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Application of unscented Kalman Filter to Sectional Model Updating in Hybrid Simulation of Frame Structures

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ABSTRACT
Hybrid simulation combines numerical computation and physical experiment for cost-effective, large-scale testing of structures. The method allows simulation of structures by representing critical components with physical specimens and the rest of the structure with numerical models. However, sometimes it is impossible to take all of the uncertain or nonlinear parts of the structure as the physical substructure. Then, concern may be raised on modeling errors of numerical part. One way to solve this problem is to update the numerical model by estimating its parameters from experimental data online. In this paper, a sectional constitutive model which considers axial flexural interaction is proposed for force-based frame element. System parameter identification is conducted using unscented Kalman Filter (UKF) to estimate the parameters of sectional constitution model with the data from the experimental substructure. Hybrid tests were performed, which included the reference test, conventional hybrid test and hybrid test with model updating. The results show that the online model updating improved the accuracy of the hybrid test of frame structure.

KEYWORDS: *hybrid simulation, sectional constitutive model, parameter estimation, model updating*

1. INTRODUCTION

Hybrid simulation, which combines the advantages of numerical computation and physical experiment, is an effective method to evaluate the dynamic performance of structures[1].With substructuring techniques, the critical components of the structures are represented with physical specimens while the rest of the structure with numerical models[2,3,4], this method has been extended to geographically distributed hybrid simulations [5,6,7].Unfortunately, large scale civil engineering structures would undergo severe nonlinearities extensively when subjected to strong dynamic loading such as earthquake. It is impossible to take all of the uncertain or nonlinear parts of the structure as the physical substructures, usually due to the limited capacity of experimental facilities. So the numerical model has to include the nonlinearities of the rest of the structure. Then, concerns may be raised on modeling errors of the numerical part.

One way to solve this problem is to update the numerical model by estimating its parameters from experimental data online. Yang et al.[8] employed model updating with a neural network in substructure hybrid simulation. Wang and Wu[9] explored model updating with least square for linear model and with constrained Unscented Kalman Filter(UKF) for Bouc-Wen model. Yang et al.[10] proposed an online optimization method to update the numerical model in bridge hybrid simulation. Kwon and Kammula[11] used several counterpart numerical models with possible variation for a experimental specimen and calibrated weighting factors for each one to implement hybrid simulation for a braced frame structure. Hashemi et al.[12] illustrated the implementation of online model updating in hybrid simulation with UKF for BoucWen models.

The numerical models to be updated in the above studies are relatively simple. They either truss elements or concentrated plastic model, and hence are not suitable for beam-column elements with distributed plasticity or axial-flexural interaction (AFI). This paper presents a new sectional constitutive model that can account for axial-flexural interaction with varying axial force, and then employs UKF to update the parameters of the model in hybrid simulation of a frame structure. The paper is organized as follows. First, a sectional constitutive model considering AFI for frame element is proposed in section 1 Then, the parameter identification with UKF are presented for online model updating hybrid test in section 3. Finally, an experimental verification of hybrid simulations for frame structure were conducted using the proposed online model updating method in section 4.

2. SECTIONAL CONSTITUTIVE MODEL FOR FRAME ELEMENT

As for beam-column element, there are three levels of constitutive models, *i.e.*, member level, sectional level and material level. Among them the constitutive model in sectional level can achieve good balance between accuracy and computational efficiency. Herein, a sectional constitutive model is proposed based on yield surface of axial force and bending moment which can account for the AFI, especially when the member is subject to varying axial force.

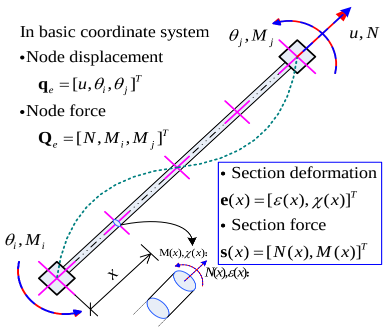


Figure 2.1 Generalized forces and deformations at the element and section level

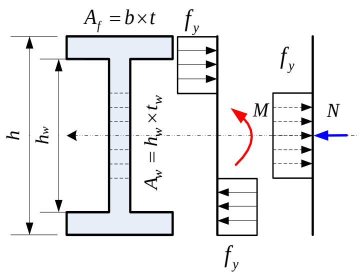


Figure 2.2 H-Section and internal force analysis

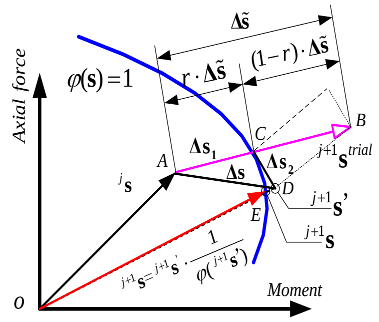


Figure 2.3 The schematic of force space and force trajectory

For a specified section in a beam-column element, see Fig. 2.1, let $\mathbf{e} = [\varepsilon_a, \chi]^T$ represent sectional deformation, in which ε_a and χ are axial deformation and curvature, respectively; $\mathbf{s} = [N, M]^T$ represent sectional force, in which N and M are sectional axial force and bending moment, respectively. To evaluate the relation between sectional deformation and sectional force, some assumptions are made as follows: (1) the plane section which is initially normal to neutral axis remains plane and remains normal to the axis after deformation; (2) the stress all over the cross-section is assumed to be yield strength for determining the yield surface.

2.1. Sectional kinematic-hardening yield surface

Chan and Chui[13] assumed that the area round neutral axis takes the axial load and the remaining of the cross-section resists the moment for I or H cross-section. Taking H section steel as an illustrative example (see Fig. 2.2), let $A_f = \alpha A_w$, where A_f denotes the area of one flange, denotes A_w the area of web. The yield force and plastic moment are defined as

$$
\begin{aligned}
N_y &= A f_y = (2\alpha + 1) A_w f_y \\
M_p &= W_p f_y = 2 A_f f_y h/2 + 2 \times 0.5 A_w f_y h_w / 4
\end{aligned}
\tag{2.1}
$$

in which f_y is the yield stress of the material. Based on Chan and Chui's assumption, the yield surface can be expressed as

$$
\varphi(\mathbf{s}) = \begin{cases} \dfrac{(2\alpha+1)^2}{4\alpha+1} \cdot \dfrac{N^2}{N_y^2} + \dfrac{|M|}{M_p} = 1, & \dfrac{N}{N_y} < \dfrac{1}{2\alpha+1} \\[2ex] \dfrac{|N|}{N_y} + \dfrac{(4\alpha+1)}{2(2\alpha+1)} \cdot \dfrac{|M|}{M_p} = 1, & \dfrac{N}{N_y} \geq \dfrac{1}{2\alpha+1} \end{cases}
\tag{2.2}
$$

where $\varphi(\mathbf{s})$ is sectional yield function, and $\alpha = A_f / A_w$.

The above formulation does not account for the hardening plasticity. Herein, we consider kinematic hardening and hence Eq. (2.2) becomes

$$\varphi(\mathbf{s},\mathbf{F}_a)=\begin{cases}\dfrac{(2\alpha+1)^2}{4\alpha+1}\cdot\dfrac{(N-F_{aN})^2}{N_y^2}+\dfrac{|M-F_{aM}|}{M_P}=1, & \dfrac{N}{N_y}<\dfrac{1}{2\alpha+1}\\ \dfrac{|N-F_{aN}|}{N_y}+\dfrac{(4\alpha+1)}{2(2\alpha+1)}\cdot\dfrac{|M-F_{aM}|}{M_p}=1, & \dfrac{N}{N_y}\geq\dfrac{1}{2\alpha+1}\end{cases}\tag{2.3}$$

where $\mathbf{F}_a=h_k\mathbf{k}_{se}\cdot\mathbf{e}^{\mathrm{p}}$ is section back force vector, $\mathbf{e}^p$ is plastic deformation vector, $\mathbf{k}_{se}$ is elastic stiffness matrix, and h_k is kinematic-hardening coefficient, $h_k=b/(1-b)$; $\mathbf{F}_{\mathrm{a}}$ contains two components, *i.e.*, $\mathbf{F}_a=[F_{aN},F_{aM}]^T$.

2.2. Sectional kinematic-hardening constitutive model

With section yield function, the relation of sectional force and deformation can be derived as follows (see Fig. 2.3).

When $\varphi(\mathbf{s},\mathbf{F}_a)\prec 1$, the generalized force increment is linearly related to deformation increment as

$$\Delta\mathbf{s}=\mathbf{k}_{se}\cdot\Delta\mathbf{e}\tag{2.4}$$

where $\mathbf{k}_{se}=\begin{bmatrix}EA & 0\\ 0 & EI\end{bmatrix}$ is the elastic stiffness matrix at the specified section, E is the material elastic modulus, A is the area of the section, and I is the moment of inertia of the section.

When $\varphi(\mathbf{s},\mathbf{F}_a)\geq 1$, the section is plastic, and the relation of generalized forces and deformation is defined in an incremental form as

$$\Delta\mathbf{s}=\mathbf{k}_{se}\cdot(\Delta\mathbf{e}-\Delta\mathbf{e}^p)\tag{2.5}$$

where $\Delta\mathbf{e}^p=\dfrac{1}{H}\dfrac{\partial\varphi}{\partial\mathbf{s}}\cdot(\dfrac{\partial\varphi}{\partial\mathbf{s}})^{\mathrm{T}}\cdot\mathbf{k}_{se}\cdot\Delta\mathbf{e}$ is the generalized plastic deformation increment; $\Delta\mathbf{e}$ is the total deformation; $H=(\dfrac{\partial\varphi}{\partial\mathbf{s}})^T\cdot\mathbf{k}_{se}\cdot\dfrac{\partial\varphi}{\partial\mathbf{s}}-h_{\mathrm{k}}(\dfrac{\partial\varphi}{\partial\mathrm{F}_a})^T\cdot\mathbf{k}_{se}\cdot\dfrac{\partial\varphi}{\partial\mathbf{s}}$.

3. PARAMETER IDENTIFICATION OF SECTIONAL CONSTITUTIVE MODEL

For the hybrid simulation in this paper, the specimen is a frame column. To estimate the parameters of the sectional model based on the displacement and force of the specimen, a finite element (FE) model is needed for the column first.

3.1. Finite element modeling of column

There are two classical formulations for beam/column elements with distributed plasticity: stiffness and flexibility methods. In stiffness method, displacement interpolation functions are used to derive the element deformation, and such elements are called displacement based elements (DBEs). DBEs may encounter serious numerical problems for structures experiencing severe nonlinearity. Often, very fine subdivision of the elements is necessary to obtain reasonable results, which can significantly increase computational effort. For flexibility method, force interpolation functions are employed to derive the force field inside the element according to the equilibrium, and corresponding elements are called force based elements (FBEs). The force interpolation functions in FBE are exact in a strict sense when element loads are not present or ignorable. So the FBE with sectional constitutive law is chosen to model the column.

For the problem of parameter identification of sectional model, the input is nodal displacements and output is nodal force, and their relation can be expressed in a general nonlinear form as

$$\mathbf{Q}_{\mathrm{e},k}=\mathbf{F}(\boldsymbol{\theta}_{k-1},\mathbf{Q}_{\mathrm{e},k-1},\mathbf{v}_{\mathrm{h},k-1},\mathbf{q}_{\mathrm{e},k})\tag{3.1}$$

where $\mathbf{Q}_{\mathrm{e}}=[N,M_i,M_j]^{\mathrm{T}}$ the element nodal force, $\boldsymbol{\theta}=[N_{\mathrm{y}},M_{\mathrm{p}},b]^{\mathrm{T}}$ the parameter of the yield surface model, $\mathbf{q}_e=[u,\theta_i,\theta_j]^{\mathrm{T}}$ the element nodal displacement, $\mathbf{v}_{\mathrm{h}}$ the history-dependent variables in element and section levels, and k the incremental step number. For simplicity, the sectional stiffness matrix is not identified in this paper.

3.2. State-space model of the column for identification

A discrete-time state-space model is needed to employ UKF for the identification of the sectional constitutive model. In general, the state transition equation can be written as

$$\mathbf{x}_k = \boldsymbol{G}(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{v}_k \tag{3.2}$$

and the observation equation can be formed as

$$\mathbf{y}_k = \mathbf{H}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k \tag{3.3}$$

where $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{u}$ are the vectors of state, measurement and input, respectively; $\mathbf{v}$ and $\mathbf{w}$ are process and measurement noises, respectively.

The choices of the state $\mathbf{x}$, measurement $\mathbf{y}$ and the input $\mathbf{u}$ are problem-dependent. For model updating in this paper, the main goal is to estimate the parameters of the sectional constitutive model. Hence, a convenient choice is to take parameter $\boldsymbol{\theta}$ as the state variable $\mathbf{x}$ in Equations (3.2) and (3.3). Straightforwardly, the measurement $\mathbf{y}$ for the sectional model is nodal force vector $\mathbf{Q}_e$. Because the nodal force at the k-th step $\mathbf{Q}_{e,k}$ is determined not only by the nodal displacement $\mathbf{q}_{e,k}$, but also by the nodal force at the (k-1)th step $\mathbf{Q}_{e,k-1}$ and history-dependent variable $\mathbf{v}_{h,k-1}$. Then the state variable and the generalized input can be written as

$$\mathbf{x}_k = \boldsymbol{\theta}_k, \quad \mathbf{u}_k = (\mathbf{Q}_{e,k-1}, \mathbf{v}_{h,k-1}, \mathbf{q}_{e,k}) \tag{3.4}$$

Correspondingly, $\mathbf{G}$ and $\mathbf{H}$ in Equations (3.2) and (3.3) are written as

$$\mathbf{G}(\mathbf{x}_{k-1}, \mathbf{u}_k) = \boldsymbol{\theta}_{k-1}, \quad \mathbf{H}(\mathbf{x}_k, \mathbf{u}_k) = \mathbf{F}(\boldsymbol{\theta}_k, \mathbf{u}_k) \tag{3.5}$$

The problem defined by Equations (3.2) to (3.5) is named pure parameter estimation.

To filter out the noise in the force measurement, which is to be fed back to the numerical part in a hybrid simulation, an alternative choice of the state variable is to include the nodal force as well, by moving the nodal force from the input into the state variable. Then state space equations (3.2) and (3.3) can be specified with

$$\mathbf{x}_k = \begin{Bmatrix} \boldsymbol{\theta}_k \\ \mathbf{Q}_{e,k} \end{Bmatrix}, \quad \mathbf{u}_k = (\mathbf{v}_{h,k-1}, \mathbf{q}_{e,k}) \tag{3.6}$$

$$\mathbf{G}(\mathbf{x}_{k-1}, \mathbf{u}_k) = \begin{Bmatrix} \boldsymbol{\theta}_{k-1} \\ \mathbf{F}(\mathbf{x}_{k-1}, \mathbf{u}_k) \end{Bmatrix}, \quad \mathbf{H}(\mathbf{x}_k, \mathbf{u}_k) = [\mathbf{0}, \mathbf{I}] \cdot \mathbf{x}_k \tag{3.7}$$

where $\mathbf{I}$ is the unit matrix. The problem defined by Equations (3.2), (3.3), (3.6) and (3.7) is called joint estimation in this paper.

3.3. UKF FOR ESTIMATION OF SECTIONAL MODEL

The UKF is an extension of Kalman filter to nonlinear systems. In the framework of Kalman filter, a recursive estimation for $\mathbf{x}_k$ can be expressed as[14].

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k \cdot (\mathbf{y}_k - \hat{\mathbf{y}}_k) \tag{3.8}$$

in which $\hat{\mathbf{x}}_k^-$ is a priori prediction of $\mathbf{x}_k$, $\hat{\mathbf{y}}_k$ is the predicted measurement, and $\mathbf{K}_k$ is Kalman filter gain; their expressions are given below.

$$\hat{\mathbf{x}}_k^- = \mathbf{E}[\mathbf{G}(\mathbf{x}_{k-1}, \mathbf{u}_k)], \quad \hat{\mathbf{y}}_k = \mathbf{E}[\mathbf{H}(\mathbf{x}_k, \mathbf{u}_k)], \quad \boldsymbol{\kappa}_k = \mathbf{P}_k^{xy}(\mathbf{P}_k^{yy})^{-1} \tag{3.9}$$

where '**E**' denotes the mathematical expectation, and '**P**' the covariance matrix. For linear systems, the further analytical expressions for Equations (3.9) can be obtained, which results in classic Kalman filter. For nonlinear functions of **G** and **H**, UKF utilizes a so-called unscented transformation (UT) to calculate the mathematical expectation and the covariance matrix. In essence, the UT is a deterministic sampling technique to compute first two order moment of nonlinear function of a random vector.

The sample points is also called sigma points, which are expressed as

$$\boldsymbol{\chi}_{k-1}^{(0)} = \hat{\mathbf{x}}_{k-1} \tag{3.10}$$

$$\boldsymbol{\chi}_{k-1}^{(i)} = \hat{\mathbf{x}}_{k-1} + \alpha\sqrt{(L+\kappa)}(\sqrt{\mathbf{P}_{k-1}})^{(i)}, \quad i = 1, 2, \cdots, L \tag{3.11}$$

$$\boldsymbol{\chi}_{k-1}^{(i+L)} = \hat{\mathbf{x}}_{k-1} - \alpha\sqrt{(L+\kappa)}(\sqrt{\mathbf{P}_{k-1}})^{(i)}, \quad i = 1, 2, \cdots, L \tag{3.12}$$

in which α and κ are constant parameters that determine the spread of the sigma points, and $0 < \alpha \leq 1$; L is the dimension of $\mathbf{x}$; $(\sqrt{\mathbf{P}_k})^{(i)}$ represents the i-th column vector of the matrix square root of $\mathbf{P}_k$. According to UT, we have

$$\hat{\mathbf{x}}_k^- = \sum\nolimits_0^{2L} W_m^{(i)} \chi_{k|k-1}^{(i)} \tag{3.13}$$

$$\hat{\mathbf{y}}_k = \sum\nolimits_{i=0}^{2L} W_m^{(i)} \mathbf{H}(\chi_{k|k-1}^{(i)}, \mathbf{u}_k) \tag{3.14}$$

$$\mathbf{P}_k^{yy} = \sum\nolimits_{i=0}^{2L} W_c^{(i)} [\mathbf{H}(\chi_{k|k-1}^{(i)}, \mathbf{u}_k) - \hat{\mathbf{y}}_k][\mathbf{H}(\chi_{k|k-1}^{(i)}, \mathbf{u}_k) - \hat{\mathbf{y}}_k]^{\mathrm{T}} + \mathbf{R} \tag{3.15}$$

$$\mathbf{P}_k^{xy} = \sum\nolimits_0^{2L} W_c^{(i)} [\chi_{k|k-1}^{(i)} - \hat{\mathbf{x}}_k^-][\mathbf{H}(\chi_{k|k-1}^{(i)}, \mathbf{u}_k) - \hat{\mathbf{y}}_k]^T \tag{3.16}$$

$$\mathbf{P}_k = \mathbf{P}_k^- - \boldsymbol{\kappa}_k \mathbf{P}_k^{yy} \boldsymbol{\kappa}_k^{\mathrm{T}} \tag{3.17}$$

in which

$$\chi_{k|k-1}^{(i)} = \mathbf{G}(\chi_{k-1}^{(i)}, \mathbf{u}_k) \tag{3.18}$$

$$\mathbf{P}_k^- = \sum\nolimits_0^{2L} W_c^{(i)} [\chi_{k|k-1}^{(i)} - \hat{\mathbf{x}}_k^-][\chi_{k|k-1}^{(i)} - \hat{\mathbf{x}}_k^-]^T + \mathbf{Q} \tag{3.19}$$

In Equations (3.13)-(3.16) and (3.19), W_m^i and W_c^i are the weights for mean and covariance, respectively, and their expressions are

$$\begin{aligned} &W_m^0 = \frac{\alpha^2(L+\kappa)-L}{\alpha^2(L+\kappa)}, \quad W_c^0 = \frac{\alpha^2(L+\kappa)-L}{\alpha^2(L+\kappa)} + (1-\alpha^2+\beta) \\ &W_m^i = W_c^i = \frac{1}{2\alpha^2(L+\kappa)}, i = 1, 2, \cdots, 2L \end{aligned} \tag{3.20}$$

where β is used to incorporate a prior knowledge of distribution of the predicted state vector, and $\beta = 2$ is optimal for Guassian distribution [14].

3.3.1. UKF for pure parameter estimation

When UKF is applied to the pure parameter estimation of the sectional model, $\mathbf{H}(\chi_{k|k-1}^{(i)}, \mathbf{u}_k)$ in Equation (3.14) is evaluated through a finite element program. It is seen from Equation (3.14), determination of the predicted nodal force requires $2L+1$ computations with the finite element program for all the parameter samples. Each computation with different sample of parameters may result in different set of new history-variables. Note the new history-variables will be used as part of the input for the next step. But it is apparent that only one set of the historical variables can be used. To cope with this problem, an additional computation is performed with the updated parameters from Equation (3.8) to generate the new history-variables. The flowchart of UKF for pure parameter estimation is shown in Fig. 3.1. The flowchart herein is similar to that in [15] where the material parameter identification was studied with fiber elements. But the former differs the latter in that the extra $(2L+2)$-th FE analysis is performed to provide updated historic variables for the next step.

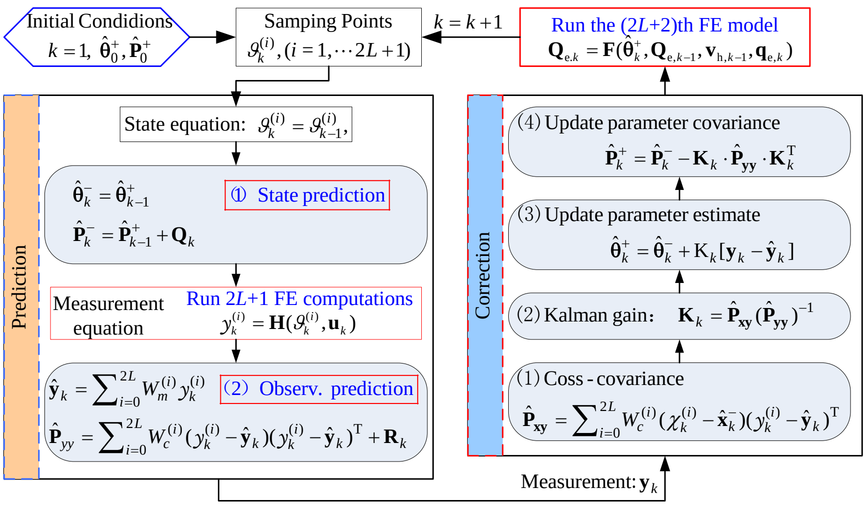


Figure 3.1 Flowchart of UKF for pure parameter estimation

3.3.2. UKF for joint estimation

The implementation of UKF for the joint estimation is similar to that for the pure parameter estimation. The only difference is the finite element program is executed at the stage of state prediction for joint estimation rather than at the stage of observation prediction for the pure parameter estimation.

4. EXPERIMENTAL VALIDATION

A hybrid test was performed on a frame structure to investigate the performance of the proposed model updating technique. The structure in this example was a one-bay one-story steel frame, as shown in Fig. 4.1. The span was 6.0m, and the height was 3.6m. The dimensions of the cross-section of the columns and beam are listed in Tab. 4.1. For each node, there were three degree of freedom, *i.e.*, two translational and one rotational. A lumped mass is placed at each joint of beam and column. The rotational inertia for each node is assumed to be 20000kgm^2 for dynamic analysis.

Table 4.1 The section dimension and features of welding h section steel

Type	dimensions (m)				Area (A)	Moment of area (I_z)
	d	b_f	t_w	t_f	(m^2)	(m^4)
HA250×250	0.250	0.250	0.006	0.012	7.35e-3	9.080e-5
HA300×200	0.300	0.200	0.006	0.012	6.45e-4	1.1010e-4

The seismic excitation to the structure is El Centro (NS, 1940) with a PGA scaled as 220gal. The classical Rayleigh damping was adopted and obtained based on damping ratio of 0.02 for the first two modes.

The left column was taken as the experimental substructure and loaded by three actuators, as seen in Fig. 4.2 at the Structural and Seismic Testing Center of Harbin Institute of Technology. The remainder of the structure was numerical substructure, in which the column was modelled by a force based beam-column element with section constitutive model described in Section II, while the beam was just modelled by an elastic beam element. The parameters to be updated were those of the column; and that of the beam remained unchanged. The column as the experimental substructure was also modelled by the nonlinear beam-column element in order to identify the sectional model parameters. For each column, five cross-sections were chosen for Gauss-Lobotto integration at the phase of element state determination.

All the numerical modelling and parameter identification was programed with MATLAB. The displacements of the interface between numerical and physical substructures calculated with MATLAB were transmitted to dSPACE through OpenFresco [16]. Then dSPACE sent the displacements to the controller of the actuators through three signal cables, and received the force measurements through other three cables, and feedback the force data for MATLAB to identify and update the sectional model parameters and calculate the next step response.

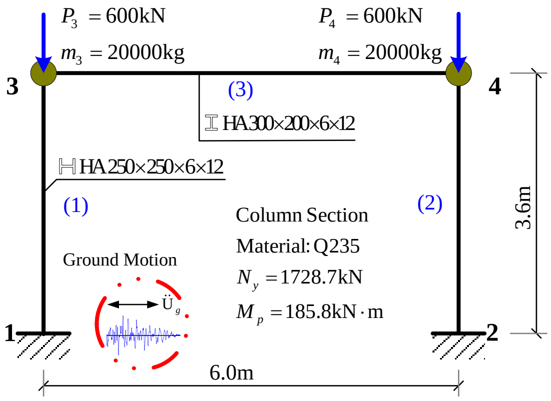


Figure 4.1 The computing model of 1 storey 1 bay steel plane frame

Figure 4.2 Tri-actuator Setup for experimental substructure: Column 1

Three cases were considered to demonstrate the effectiveness of sectional model updating with UKF: (A) with fixed parameters that were calculated based on the data of material test, (B) with the fixed parameters of which the yielding forces were calculated based on the nominal yield strength of 235MPa of the steel, and the kinematic hardening coefficient took value of 0.015, and (C) with online updating with the initial parameters that were the same as in Case B. Case A served as the reference solution.

Tab. 4.2 lists the initial guess of the sectional model parameters for both Cases B and C. In Case C, the variation of the estimated parameters was constrained by their limits. The lower and upper limits of N_y and M_p were calculated based on the yield strength of steel material of 215 MPa and 315 MPa, respectively. The range of b was set by experiences. When the estimated parameters exceeded their limits, they would be replaced by the limits.

In Case C, the joint estimation with UKF was adopted. The parameters of scaled unscented transform[14] of UKF were $\alpha = 0.5$, $\beta = 2$ and $\kappa = -3$. The covariance matrices of process and measurement noises were $10^{-2}\mathbf{I}_{6\times 6}$ and $0.5\mathbf{I}_{3\times 3}$, respectively. The initial covariance matrix of the state vector was

$$\mathbf{P}_0 = \mathrm{diag}([\sigma_{N_y}^2, \sigma_{M_p}^2, \sigma_b^2, \sigma_N^2, \sigma_{M_i}^2, \sigma_{M_j}^2]) = \mathrm{diag}([10^3, 10^3, 10^{-4}, 10^{-8}, 10^{-8}, 10^{-8}]) \tag{4.1}$$

To reduce the computational effort in case C, the (2L+2)-th FE calculation was canceled, whose consequence was the loss of consistent historical variable for the next step. In this way, the (2L+1) sets of history-variables resulted from the related parameter samples were transferred to the next step as the inputs for computation with corresponding new parameter samples. Although the mathematical rigor was lost to some extent in doing so, the effects of the identification appeared satisfactory as will be seen later on.

Table 4.2 Parameters of the sectional constitutive model for left column

Parameters	Initial guess	Lower limit	Upper limit	Finial estimation	Reference Value
N_y	1728.7	1581.5	2322.9	2075.5	2059.7
M_p	185.8	170.0	249.4	223.3	221.4
k_h	0.015	0.010	0.040	0.208	0.030

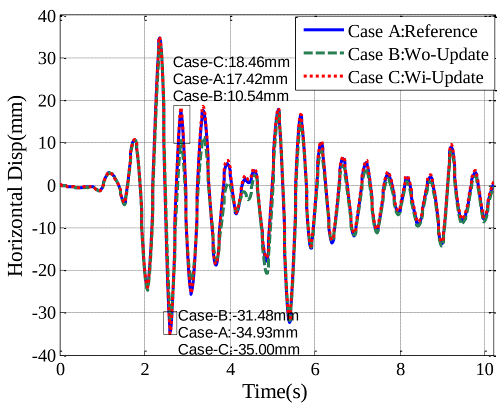


Figure 4.3 Time histories of roof horizontal displacement . (Case A vs Case B vs Case C)

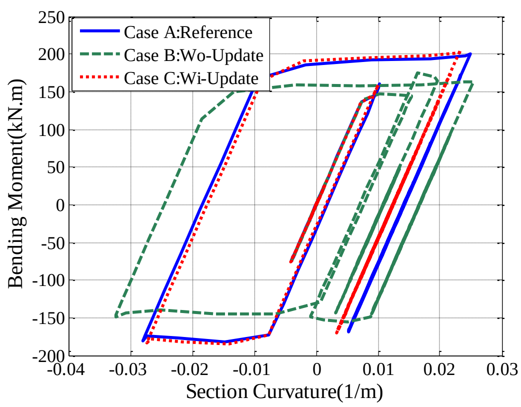
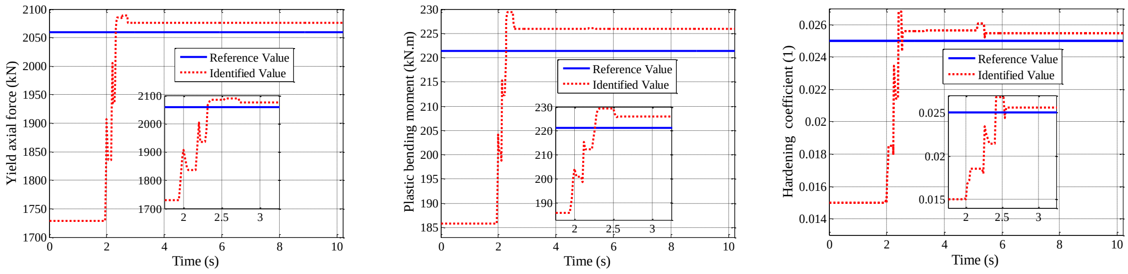


Figure 4.4 Moment vs. curvature at bottom section of column 2 (Case A vs Case B vs Case C)

Figs. 4.3 and 4.4 show the horizontal displacement responses at top of the column of the numerical substructure and moment-curvature hysteretic loops at the bottom cross-section of the column. In Fig. 4.3, we see that the displacement response with online model updating (Case C) was closer to reference solution (Case A) than that without parameter updating (Case B). The differences of Case B and the other two cases can be seen more clearly from the hysteretic loops as shown in Fig. 4.4. These indicate the advantage of the proposed model updating method over conventional hybrid test without updating. The time-histories of the model parameters are shown in Fig. 4.5. It is seen that all the parameters converged to the final values after around 2.2 s. The errors of axial yielding force, plastic moment and kinematic hardening coefficient relative to the references were 0.768%, 0.894%, and 4.67%, respectively; they were much improved than the initial parameters.



(a) Evolution of yield axial force N_y

(b) Evolution of plastic bending moment M_p

(c) Evolution of hardening coefficient

Figure 4.5 Time history of the updating parameters

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