B\dot{D}^m - \beta_B |\dot{D}^m| |z|^{n_B-1} z - \gamma_B \dot{D}^m |z|^{n_B} \tag{3.2}$$

where k_B is the stiffness coefficient and 0 $\leq$ α_B $\leq$ 1 determines the level of nonlinearity, $\alpha_B = 1$ indicates the system is purely linear and $\alpha_B = 1$ indicates the system is purely hysteretic. A_B, β_B, n_B, γ_B governs the shape of the hysteresis loop. The exact and initial value of each parameter is given in Table 3.1, the initial Bouc-Wen parameter set yields 19.7% error in the restoring force under this simulation loading condition.

Table 3.1 Parameters in the Bouc-Wen Model

Case	α_B	β_B	γ_B	k_B (kN/mm)	n_B	A_B
Exact Value	0.35	1.7	0.5	35	1.7	15
Initial Value	0.63	3.9	0.7	70	2.38	21
Updating Constraints	[0 1]	[0 ∞]	[0 ∞]	[0 ∞]	[1 5]	[0 ∞]

3.2. Distributed Real Time Simulation Platform

The exchange of measured inputs and outputs, as well as updated parameters, between master and slave computers is accomplished in real-time using User Datagram Protocol (UDP) blocks by MATLAB. Real-time UDP transmission block requires a dedicated Ethernet board installed on each distributed xPC to utilize a reliable connection over dedicated Ethernet link that does not share bandwidth with the host computer. To further reduce the chance of interference with other network devices, xPC computers are connected to each other directly with a crossover Ethernet cable. The data produced by a xPC computer at each time step is delivered to the remote xPC within the real-time constraints, hence sampling rate and transmission rate are treated the same. To minimize loss of data during transmission, a two time step buffer is added on the remote xPC. It should be noted that the tradeoff of using buffer manifests as transmission time delay.

Due to the different computational cost and execution priority in RTHSMU, the master and slave xPCs are sampled at different rates with multi-rate communication logic illustrated in Figure 3.2. In this logic, each xPC transfers the data out (Tx) and the remote counterpart receives the incoming package (Rx). In the RTHSMU case, the master produces and sends data (experimental substructure input and output measurement) to the slave at a higher rate. Only the most recent master data is captured by the slave and any other intermediate data is omitted. Likewise, slave transmits data at a reduced sampling rate, thus the previous slave data (estimated parameter set) is used in the next time steps by master until a new slave data is received. The results presented in the following section consider how the presence of model updating and the multi-rate ratio between master sampling rate f_m and slave xPCs rate f_s impacts RTHS accuracy.

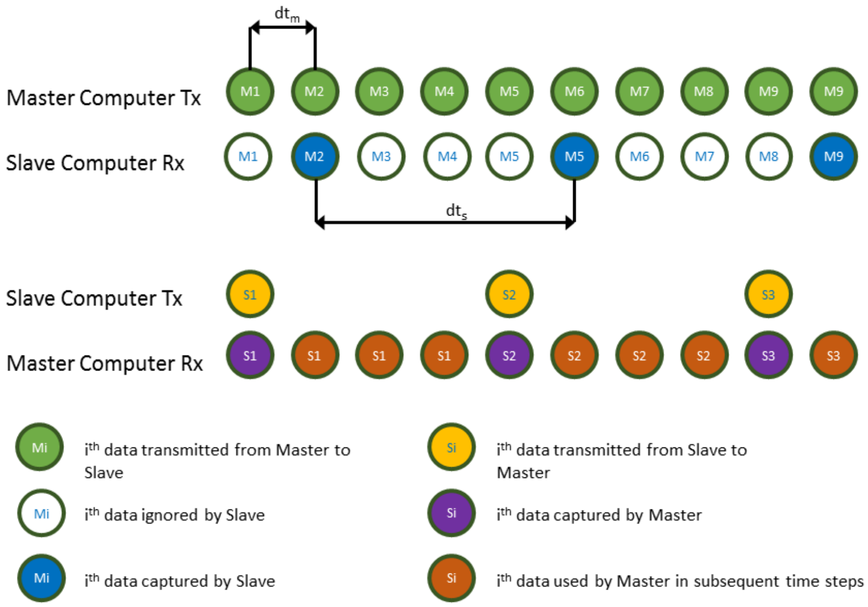
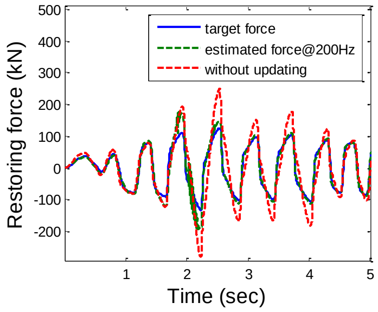
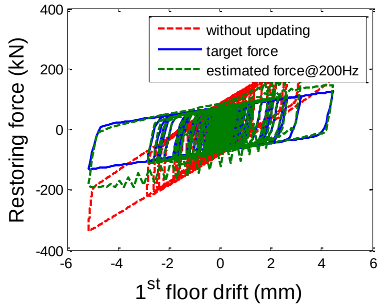


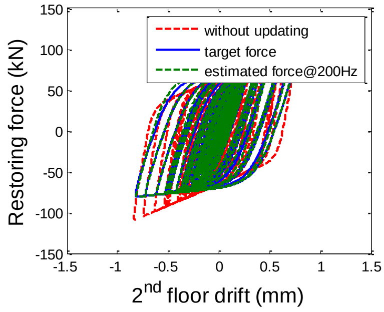
Figure 3.2 Multi-rate communication logic in distributed RTHSMU

3.3. Results

Performance of RTHSMU with multi-rate is evaluated at both local and global levels. Local estimation error is defined as the RMS error between the measured restoring force from the loaded brace and the estimated restoring force from model updating algorithm. Local response error at each floor is the RMS error between the restoring force of the updated braces model and the restoring force of the exact model under the same loading path. Global response error is defined as the summation of the RMS error of each story drift between the baseline solution (simulation with exact Bouc-Wen parameters) and RTHSMU.



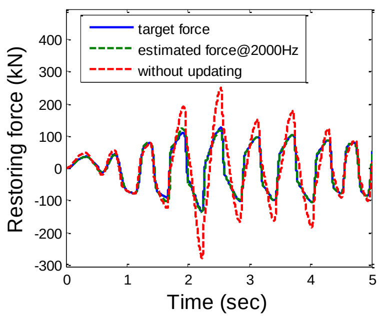


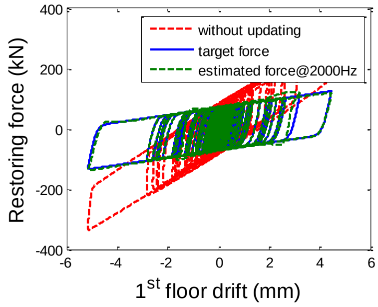


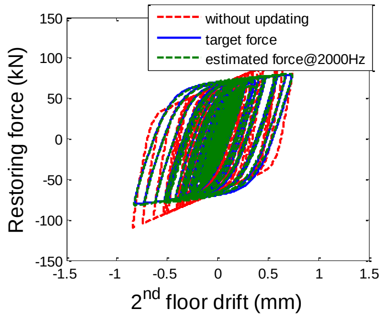
(a) Model updating time history with f_s at 200 Hz

(b) Updated specimen hysteretic behavior with f_s at 200 Hz

(c) 2nd story hysteresis behavior with f_s at 200 Hz







(d) Model updating time history with f_s at 2000 Hz

(e) Updated specimen hysteretic behavior with f_s at 2000 Hz

(f) 2nd story hysteresis behavior with f_s at 2000 Hz

Figure 3.3 Local response with RTHSMU at different model updating rates

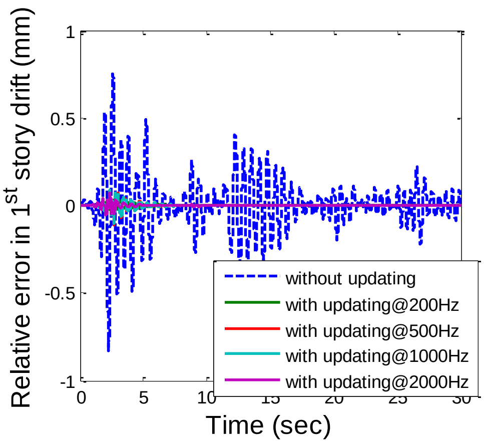


Figure 3.4 Global response with RTHSMU at different updating rates

Figure 3.3 provides the local responses in RTHSMU, and the estimation steady state solution is reached after 3 secs when updated at 200 Hz (Figure 3.3.a) and in 2 sec when the updating algorithm implemented at 2000 Hz (Figure 3.3.d). The hysteretic loop of the nonlinear brace at 2nd floor is modified with the identified parameter and is shown to have more accurate behavior as in Figures 3.3.b and 3.3.e. The improvement in the nonlinear braces model at higher floors also affects the global response, as illustrated in Figure 3.4, the relative error of the first story drift is significantly reduced with RTHSMU.

The quantitative analysis in Table 3.2 assesses the fidelity of RTHSMU at different updating rates as compared to the conventional RTHS. The results indicate that 2000Hz samping rate yields slightly less error in local estimation. However, on the global response level, the differences are not so critical as all RTHSMU cases largely improve the RTHS fidelity with residual response errors under 0.5% as compared to 7.8% without updating case. Differences between global response error with f_m at 200 Hz and 2000 Hz is less than 0.04%.

Table 3.1 RTHSMU accuracy with different model updating rate

Case	Local Estimation Error	Floor 2 Local Response Error	Floor 3 Local Response Error	Global Response Error
Without Updating f_m 2000Hz	Not Available	16.85%	16.53%	7.81%
Updating f_s 200 Hz	1.03%	1.35%	1.25%	0.34%
Updating f_s 500 Hz	0.67%	1.50%	1.38%	0.39%
Updating f_s 1000 Hz	0.69%	1.87%	1.92%	0.38%
Updating f_s 2000 Hz	0.52%	1.24%	1.11%	0.30%

4. CONCLUSION

RTHSMU performance is analyzed using a numerical example with three identical nonlinear braces in a structural system. To minimize overhead in the computational resources, the model-updating algorithm is successfully implemented on a supplemental real-time operating system with a reduced rate. Results are provided to evaluate the fidelity of RTHSMU with different updating rates, and all cases are shown to yield improved local and global response accuracy largely with similar performance. This distributed platform with multi-rate concept is believed to be useful for model-updating algorithms with larger computational cost, and nonlinear model with extended degree of freedoms or more complex behavior.

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REFERENCES

1. Kwon, O.S. and Kammula, V., (2013). "Model updating method for substructure pseudo-dynamic hybrid simulation", *Earthquake Engineering and Structural Dynamics*, 134:42, 1971-1984.
2. Wu, B. and Wang, T. (2015). "Model updating with constrained unscented Kalman filter for hybrid testing", *Smart Structures and Systems,* **14:6**, 1105-1129.
3. Hashemi, M.J., Masroor,A., and Mosqueda, G., (2014). "Implementation of online model updating in hybrid simulation", *Earthquake Engineering and Structural Dynamics*, **134:43**, 395-412.
4. Muller, A., (2014). "Real-Time Hybrid Simulation with Online Model Updating", *Master Thesis, Western Michigan University,* Paper 506.
5. Teixeira, B., Torres, L., Aguirre, L., and Bernstein, D., (2010). "On unscented Kalman filtering with state interval constraints" *Journal of Process Control*, **20:1**, 45-57.
6. Wen, Y.K. (1976). "Method of Random Vibration of Hysteretic Systems" *Journal of Engineering Mechanics Division*, ASCE, **102:EM2**, 249–263.