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Hybrid Testing System with General Interfaces to Coordinate Substructures

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ABSTRACT
An interface element method is proposed to couple multiple subdomains. The overall stiffness matrix of a coupled structure is first formulated separately considering the stiffness matrices of subdomains and the stiffness matrix of interface elements. The method essentially introduces boundary confinement forces to substructures, and builds up the equilibrium and compatibility on the common boundaries between subdomains. Considering the tested substructure in an online hybrid test, the boundary stiffness matrix cannot be obtained explicitly. The static condensation technique and the BFGS method are further introduced. Only boundary forces and displacement are implemented to realize the coordinating between subdomains, while the common nodes between subdomains are not necessary. Finally, the nonlinear time history analysis was conducted on a cantilever beam with a concentrated mass, which fully demonstrate the feasibility and accuracy of the interface element method.

KEYWORDS: *Interface element; Coupled subdomains; Static condensation technique; BFGS method*

1. INTRODUCTION

In the past decade, various geographically distributed hybrid tests have been conducted between different laboratories including remote experimental and numerical substructures. Park et al. [1] conducted several rounds of distributed tests on a base-isolated bridge with multiple piers. Two rubber bearings were tested at two distributed laboratories, each bearing loaded by two hydraulic actuators. More recent efforts have focused on developing a general software framework for hybrid experimental-computational simulation such as OpenFresco at the University of California, Berkeley [2]. Demonstration tests included a base-isolated bridge pier tested collaboratively between Japan and U.S with only one rubber bearing tested using three quasi-static jacks. Another hybrid test framework, Internet-based Simulation for Earthquake Engineering (ISEE), was developed at National Center for Research on Earthquake Engineering (NCREE), Taiwan [3]. A three-site hybrid test was conducted on three piers of a multi-span continuous bridge. Each laboratory handled one pier using two hydraulic actuators. Mosqueda et al. [4] conducted a five-site collaborative test within the George E. Brown Network for Earthquake Engineering Simulation (NEES) [5] laboratories in the U.S. Five piers of a six-span bridge were taken as the substructures, in which two were physically tested, each using one hydraulic actuator, while the others were numerically simulated. Also as part of NEES, a hybrid test framework UI-SIMCOR was developed at the University at Illinois at Urbana-Champaign [6]. Recently, a three-site large scale bridge hybrid test was conducted with two experimental sites, each loading one pier by two hydraulic actuators. Most past distributed hybrid tests have been on multi-span bridges with the piers as substructures. This model provides relatively simple boundary conditions that could be controlled by limited number of actuators. Further, few hybrid tests have examined structures up to collapse with significant geometric and material nonlinearities [7, 8]. A key step in a hybrid test to collapse is partitioning the structure to capture the collapse mechanism experimentally while properly enforcing boundary conditions.

2. THEORY OF INTERFACE ELEMENT METHOD

Interface elements are virtual elements without material definitions. Only virtual nodes and interpolation functions are defined for interface elements, so that they serve as coordinating components between subdomains. Considering a generalized structure shown as Fig2.1, the structure is partitioned into two independent subdomains, denoted as Ω_1 and Ω_2. Suppose q is the displacement field, then each subdomain is spatially discretized into finite elements, where superscript i represents the boundary degrees of freedom, while superscript o stands for internal degrees of freedom of each subdomain. To couple the two subdomains, interface elements are defined as the domain v where all variables are denoted by the subscript s. Given two sets of shape functions, the displacement fields of each subdomains can be represented by Eqs. 2.1 and 2.2:

$$v = Tq_s \tag{2.1}$$

$$u = Nq^i \tag{2.2}$$

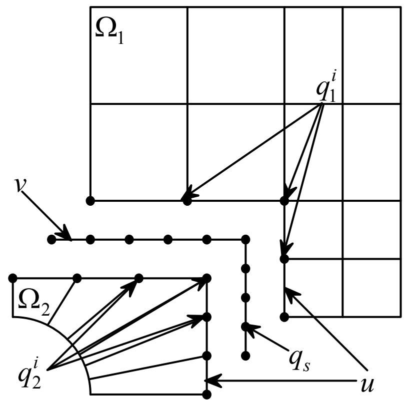


Figure 2.1 Interface elements and subdomains

Then the potential energy of the overall structure can be formulated as Eq.2.3 by introducing the Lagrangian multiplier, where K is the stiffness matrix, f is the external force, and λ is the Lagrangian multiplier. It is also possible to be reformulated as Eq. 2.4 where R is the shape function and α is the confinement force vector. Then conducting the variational principle on the potential energy with regards to the displacements of each subdomain and the confinement force vector, the equilibrium equation can be obtained, as Eq.2.5. The matrices M and G are expressed as Eqs. 2.6 and 2.7, respectively.

$$\begin{aligned}\Pi = {} & \Pi_{\Omega 1} + \Pi_{\Omega 2} + \int_s \lambda^T (\mathrm{v} - \mathrm{u})\,\mathrm{ds} = \frac{1}{2} q_1^{iT} K_1^{ii} q_1^i + \frac{1}{2} q_1^{oT} K_1^{oi} q_1^i + \frac{1}{2} q_1^{iT} K_1^{io} q_1^o + \frac{1}{2} q_1^{oT} K_1^{oo} q_1^o \\ & + \frac{1}{2} q_2^{iT} K_2^{ii} q_2^i + \frac{1}{2} q_2^{oT} K_2^{oi} q_2^i + \frac{1}{2} q_2^{iT} K_2^{io} q_2^o + \frac{1}{2} q_2^{oT} K_2^{oo} q_2^o - q_1^{iT} f_1^i - q_1^{oT} f_1^o - q_2^{iT} f_2^i - q_2^{oT} f_2^o \\ & + \int_{S_1} \alpha_1^T R_1^T (\mathrm{T_1 q}_s - \mathrm{N_1 q_1^i})\,\mathrm{ds} + \int_{S_2} \alpha_2^T R_2^T (\mathrm{T_2 q}_s - \mathrm{N_2 q_2^i})\,\mathrm{ds}\end{aligned} \tag{2.3}$$

$$\lambda = R\alpha \tag{2.4}$$

$$\begin{bmatrix} K_1^{ii} & K_1^{io} & 0 & 0 & 0 & M_1 & 0 \\ K_1^{io} & K_1^{oo} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_2^{ii} & K_2^{io} & 0 & 0 & M_2 \\ 0 & 0 & K_2^{io} & K_2^{oo} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_1 & G_2 \\ M_1^T & 0 & 0 & 0 & G_1^T & 0 & 0 \\ 0 & 0 & M_2^T & 0 & G_2^T & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1^i \\ q_1^o \\ q_2^i \\ q_2^o \\ q_s \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} f_1^i \\ f_1^o \\ f_2^i \\ f_2^o \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.5}$$

$$M_j = -\int_S \mathrm{N}_j^T R_j\,\mathrm{ds}\ (j\text{=1,2}) \tag{2.6}$$

$$G_j = \int_S T_j^T R_j\,\mathrm{ds}\ (j\text{=1,2}) \tag{2.7}$$

3. STATIC CONDENSATION TECHNIQUE AND BFGS METHOD

The objective of employing interface elements is to coordinate subdomains in a hybrid test framework. However, one issue has to be solved before the hybrid test application, which is the effective searching of boundary displacements. It is traditionally found by using of Newton method where the stiffness matrix is repeatedly used to find the accurate solution. Meanwhile, the stiffness matrix is updated considering the material and geometric nonlinearities. This is not feasible in a hybrid test since the encapsulated subdomains does not provide the stiffness matrix to the coordinator. What can be used are just the displacements and forces of common boundaries, as shown in Fig.3.1.

Given a structure external excitation and prescribed boundary conditions, the state variables can be fully determined. The reaction forces at the given boundaries can be obtained by the static condensation procedure and the Newton solution process. The static condensation is deemed as the feasible way to find the dynamic states of each subdomain for the condensed boundary stiffness and effective forces actually represent the entire subdomain. Take one subdomain as the example. The govern equation is shown as Eq. 3.1. With the static condensation procedure, only items associated with boundary degrees of freedom remains, as Eq. 3.2, where the left stiffness matrix can be defined as $K^{boundary}$, while the left can be denoted as effective force $f^{boundary}$. Therefore, Eq. 2.5 can be reformulated as Eq. 3.3.

Finally, the issue becomes to be how to get the boundary stiffness matrix $K^{boundary}$, and nonlinearities shall be considered in this process. Considering only boundary displacements and forces can be implemented, it is thus feasible to employ quasi-Newton procedure to solve this nonlinear problem unnecessarily formulating the stiffness matrix of the entire subdomain. Considering a general structure as shown in Fig.3.1, it is divided into two substructures. The initial stiffness matrix associated with boundary degrees of freedom of each substructure is given but not necessarily accurate. Given an external load and prescribe boundary condition, each substructure is analyzed in a selected finite element program, and the reaction forces corresponding to the boundary can be obtained. This provides enough information to update the boundary stiffness by using of the quasi-Newton procedure, using the incremental displacement and force vectors, denoted as $\Delta^1 q_n{}^i$, $\Delta^2 q_n{}^i$, $\Delta^1 \Psi_n{}^i$, and $\Delta^2 \Psi_n{}^i$, respectively for substructure 1 and 2 as shown in Fig.3.2, where a typical BFGS method is used. Once the matrix of each substructure is obtained, the global stiffness matrix is assembled with the displacement vector of interfaces q_s and the constraint force vectors α. Then this equation is solved to obtain the displacement increment which is further used to update the boundary displacement for the next step iteration. The iteration process stops until the convergence criterion is reached.

$$\begin{bmatrix} K^{ii} & K^{io} \\ K^{oi} & K^{oo} \end{bmatrix} \begin{bmatrix} q^i \\ q^o \end{bmatrix} = \begin{bmatrix} f^i \\ f^o \end{bmatrix} \tag{3.1}$$

$$(K^{ii} - K^{io} K^{oo-1} K^{oi}) q^i = f^i - K^{io} K^{oo-1} f^o \tag{3.2}$$

$$\begin{bmatrix} K_1^{boundary} & 0 & 0 & M_1 & 0 \\ 0 & K_2^{boundary} & 0 & 0 & M_2 \\ 0 & 0 & 0 & G_1 & G_2 \\ M_1^T & 0 & G_1^T & 0 & 0 \\ 0 & M_2^T & G_2^T & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1^i \\ q_2^i \\ q_s \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} f_1^{boundary} \\ f_2^{boundary} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.3}$$

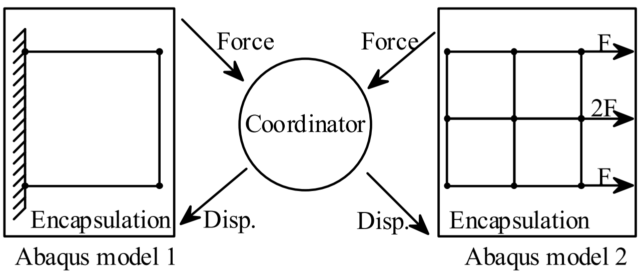


Figure 3.1 Cantilever beam collaboratively solved using interface elements and BFGS method

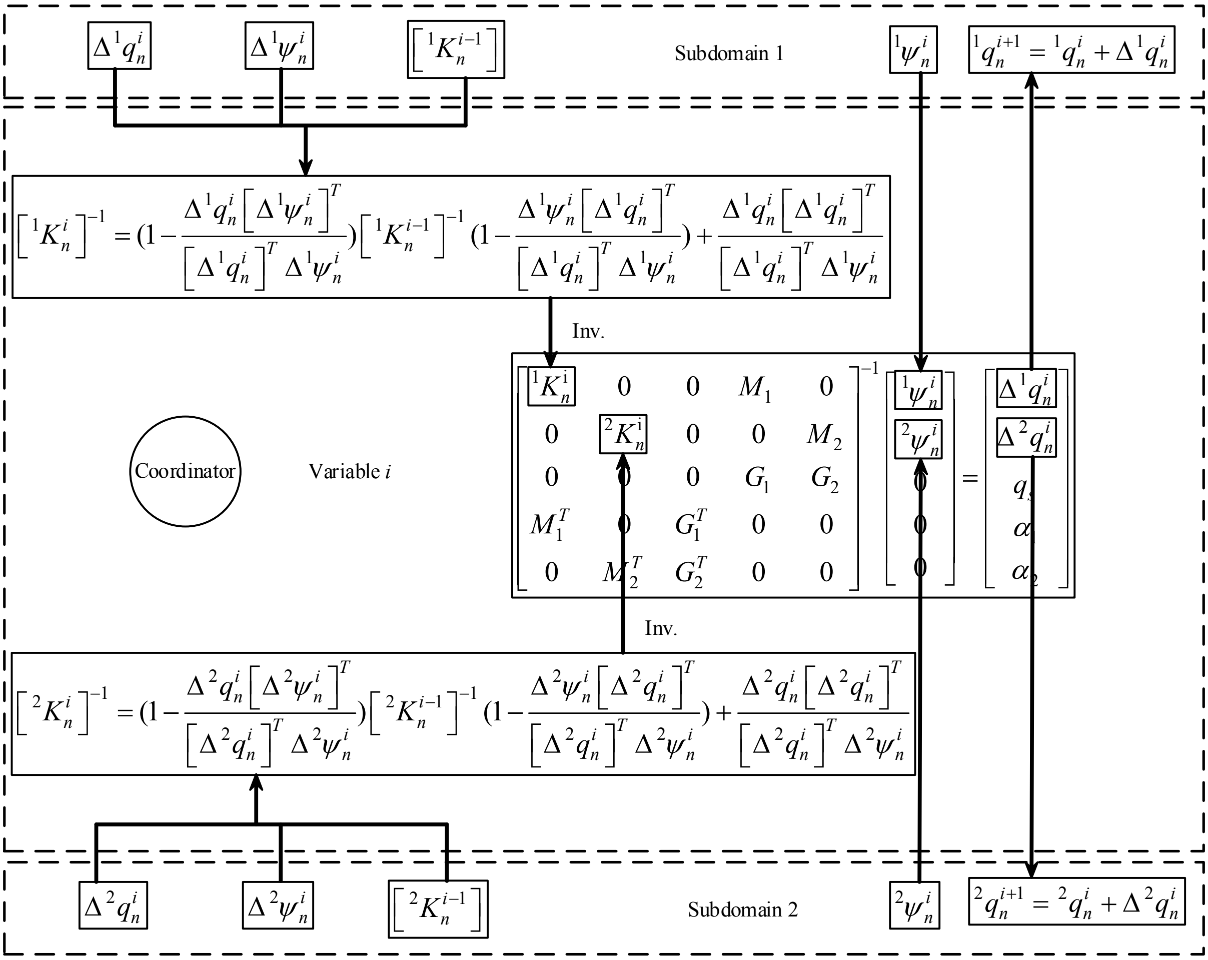


Figure 3.2 Iterative procedure of BFGS method combined with interface element technique

4. DYNAMIC EXAMPLE

To demonstrate the feasibility, the dynamics of a cantilever plate is examined, as shown in Fig.4.1. The height is 1 m, and the length is 2 m. It is a plane stress plate model intentionally meshed with different densities. The left part was fixed and only one element was used, while the right part was meshed with 2 x 2 grid. Each node has two degrees of freedom, vertical and horizontal displacements specifically. Node 2 was coupled with node 5 in both vertical and horizontal directions, and node 4 was coupled with node 11 in the same way. Node 8 belongs to the right part, and has identical horizontal displacement as Nodes 5 and 11. The young's module is 21000 Pa, the poisson ratio is 0.3, and the thickness of the plate is 0.025m. The strength is 300Pa. There's a concentrated mass of 31.2kg associated with Node 10.

The cantilever plate was then partitioned into two subdomains. One only has one element, while the other has four elements. Two interface elements are used with a linear interpolation function, which coordinate the two subdomains perfectly. ABAQUS is used to simulate the two substructures independently considering a bilinear material model with the isotropic hardening rule. To form the coordinated govern equation, the boundary stiffness matrix and force vector associated with each substructure are identified by use of quasi-Newton method.

Ten seconds of simulation was conducted. The horizontal responses of the hybrid simulation matched well with the overall numerical simulation, as shown in Fig.4.3 where both horizontal and vertical displacements were compared. The force-displacement relationship of the entire structure is shown in Fig.4.4. It is demonstrated that the responses were well reproduced by the hybrid simulation using interface elements. Further, the displacement in the vertical direction at Node 8 is given in Fig.4.5. Considering the structure is symmetric, the theoretical response of Node 8 in the vertical direction shall be zero, while the maximum displacement of the vertical degree of freedom of Node 8 is about 1e-5m, much smaller than the vertical displacement at Node 5, 1.0e-2m. The error is so small that the global response is deemed accurate.

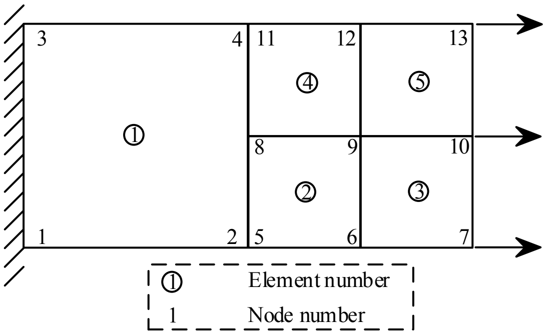


Figure 4.1 Overall FEM model using Abaqus

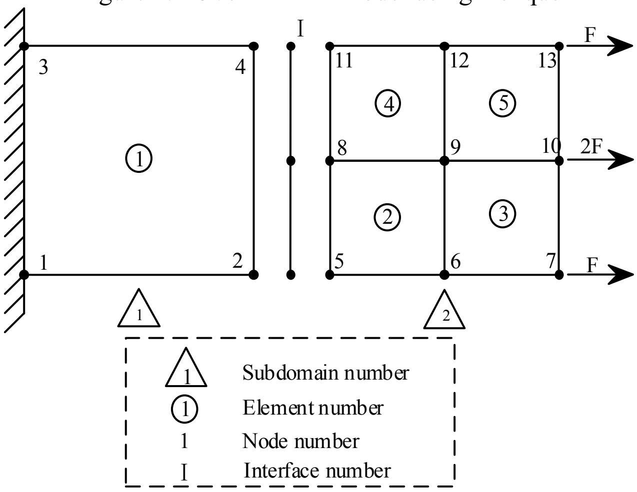
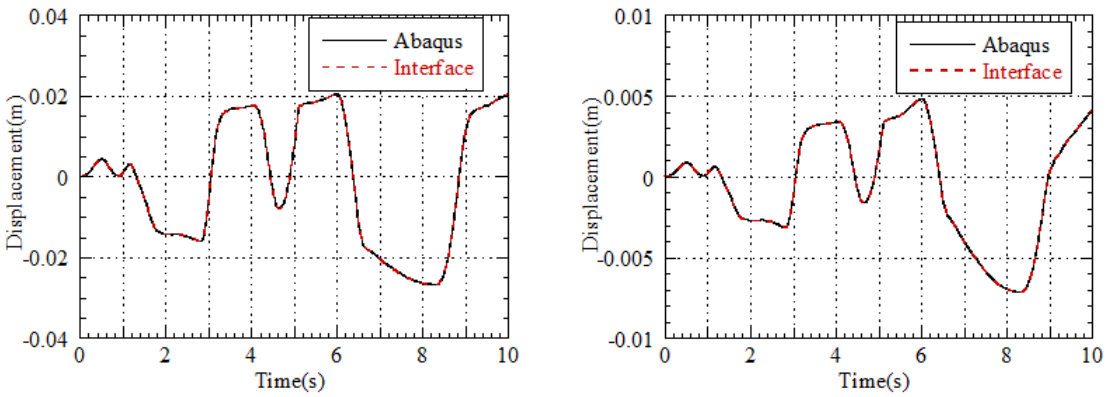


Figure 4.2 Substructures and interface elements for static analyses



(a) X-direction (b) Y-direction

Figure 4.3 Displacement response of Node 5

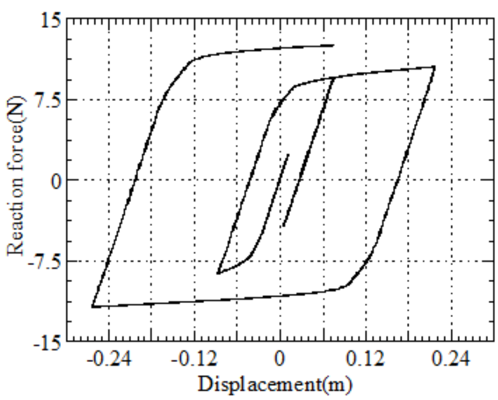


Figure 4.4 Force displacement relationship

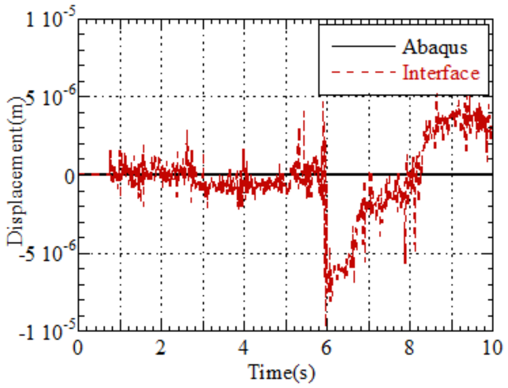


Figure 4.5 Displacement of node 8 in *y* direction

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