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An Improved System Identification Approach to Enable Structural Health Monitoring

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ABSTRACT

The objective of this study is to improve the conventional stochastic subspace system identification method in order to advance structural health monitoring. To obtain qualitative identified modal properties, this conventional method requires expensive computational power to deal with a large set of measurements. Therefore, this study proposes an automated stochastic system identification framework that improves the computational efficiency and quality. This framework consists of 1) signal preprocessing, 2) order determination, 3) efficient system identification, and 4) mode assurance. Signal preprocessing removes some noise and uncertainties from the measured responses of a structure. The order determination attempts to give the number of possible natural frequencies that is equivalent to the number of states used in system identification. The efficient system identification modifies the conventional method by integrating a reduced-dimension Hankel matrix and efficient orthogonal projection with the state-space realization and modal extraction. Consequently, the mode assurance utilizes the kernel density estimation and singular value decomposition to categorize the assured modes. The proposed identification framework is applied to the field measurements of a cable-stayed bridge. The results show that the automated framework saves a large amount of computational time, while qualitative modal parameters are still attained.

KEYWORDS: *Efficient Stochastic System Identification, Mode Assurance, Operational Modal Analysis*

1. INTRODUCTION

Performance of civil infrastructure directly affects public safety and society cost. Civil infrastructure refers to the integration of various systems such as buildings, bridges, transportation networks, lifeline, etc. Components in such systems need to be functional; otherwise, a huge amount of economic loss and dead lives would occur and impact the entire society. For example, the I-35W Mississippi River Bridge collapsed on August 1, 2007, resulting in 13 people killed and 145 people injured. A replacement bridge, the I-35W Saint Anthony Falls Bridge, was then constructed and opened on September 18, 2008. The estimated users' economic loss was US$71,000 to US$220,000 a day, while more than 50,000 users needed to reroute [1]. This example demonstrates that deficient and aging infrastructure systems require diagnosis of present conditions to prevent catastrophic failures. Thus, structural health monitoring (SHM) is of need to early identify these problems in structures [2, 3].

Operational modal analysis is to extract the dynamic characteristics of structures based on the structural vibration responses. These dynamic characteristics are composed of natural frequencies, damping ratios, and mode shapes. Deviations in these dynamic characteristics reflect the changed properties of structures. Detailed inspection may be required for the structures. A number of researchers applied system identification to real-world structures for operational modal analysis such as Farrar and James [4], Brownjohn [5], Lynch et al. [6], Siringoringo and Fujino [7], Weng et al. [8] and Jang et al. [9]. With growth of sensing technology, structural health monitoring has drawn a lot of attentions to researchers in order to assure structures of their serviceability and safety. Structural integrity can be then studied through operational modal analysis to the sensor measurements of structures.

Due to unavailable input excitation, the operational modal analysis is directed to the stochastic system identification. This type of system identification methods is emphasized on assessment of structures through the structural response outputs. In terms of time-domain approaches, one popular method is stochastic subspace system identification (SSI) proposed by Van Overschee and De Moor in 1991 [10]. This method utilizes the orthogonal projection to obtain the extended observability. By solving the extended observability matrix, the system and measurement matrices in a stochastic state-space model are derived. Information about the modal parameters are obtained from these two matrices. The SSI method has been proven to be numerically stable, robust to noise, and applicable for non-stationary excitations. Peeters and De Roeck in 1999 further extended the SSI method to address the shortcoming of computational efficiency [11]. Peeters and De Roeck in 2001 also proposed the stabilization diagram for SSI to enhance the quality of identification results [12]. Even though such methods are available for operational modal analysis, both identification efficiency and quality are still a concern.

One of commonly seen problems in SSI is the noise modes obtained in results. This noise modes may be reduced or removed by means of time series analysis prior to system identification. A novel technique of time series analysis is the singular spectrum analysis (SSA). By forming delayed coordinates of measured signals, a Hankel matrix is established. Utilizing SVD, the principle components in a time series can be distilled. A number of fields has employed SSA to investigate the trend and periodic components to various components. The early development of SSA includes the publications by Broomhead and King in 1986, Fraedrich in 1986, and Broomhead et al. in 1987 [13-15]. In 1996, Elsner and Tsonis reorganized the content and prepared a textbook regarding SSA [16]. Moreover, SSA is applicable to not only a single time series but also multi-channel time series. For structural health monitoring applications, a large amount of data are measured from structures. When applying SVD to the Hankel matrix, the loading in computation becomes a problem. Further reductions on the dimension of the Hankel matrix is a solution to expedite the computational rate as well as to be suitable for structural health monitoring problems.

Another possible reason to result in noise modes in identification results is the orthogonal projection. Yang and Nagarajaiah in 2014 found that the outliers in measure signals can contaminate identification results and introduce noise modes to be obtained [17]. Moreover, the low-amplitude signals in measurements can also induce the difficulty in the calculation of a subspace through the orthogonal projection. To effectively eliminate the noise modes in identification results, these unfavourable components need to be removed in advance.

As introduced in [12], a stabilization diagram is helpful for determining true modes in identification. However, establishing a stabilization diagram may take a large amount of time, especially state-space realization. Many studies developed a series of methods that allow determining the number of states [18-20]. The resulting number of states from these methods is tended to be overestimated. Thus, these methods can be employed in a stabilization diagram to limit the numbers of states used. To accelerate the determination of true modes, modifications should be made to a stabilization diagram.

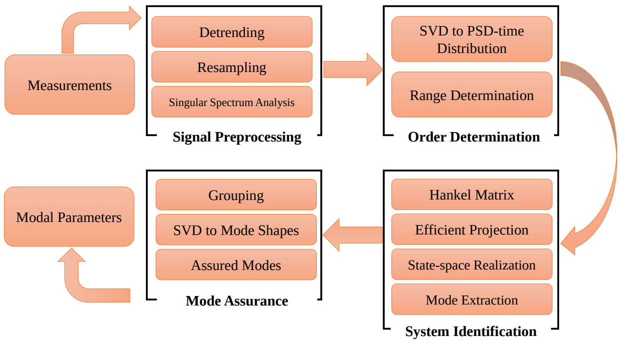

Figure 1. Framework of proposed stochastic subspace system identification.

In this study, an automated stochastic subspace system identification framework is developed. This framework consists of 1) signal preprocessing, 2) order determination, 3) efficient system identification, and 4) mode assurance. The signal preprocessing preliminarily processes the measured signals using the detrending, resampling, and singular spectrum analysis techniques, and renders reduced noise and uncertainties in signals. The order determination recognizes the possible natural frequencies from the multi-channel power spectral density (PSD) distribution over time, resulting in a range of orders (or numbers of states) that will be used in the stochastic state-space realization. The efficient system identification utilizes a reduced-dimension Hankel matrix and modified orthogonal projection to efficiently realize multiple stochastic state-space models and then to extract numerous sets of modal parameters. The mode assurance consequently distills most likely modes from these identification sets using the kernel density estimation to natural frequencies and the singular value decomposition to a matrix set of mode shapes. By integrating these four components, the framework is sought to automate stochastic system identification using minimized user-defined parameters. The proposed stochastic subspace system identification framework is evaluated by the field measurements to a cable-stayed bridge in Taiwan. The identification results illustrate that the efficiency and quality of the proposed method is superior to those of the conventional stochastic system identification approach.

2. FRAMEWORK OF IMPROVED SYSTEM IDENTIFICATION METHOD

Figure 1 illustrates the automated stochastic subspace system identification framework. This improved method contains four sequential components: 1) signal preprocessing, 2) order determination, 3) efficient system identification, and 4) mode assurance. In the following, each component are described in detail.

In this study, the proposed system identification method is developed in accordance to the stochastic state-space model for a structure. The stochastic discrete-time state-space model is defined by

$$\begin{aligned} &\mathbf{x}[k+1] = \mathbf{A}_{\mathrm{d}}\mathbf{x}[k] + \mathbf{w}[k] \\ &\mathbf{y}[k] = \mathbf{C}_{\mathrm{d}}[k] + \mathbf{v}[k] \\ &\mathbf{x} \in \mathbb{R}^{n_s \times 1}, \mathbf{y} \in \mathbb{R}^{n \times 1},\ \mathbf{w} \in \mathbb{R}^{n_s \times 1}, \mathbf{v} \in \mathbb{R}^{n \times 1} \end{aligned} \tag{1}$$

where $\mathbf{x}$ and $\mathbf{y}$ are the state and output measurement vectors at time step, k; $\mathbf{A}_{\mathrm{d}}$ and $\mathbf{C}_{\mathrm{d}}$ are the system and measurement matrices in the stochastic state-space representation; and $\mathbf{w}$ and $\mathbf{v}$ are the input and output noise. The time span is defined as $k \in [1, N]$ where N is the total number of samples.

2.1 Signal Preprocessing

Noise is a critical issue in signal processing. The stochastic subspace system identification can account for the Gaussian white noise; however, non-white noise may be irreducible that could induce unanticipated modes in the identification result. Structures usually have the components with high energy in the range of low frequencies. Resampling signals to the frequency range of interest is an approach to limit the frequency content in low frequencies. When decimating signals in resampling, a polyphase filter should be employed to avoid the aliasing effect [21]. An appropriate resampling rate can be determined by observing the power spectral density of signals.

The last approach considered in the preprocessing is to utilize the singular spectrum analysis. SSA is a novel and powerful tool that makes a decomposition of the original series into a sum of a small number of independent and interpretable components [13, 16]. Moreover, SSA can be extended to Multi-channel Singular Spectrum Analysis (MSSA) for a multivariate time series of vectors at concurrent moments at a same or different location(s) [15]. SSA or MSSA consists of three steps: embedding, singular value decomposition, and reconstruction. SSA has been widely studied by researchers, and the details about the SSA theory are available in Elsner and Tsonis 1996. In the following, MSSA is briefly summarized. The basic concept of MSSA is to find a transformation matrix that satisfies

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}[1] & \mathbf{y}[1+h] & \cdots & \mathbf{y}[1+(n_c-1)h] \\ \mathbf{y}[2] & \mathbf{y}[2+h] & \cdots & \mathbf{y}[2+(n_c-1)h] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}[l] & \mathbf{y}[l+h] & \cdots & \mathbf{y}[l+(n_c-1)h] \end{bmatrix} = \mathbf{TY} + (\mathbf{I}-\mathbf{T})\mathbf{Y} \tag{2}$$

where $\mathbf{Y}$ is the extended Hankel matrix (or the delay coordinates) with a specific delay, h; l is the window length each column vector; n_c is the number of delays in the row; $\mathbf{T}$ is the transformation matrix; and ($\mathbf{I}$-$\mathbf{T}$) indicates the noise or less significant components in the $\mathbf{Y}$ space. The transformation matrix represents the mapping of most significant components in the space of the measurements, $\mathbf{y}[k]$.

Embedding in SSA is to form an extended Hankel matrix such as $\mathbf{Y}$ in Eq. (2). In the SSA or MSSA theory, h in Eq. (2) is typically equal to 1; l should be in a range of 2~N/2; and nc is equal to N-l+1. To reduce the dimension in embedding, an additional parameter, h, is introduced. A smaller size of the extended Hankel matrix can expedite the rest of the SSA process (e.g., singular value decomposition and reconstruction). When using h, some rules should be satisfied such as

$$\begin{aligned} &1 \le h \le l \\ &\left(1 + (n_c - 1)h\right) > nl \\ &2 < l < N/2 \end{aligned} \tag{3}$$

Singular value decomposition in SSA is to obtain a transformation matrix. When $\mathbf{Y}$ in Eq. (2) is decomposed by SVD, the transformation matrix can be represented by

$$\mathbf{Y} = \mathbf{U\Lambda V}^T = \mathbf{U}_m\mathbf{U}_m^T\mathbf{Y} + \left(\mathbf{I} - \mathbf{U}_m\mathbf{U}_m^T\right)\mathbf{Y}$$
$$\mathbf{T} = \mathbf{U}_m\mathbf{U}_m^T \tag{4}$$

where $\mathbf{\Lambda}$ is a matrix which contains nonnegative singular values of **Y** in diagonal terms; **U** and **V** are unitary matrices. The singular values in $\mathbf{\Lambda}$ are in a decreasing order so that the first few column vectors in $\mathbf{U}_m$ indicate the most significant components in the space of **Y**. Assume that the number of modes in noise-free structural measurements is known, and the number of significant components are equal to two times of number of modes; the rest of singular values are very close to zero. Consider structural measurements with noise, and the rest of singular values will be nonzero.

The final step in MSSA is the reconstruction that averages out the processed measurements (e.g., **TY**). When single-channel measurements are considered and *h* equals 1, the reconstruction can be completed by the diagonal averaging of **TY** [16]. For MSSA, a block-diagonal averaging method can be applied if *h* is equal to 1. When *h* is greater than 1, the reconstruction can be completed by a similar approach, counting the number of occurrences per step in **Y** and then averaging the signals at a step by this number. The reconstructed signals contains the most dominate components in a structure.

2.2 Order Determination

This study proposes a quick approach that employs the SVD to multi-channel PSD-time distribution to first determine the order number. A PSD-time distribution exploits a short-time Fourier transform to generate a matrix of which each column represents the short-term, time-localized frequency content of a single-channel signal. In contrast to a frequency-time distribution, the PSD-time distribution is focused on the signal magnitude of the frequency content, given by

$$\mathbf{G}_1(\boldsymbol{\omega},\mathbf{t}) = \begin{bmatrix} \left\|g_1(\omega_0,t_0)\right\| & \left\|g_1(\omega_0,t_1)\right\| & \cdots & \left\|g_1(\omega_0,t_s)\right\| \\ \left\|g_1(\omega_1,t_0)\right\| & \left\|g_1(\omega_1,t_1)\right\| & \cdots & \left\|g_1(\omega_1,t_s)\right\| \\ \vdots & \vdots & \ddots & \vdots \\ \left\|g_1(\omega_f,t_0)\right\| & \left\|g_1(\omega_f,t_1)\right\| & \cdots & \left\|g_1(\omega_f,t_s)\right\| \end{bmatrix} \tag{5}$$

where $\mathbf{G}_1$ represents a PSD-time distribution from the channel 1; $\left\|g\right\|$ is a power spectral density at a frequency and time; $\boldsymbol{\omega}$ and **t** are frequency and time vector. Each column of $\mathbf{G}_1$ denotes a small segment of a time series. Each time segment can have an overlapped region with other segments. Windowing is recommended to use in the Fourier transform. The time in each column corresponds to the center of each segment. Presenting a PSD-time distribution in a colored plot can clearly show the major frequency components in a time series.

For multi-channel signals, a PSD-time distribution can be formed by stacking Eq. (5) in a row. This is because the focus of this approach in this study is to observe the number of natural frequencies. Similar to the principle component analysis, applying SVD to a PSD-time distribution can derive a principle component that corresponds to the most significant frequency content of multi-channel signals such as

$$\bar{\mathbf{G}} = \begin{bmatrix}\mathbf{G}_1 & \mathbf{G}_2 & \cdots & \mathbf{G}_n\end{bmatrix} = \mathbf{u}_1\lambda_1\mathbf{v}_1^T + \mathbf{U}_r\mathbf{\Lambda}_r\mathbf{V}_r^T \tag{6}$$

where $\bar{\mathbf{G}}$ is a multi-channel PSD-time distribution with n-channel signals; u1 is the first (or principle) component in terms of frequency, corresponding to the largest singular value, λ_1, with a right-singular vector, v1; and Ur, $\mathbf{\Lambda}_r$, Vr are the less significant singular vectors and values. u_1 will be employed to preliminary find the number of natural frequencies using the peak-picking method.

2.3 Efficient System Identification

The efficient system identification improves the conventional data-driven stochastic subspace system identification method [10]. The improvements include a reduced-dimension Hankel matrix and efficient orthogonal projection. With this efficient system identification, the computational time is significantly lessened.

Reduced-dimension Hankel Matrix

As described in [10], a Hankel matrix is formed in the beginning of the SSI method. Similar to Eq. (2), the Hankel matrix, **H**, is composed of the processed measurements, given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{Y}_\mathrm{p} \\ \hline \mathbf{Y}_\mathrm{f} \end{bmatrix} = \left[\begin{array}{cccc} \bar{\mathbf{y}}[1] & \bar{\mathbf{y}}[2] & \cdots & \bar{\mathbf{y}}[n_\mathrm{h}] \\ \bar{\mathbf{y}}[2] & \bar{\mathbf{y}}[3] & \cdots & \bar{\mathbf{y}}[n_\mathrm{h}+1] \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\mathbf{y}}[l] & \bar{\mathbf{y}}[l+1] & \cdots & \bar{\mathbf{y}}[n_\mathrm{h}+l] \\ \hline \bar{\mathbf{y}}[l+1] & \bar{\mathbf{y}}[l+2] & \cdots & \bar{\mathbf{y}}[n_\mathrm{h}+l+1] \\ \bar{\mathbf{y}}[l+2] & \bar{\mathbf{y}}[l+3] & \cdots & \bar{\mathbf{y}}[n_\mathrm{h}+l+2] \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\mathbf{y}}[2l] & \bar{\mathbf{y}}[2l] & \cdots & \bar{\mathbf{y}}[n_\mathrm{h}+2l] \end{array} \right] \tag{7}$$

where the subscripts, "p" and "f", denote the "past" and "future" delay coordinates, and $\bar{\mathbf{y}}$ is the processed measurements using the preprocessing methods. The purpose of forming the Hankel matrix is to obtain a column subspace through projection. However, the subspace may be derived a low-resolution projection if the column vectors in **H** has some small numbers. Thus, the improved method introduces a criterion that eliminates "ill-conditioned" column vectors in **H**. These ill-conditioned column vectors can be viewed as those have lower norms. Define that the column vector has the maximum norm in **H** as h_{max}, and then only those column vectors having a norm greater than $\alpha_h h_{\max}$ are retained, given by

$$\begin{gathered} \bar{\mathbf{H}} = \begin{bmatrix} \bar{\mathbf{Y}}_\mathrm{p} \\ \hline \bar{\mathbf{Y}}_\mathrm{f} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_{m_\mathrm{h}} \end{bmatrix} \\ \|\mathbf{h}_i\| \geq \alpha_h h_{\max},\ i \in [1, m_\mathrm{h}] \end{gathered} \tag{8}$$

where m_h is the number of columns in $\bar{\mathbf{H}}$. Utilizing Eq. (8), $\bar{\mathbf{H}}$ is derived with a reduced dimension, as well as the column vectors which may induce the projection errors are deleted.

Efficient Orthogonal Projection

The second step in the proposed SSI method is projection that allows one or multiple subspace(s) to be obtained. In Eq. (7), the Hankel matrix is divided into the past and future portions. As given in [10], the projected subspace is calculated using the past and future matrices in Eq. (8) and defined as

$$\mathbf{O} = \bar{\mathbf{Y}}_\mathrm{f} / \bar{\mathbf{Y}}_\mathrm{p} = \bar{\mathbf{Y}}_\mathrm{f} \bar{\mathbf{Y}}_\mathrm{p}^T \left(\bar{\mathbf{Y}}_\mathrm{p} \bar{\mathbf{Y}}_\mathrm{p}^T \right)^{-1} \bar{\mathbf{Y}}_\mathrm{p} \tag{9}$$

where **O** is the projected subspace. Consider using SVD to represent $\mathbf{Y}_\mathrm{p}$ and assume that $\mathbf{Y}_\mathrm{p}$ is a low-rank matrix, and then Eq. (9) can be simplified as

$$\begin{gathered} \bar{\mathbf{Y}}_\mathrm{f} / \bar{\mathbf{Y}}_\mathrm{p} \cong \bar{\mathbf{Y}}_\mathrm{f} \mathbf{V}_\mathrm{p,m} \mathbf{V}_\mathrm{p,m}^T \rightarrow \bar{\mathbf{Y}}_\mathrm{f} \mathbf{V}_\mathrm{p,m} \\ \bar{\mathbf{Y}}_\mathrm{p} = \mathbf{U}_\mathrm{p,m} \mathbf{\Lambda}_\mathrm{p,m} \mathbf{V}_\mathrm{p,m}^T + \mathbf{U}_\mathrm{p,0} \mathbf{\Lambda}_\mathrm{p,0} \mathbf{V}_\mathrm{p,0}^T \end{gathered} \tag{10}$$

where the subscripts, "m" and "0", denote the main and null spaces. All singular values in $\mathbf{\Lambda}_\mathrm{p,0}$ are assumed to be close to zero. $\mathbf{V}_\mathrm{p,m}^T$ in Eq. (10) is neglected because this matrix can be a similarity matrix to both extended observability and controllability matrices in the SSI theory. Therefore, a projected subspace is derived from $\bar{\mathbf{Y}}_\mathrm{f} \mathbf{V}_\mathrm{p,m}$.

In [10], a Hankel matrix in Eq. (7) only supports one subspace to be obtained. The separation line in this Hankel matrix yields an identical dimension of the past and future submatrices. Thus, the two submatrices result in the extended observability and controllability matrices with an equal number of rows and columns, respectively. However, the separation line can be shifted as long as the resulting past and future submatrices have a number of time lags greater than the number of system states. Shifting the separation line results in multiple subspaces to be obtained and multiple sets of modal parameters to be compared.

State-space Realization

After a projected subspace is obtained, the next step in the efficient system identification is state-space realization. Because the projected subspace represents the product of the extended observability and controllability matrices, SVD is employed to separate these two matrices. Then, the system and measurement matrices, $\mathbf{A}_d$ and $\mathbf{C}_d$, are calculated by

$$\begin{aligned}
&\bar{\mathbf{Y}}_f \mathbf{V}_{p,m} = \mathbf{U}_{id,m}\boldsymbol{\Lambda}_{id,m}\mathbf{V}_{id,m} + \mathbf{U}_{id,0}\boldsymbol{\Lambda}_{id,0}\mathbf{V}_{id,0} \\
&\boldsymbol{\Gamma} = \mathbf{U}_{id,m} = \begin{bmatrix} \mathbf{C}_d \\ \mathbf{C}_d\mathbf{A}_d \\ \mathbf{C}_d\mathbf{A}_d^2 \\ \vdots \\ \mathbf{C}_d\mathbf{A}_d^{n_s} \end{bmatrix}, \quad \mathbf{A}_d = \begin{bmatrix} \mathbf{C}_d \\ \mathbf{C}_d\mathbf{A}_d \\ \mathbf{C}_d\mathbf{A}_d^2 \\ \vdots \\ \mathbf{C}_d\mathbf{A}_d^{n_s-1} \end{bmatrix}^{\dagger} \begin{bmatrix} \mathbf{C}_d\mathbf{A}_d \\ \mathbf{C}_d\mathbf{A}_d^2 \\ \mathbf{C}_d\mathbf{A}_d^3 \\ \vdots \\ \mathbf{C}_d\mathbf{A}_d^{n_s} \end{bmatrix}
\end{aligned} \tag{11}$$

where the subscripts, "m" and "0", denote the main and redundant components in the product of the extended observability and controllability matrices; $\boldsymbol{\Gamma}$ represents the extended observability matrix; and $(\ )^{\dagger}$ denotes the pseudo inverse. $\mathbf{A}_d$ is derived from Eq. (11), and $\mathbf{C}_d$ is obtained from the first block matrix in $\boldsymbol{\Gamma}$.

Mode Extraction

The objective of the efficient system identification is to derive the modal parameters by a given number of states. Applying eigen analysis to $\mathbf{A}_d$ renders complex-valued natural frequencies and mode shapes with respect to the states. Thus, all modal parameters in each mode are represented by

$$\begin{aligned}
&\mathbf{A}_d\boldsymbol{\eta} = \lambda_d\boldsymbol{\eta} \\
&\frac{1}{\Delta t}\ln\left(\lambda_d, \lambda_d^*\right) = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \\
&\boldsymbol{\varphi} = \mathbf{C}_d\boldsymbol{\eta}
\end{aligned} \tag{12}$$

where λ_d is one of the eigenvalues in $\mathbf{A}_d$; λ_d^* is the complex conjugated λ_d; $\boldsymbol{\eta}$ is the eigenvector with respect to the state vector; ω_n and ξ are the natural frequency and damping ratio; Δt is the sample time; and $\boldsymbol{\varphi}$ is the mode shapes with respect to the outputs. By calculating all eigenvalues and eigenvectors from $\mathbf{A}_d$, the modal parameters can be extracted.

2.4 Mode Assurance

The mode assurance is to acquire most likely modal parameters from a variety of identification results. In the proposed framework, various sets of modal parameters are extracted by a number of separation lines to Eqs. (7, 10) and a range of orders to Eq. (11). Using the kernel density estimation and SVD to a grouped mode shapes matrix, the most significant modal parameters are obtained.

Grouping

The kernel density estimation [23] serves as a role to group various natural frequencies into several sets. This grouping process is to isolate stable natural frequencies from spurious ones. The kernel density estimation is a non-parametric approach to estimate the probability density function of an array as well as to cluster an array into several segments. Thus, the kernel density estimation to an array of natural frequencies is written by

$$f\left(\omega_k\right) = \frac{1}{m_f h_k}\sum_{i=1}^{m_f} K\left(\frac{\omega_k - \omega_{id}}{h_k}\right) \tag{13}$$

where the kernel function is defined as

$$K\left(x\right) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} \tag{14}$$

$f\left(\omega_k\right)$ is a probability density function of ω_k; h_k is a bandwidth to the kernel function; m_f is the number of natural frequencies in an array; and ω_{id} is identified natural frequencies. If multiple sets of modal parameters show up the same natural frequency, the probability density will be large at this frequency. Note that this study employs the Gaussian normal distribution with zero mean as the kernel function; different kernel functions can be used. A group of modes are initially determined through the peak-picking method to Eq. (13).

SVD to Grouped Mode Shapes

After grouping modes using Eq. (13), a mode shape in a specific mode is determined by applying SVD to a grouped mode shapes matrix. Ideally, if a matrix contains identified mode shapes to a specific mode, applying SVD to these mode shapes will show up the mode shape in the first singular vector, such as

$$\begin{bmatrix} \boldsymbol{\varphi}_{\mathrm{id}}^{1} & \boldsymbol{\varphi}_{\mathrm{id}}^{2} & \cdots & \boldsymbol{\varphi}_{\mathrm{id}}^{m_{\mathrm{g}}} \end{bmatrix} = \mathbf{u}_{\mathrm{ms}} \lambda_{\mathrm{ms}} \mathbf{v}_{\mathrm{ms}}^{T} \tag{15}$$

where $\mathbf{u}_{\mathrm{ms}} \in \mathbb{R}^{n\times 1}$ and m_{g} is the total number of mode shapes in a group. To determine whether the singular vector, $\mathbf{u}_{\mathrm{ms}}$, is the mode shape, the following criteria is used as

$$\begin{aligned} &\mathbf{u}_{\mathrm{ms}}^{T} \begin{bmatrix} \boldsymbol{\varphi}_{\mathrm{id}}^{1} & \boldsymbol{\varphi}_{\mathrm{id}}^{2} & \cdots & \boldsymbol{\varphi}_{\mathrm{id}}^{m_{\mathrm{g}}} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} \\ &\left\| \boldsymbol{\varphi}_{\mathrm{id}}^{i} \right\| = 1,\ i \in \left[1, m_{\mathrm{g}}\right] \end{aligned} \tag{16}$$

Repeating the same procedure, the most likely modes are determined by utilizing Eqs. (13-16) to all sets of modal parameters.

3. APPLICATION TO FIELD TEST MEASUREMENTS

This study applied the proposed system identification framework in a structural health monitoring problem of a cable-stayed bridge in Taiwan. This cable-stayed bridge, named as the Gi-Lu Bridge, was damaged during construction by the Chi-Chi earthquake in 1999. Then, this bridge was retrofitted and completed with the construction in 2004 [8, 22]. The structural engineers who designed and constructed this bridge were quite concerned about the current condition of this bridge. To ensure the users safety and bridge integrity, a series of structural health monitoring campaigns were carried out in 2006-2008.

The parameters used in the proposed system identification framework is first presented. The structural responses collected are velocities, and the total duration of measurements is 150 seconds. Ten sensor locations with respect to the bridge dimension are exhibited in Figure 2. These 10 sensors are denoted as “V01-V10” in the figure. The vertical velocity responses are used in the system identification. These measurements are recorded at 100 Hz.

Figure 3 illustrates the assured modes after performing the proposed system identification framework. The total number of successfully identified modes is 10.The results are quite comparable to the study in [8], though two of modes are deviated. In comparison of computational time, 341 runs performed using the proposed method is about 1-2 minutes, while the conventional approach including the stabilization diagram needs at least 10 minutes using the same computer. For system identification, the proposed framework is able to attain the computational efficiency and qualitative results.

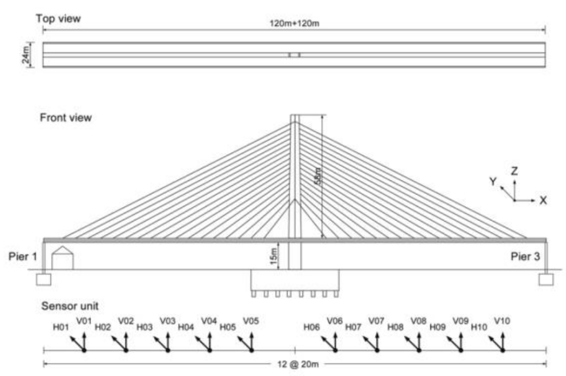

Figure 2. Sensor setup for bridge deck measurements.

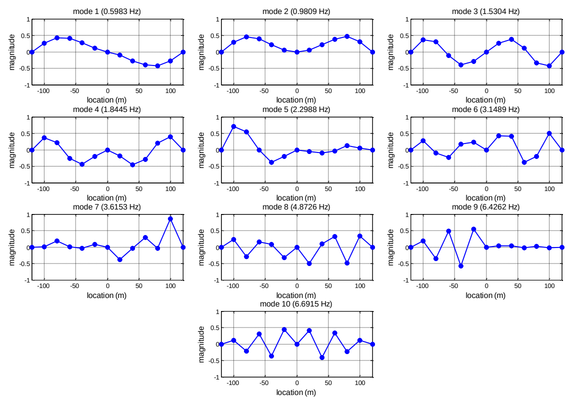

Figure 3. Identified natural frequencies from the field measurements of a cable-stayed bridge.

4. CONCLUSIONS

This study proposed an automated stochastic subspace system identification framework that can enhance the efficiency and quality in operational modal analysis. In this framework, all measured signals were preprocessed to reduce noise levels and uncertainties. Orders to the state-space realization were determined into a range by the singular value decomposition to a multi-channel PSD-time distribution. The efficient system identification was realized by introducing a reduced-dimension Hankel matrix combined with an efficient orthogonal projection. This combination rendered multiple subspaces that allowed more sets of modal parameters to be collected. Assured identified modes were consequently determined by the mode assurance process of which a kernel density estimation was employed along with the singular value decomposition to a grouped mode shapes. By integrating all these components, operational mode analysis was efficiently and qualitatively performed.

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