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Broadband Dynamic Load Identification Using Augmented Kalman Filter

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ABSTRACT
Knowledge of the input forces to systems is crucial for system identification, structural control and structural health monitoring. However, in many engineering structures, direct measurement of the applied input forces, e.g. wind loading, earthquake loads, forces from traffic on a bridge, etc. is not feasible. In this study, an indirect model-based method is developed by means of state augmentation in Kalman filter to estimate the input loading from dynamic characteristics and measured responses of the structural systems. The effectiveness of the proposed method is numerically validated with a truss bridge model; the augmented Kalman filter used along with multimetric measurements of acceleration and strain shows accurate results in estimating both low- and high-frequency components of the input excitation.

KEYWORDS: *Dynamic Force Identification, Strain, Kalman Filtering, Truss Bridge*

1. INTRODUCTION

Knowledge of the input forces to systems is crucial for system identification, structural control and structural health monitoring; design, safety, and performance of systems and structures can be enhanced if the forces applied to them are known. Determining applied forces can be accomplished either using direct measurement methods or indirect estimation of the input loading. Direct measurements are in most cases not possible due to several reasons such as hardware limitation, cost, etc. Therefore, indirect methods have been proposed and developed [1, 2]. That is, instead of measurement of the forces directly, they are estimated based on the measured responses and dynamic properties of the system. It may seem straight forward and comparable with system identification methods where the dynamic properties are estimated using outputs and inputs of the system; theoretically it is possible to find the inputs if Frequency response functions and outputs of the system are known. However, FRF matrix suffers from rank deficiency and the input estimation is an ill-posed inverse problem where presence of small noises and deviations causes significant errors; therefore the results can be far from reality and misleading [3, 5].

In structural engineering, Kalman Filtering (KF) based approaches have proven to be effective and promising way of identification of input loadings [6] also response estimation at unmeasured locations [7, 8]. KF is a recursive algorithm that models the system linearly in a set of state equations. Data which are polluted by Gaussian distributed errors can be processed and the states are estimated in an optimal manner. It means that the error covariance matrices are minimized. Applications of the K-F are broad and include for example navigation, object tracking, economics, signal processing, etc [9]. There are different variants of KF based force estimation methods. One technique requires all the states to be measured which is not practical in many cases [5, 6]. Another approach which is called Augment Kalman Filtering (A-KF) [10]. A-KF has the stability problem if the accelerations are the only measured responses and since the error covariance matrix of A–KF has simple form of Riccati equations, using analytical arguments it is shown that estimations based on solely acceleration measurement are inherently unstable [11]; and other measurements such as displacement or velocity in addition to the accelerations would solve the problem. In the same paper it is suggested to use the dummy measurement but it seems there would be difficulties in estimation of low varying function with nonzero means.

One of the reasons that acceleration measurement is widely used is structural engineering, system identification and load identification is that they are usually the cheapest and easiest one to measure. Despite development of very accurate displacement measurement devises [12] measurement of strain is in many cases, especially civil engineering problem where deformations are small, much easier, practical, and cost effective compared to displacement or velocity measurements.

In this paper, AKF method is used to estimate the inputs to the truss bridge model when both strain and acceleration measurements are used together. The effectiveness and reliability of the method is investigated when the system is subject to different simultaneous forces and the results are compared with those based on only acceleration measurements. It is shown that when there are forces with nonzero mean values the acceleration results can be misleading even when there is no measurement noise and no modeling error. In order to use the strain measurement in KF technique, linear expression between strains and displacements for planar truss is obtained. The results show that combination of limited number of strain and acceleration measurements provides stable and accurate results for the load estimation in the whole frequency range even when there are modeling errors and measurement noises.

2. KALMAN FILTER FORMULATION

Linear second order differential equation describes motion of a linear discrete system.

$$\mathrm{M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = S_f f(t)} \tag{2.1}$$

Where M, C, and K are mass, damping, and stiffness matrices of size $n \times n$ and $\boldsymbol{S_f}$ is force selection matrix of size $\mathrm{n \times n_f}$. The number of degrees of freedom is denoted by “n” and “$\mathrm{n_f}$ ($\leq$ n)” is the number of degrees of freedom subjected to the nonzero excitations.
State space representation of the equation of motion, which is rewriting the second order equation in terms of set of first order equations [13], for discrete case in presence of process error, $\boldsymbol{w_k}$, and measurement noise, $\boldsymbol{v_k}$, the state equation and observation equations become:

$$\mathrm{\dot{x}_{k+1} = Ax_k + Bf_k + w_k} \tag{2.2}$$

$$\mathrm{y_k = Hx_k + Df_k + v_k} \tag{2.3}$$

2.1 AUGMENTED KALMAN FILTER

In augmented KF (A-KF), the state and force vectors are placed into the single vector [10]:

$$\mathrm{X_{k+1} = AX_k + Bf_k + w_k} \tag{2.4}$$

$$\mathrm{f_{k+1} = f_k + \eta_k} \tag{2.5}$$

$$\mathrm{X_k^a = \begin{Bmatrix} X_k \\ f_k \end{Bmatrix}_{(n_s+n_p)\times 1}} \tag{2.6}$$

Then, the state equation:

$$\mathrm{X_{k+1}^a = A_a X_k^a + \zeta_k} \tag{2.7}$$

$$\mathrm{A_a = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}} \tag{2.8}$$

And observation Equation is:

$$\mathrm{d_k = G_a X_k^a + v_k} \tag{2.9}$$

$$\mathrm{G_a = [G \;\; J]} \tag{2.10}$$

Matrices G and J are given in equation 2.15. Then the time and measurement update equations in the A-KF method become:

Measurement update

$$\begin{gathered} L_k = P_{k|k-1} G_a^T \left(G_a P_{k|k-1} G_a^T + R \right)^{-1} \\ \hat{X}_{k|k}^a = \hat{X}_{k|k-1}^a + L_k (d_k - G_a \hat{X}_{k|k-1}^a) \\ P_{k|k} = P_{k|k-1} - L_k G_a P_{k|k-1} \end{gathered} \tag{2.11}$$

Time update

$$\begin{gathered} \hat{X}_{k+1|k}^a = A_a \hat{X}_{k|k}^a \\ P_{k+1|k} = A_a \, P_{k|k} A_a^T + Q_a \end{gathered} \tag{2.12}$$

In the A-KF, since the force vector is augmented in state vector, the modeling error covariance matrix together with regularization matrix S form Q_a , which is augmented covariance matrix:

$$Q_a = \begin{bmatrix} Q & 0 \\ 0 & S \end{bmatrix} \tag{2.13}$$

Observation (measurement) matrix, G_a, in A-KF can be derived starting from Eq. 2.14:

$$d(t) = S_a \ddot{u}(t) + S_v \dot{u}(t) + S_d u(t) \tag{2.14}$$

S_a, S_v, and S_d are "$n_d \times n$" matrices. "n_d" is number of measurements. We have:

$$G = [\, S_d - S_a M^{-1} K \,,\; S_v - S_a M^{-1} C \,], \qquad J = S_a M^{-1} S_p \tag{2.15}$$

2.2 AUGMENTED KALMAN FILTER WITH STRAIN MEASUREMENT

When strain is measured, if we can linearly relate the strains to the displacements, then it is possible to use augmented Kalman filter; therefore Eq. 2.14 is written as:

$$d(t) = S_a \ddot{u}(t) + S_v \dot{u}(t) + S_d u(t) + S_s u(t) \tag{2.16}$$

We need to relate the strains to the displacements linearly which is in the form of Eq. 2.9; the linear relation of strains and displacements (states) is convenient for A-KF. Then to incorporate the strain, Eq 2.15 becomes:

$$G = [\, S_s + S_d - S_a M^{-1} K \,, \quad S_v - S_a M^{-1} C \,], \quad J = S_a M^{-1} S_p \tag{2.17}$$

In the last equation, S_s is a $n_d \times n$ matrix.

3. SIMULATION AND RESUTLS

A truss bridge model with 20 joints and 37 members used for simulation is shown in Figure 3.1. All the vertical members have the length of 8m and the horizontal ones 10m. Fourth order Runge-Kuta method [14] is used for simulation of the response of the system subjected to the loading. The strain is also simulated using Eq. 2.17 at each time step. All the members have the same cross section of 39 cm^2 and modal damping ration of 2% was considered for all the modes of the structure. Black arrows applied in vertical direction at joints (nodes) number 7, 9, 11, and 15 show the applied force locations in the structure; four different excitations are simultaneously applied to the system. Time step for simulation is 1/4096 seconds. Frequency response function (FRF) of DOF "20" and excitation at same DOF is shown in Fig. 3.1 by black dotted line. The frequency range is same as the bandwidth of the applied forces to the structure.

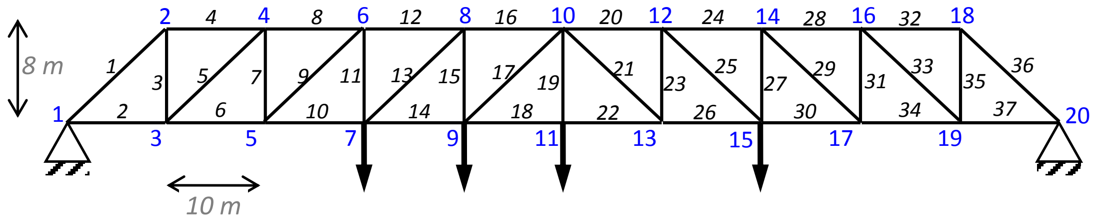

Figure 3.1 Truss bridge model used for simulation. There are 20 joints (nodes) and 37 members (truss elements). Downward bold arrows shows the locations of four forces which are applied simultaneously. Joint and member numbers are depicted.

In order to have more realistic simulation results for force identification, modeling error (5%) is also considered; this is done by using a 5% stiffer system used for K-F in order to reconstruct the applied forces and the FRF is shown by red solid line.

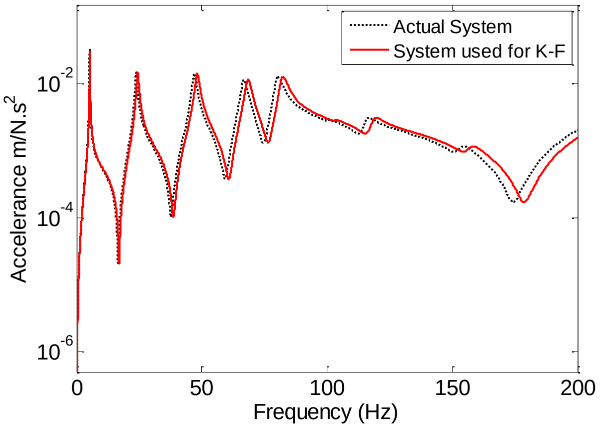

Figure 3.2 FRF of the system that is used for simulation of response (Dotted black); and FRF of the perturbed system, to represent modeling error (5%) used for input estimation in A-KF (solid red).

Four different forces are simultaneously applied to the structure: Random force, impulsive force, bump with random force, and ramp shape force with added random force. The applied forces are shown with black dotted lines in Fig. 3.3.

The applied forces are estimated using AKF for two different cases: when only acceleration is measured as the response; and when both strain and acceleration are used. Each case is considered when there is no additional measurement and modeling error and when both are present; measurement noise of 2% (of RMS magnitude) and modeling error of 5% stiffer system is considered. To summarize the scenarios:

- Case 1: Only acceleration measurement is used
  - No additional measurement noise and No modeling error
  - 2% measurement noise and 5% modeling error
- Case 2: Both Strain and acceleration are used.
  - No additional measurement noise and No modeling error
  - 2% measurement noise and 5% modeling error

3.1. CASE 1: ACCELERATIONS ARE MEASURED ONLY

The system shown in Fig. 3.1 subjected to the aforementioned loading and its acceleration is measured (simulated) in vertical direction of joints 3, 5, 7, 9, 11, 13, 14, 15, 16 and in horizontal direction of Joint 6, 9 and 15.

Using A-KF method, when there are no additional measurement noises or modeling errors, the applied and estimated forces are shown in Fig. 3.3. In practice it is impossible to have perfect model or measurements; therefore, to have more realistic results from simulation, measurement noises and modeling error are considered. The applied forces and the reconstructed ones are depicted in Fig. 3.4.

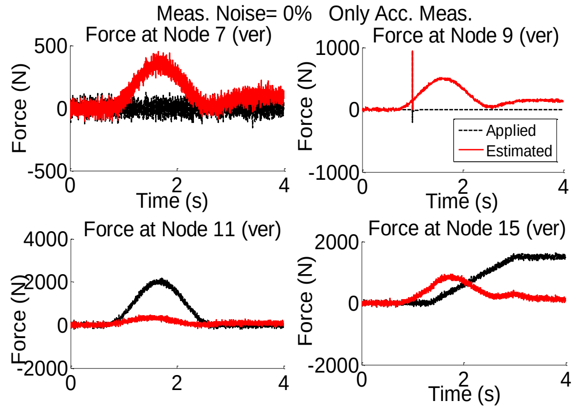

Figure 3.3 Estimated and applied four simultaneous forces when accelerations are the only response measurements; no additional noise and without modeling errors

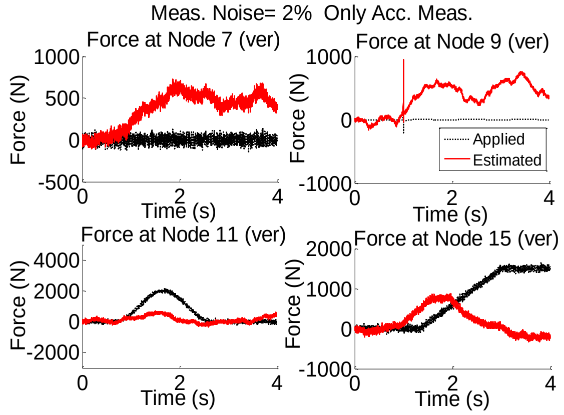

Figure 3.4 Estimated and applied forces; Accelerations are the only response measurements; additional noise and modeling errors are present.

3.2. CASE 2: STRAINS AND ACCELERATIONS ARE MEASURED

There are four different forces simultaneously applied to the structure same case “1”. In addition to the accelerations, strains are also used in order to estimate the applied forced. Accelerations are measured in vertical direction of joints 5, 7, 9, 11, and 15; strain in the members 10, 17, 22, 25, and 30 are measured. When neither measurement error, nor modeling error is considered, the estimated forces are shown in Fig. 3.5.

In presence of 2% measurement noise, and 5% modeling error, the input loadings are estimated using both strains and accelerations and the results are shown in Fig. 3.6.

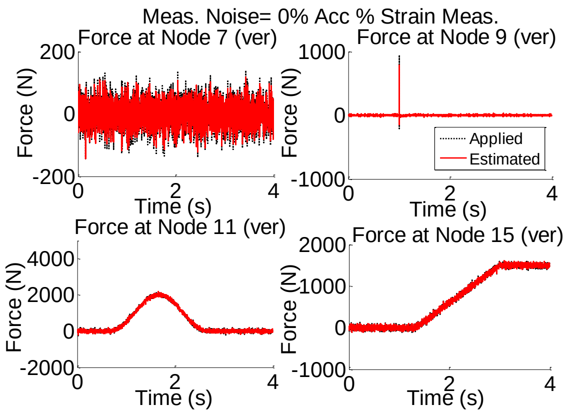

Figure 3.5 Estimated and applied four simultaneous forces when accelerations and strains are used together as response measurements; no additional noise and without modeling errors

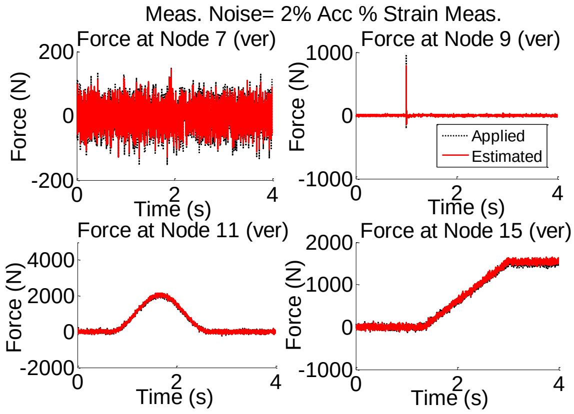

Figure 3.6 Estimated and applied four simultaneous forces when accelerations and strains are used together as response measurements; additional noise and modeling errors are present

In order to better see the quality of estimated forces using acceleration and strain measurements when both modelling error and measurements noises are present, Fig. 3.6 is plotted for shorter time intervals and shown in Fig. 3.7.

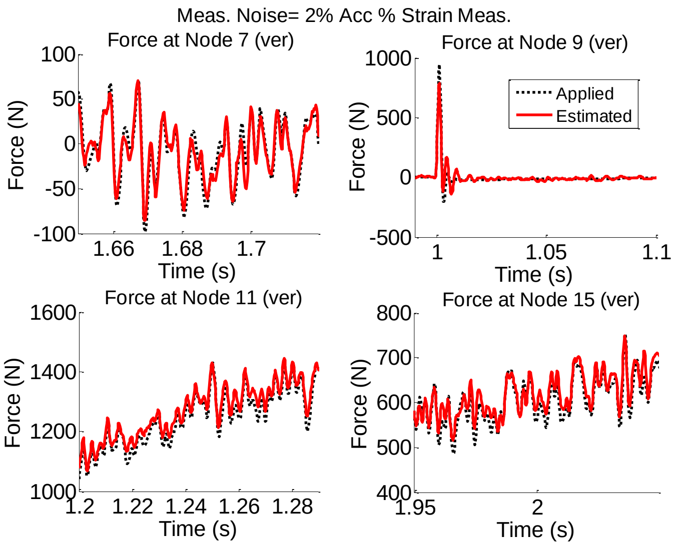

Figure 3.7 to observe the estimated loads in more detail, in the case of both acceleration and strain measurements and in presence of both modeling error and measurement noises, Fig. 3.6 is re-plotted for shorter time intervals.

Quality of the input force estimation for when accelerations are the only measured responses and when accelerations and strains are measured together for both cases of without and with additional measurement noises and modelling errors are compared by plotting the RMS error of estimated and applied forces in Fig. 3.8.

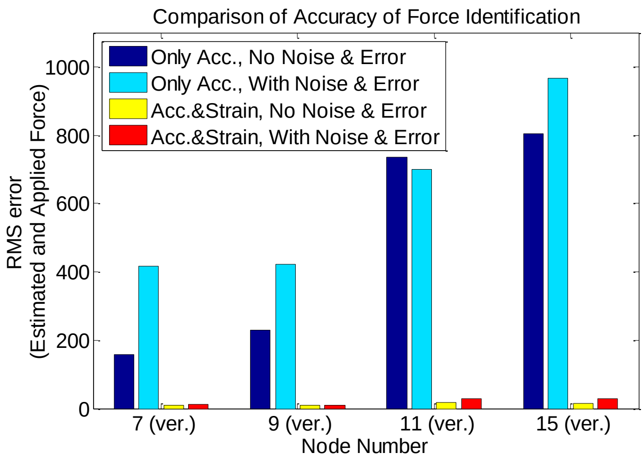

Figure 3.8 RMS error comparisons between the estimated and applied forces for the four cases given in Fig. 3.3 to Fig.3.6.

It is seen that A-KF has superior performance for input estimation when strain and acceleration measurements are used compared to the case when acceleration is the only measured responses.

4. SUMMARY

Using numerical simulation of the bridge truss model subjected to four different forces applied simultaneously at four different points, using Augmented Kalman Filter (A-KF) method the inputs were estimated using two different response measurements: 1) only acceleration measurements; and 2) for the case when strains of few members were measured in addition to the acceleration; for the bridge truss model strains are linearly related to displacement which is suitable for A-KF. Also to have more realistic simulation results, additional noises and modeling error for the system to be used by A-KF was also considered. It is seen that for multi Input cases which is the case in reality, especially when there are low frequency, DC components and nonzero forces, force reconstruction methods relying on only acceleration measurements is unstable and misleading. On the other hand, incorporation of strain of few members in the estimation would highly stabilize and improve the quality of estimation. To have fair comparisons, the number of measured responses is similar for all cases.

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