6th International Conference on Advances in Experimental Structural Engineering
11th International Workshop on Advanced Smart Materials and Smart Structures Technology
August 1-2, 2015, University of Illinois, Urbana-Champaign, United States

Automated Flexibility-based Damage Identification Strategy for Cable Structures

R. E. Martínez-Castro[1], S. Jang[2]

1 *Graduate Student, Dept. of Civil and Environmental Engineering, University of Connecticut, Storrs, United States. E-mail: rem13005@engr.uconn.edu*
2 *Assistant Professor, Dept. of Civil and Environmental Engineering, University of Connecticut, Storrs, United States. E-mail: sjang@engr.uconn.edu*

ABSTRACT
The structural integrity of cable-supported bridges relies on the condition of the main tension-resisting members. Monitoring and damage detection strategies are particularly challenging to implement in cables, especially in the critical anchor zone. The eigenparameter decomposition of modal flexibility damage identification method is examined for applications in damage detection near anchor zones in cable structures. The cable structure is modeled as a multi-degree of freedom system with acceleration sensors. The flexibility matrices of damaged and undamaged conditions are determined by performing a power spectral analysis. The effectiveness of this method to determine existence, location and extent of damage is evaluated. The effects of damage severity and critical damping ratio are considered. A set of additional formulas to automate the decision-making processes of the method is proposed. The effectiveness of the method to determine damage location is validated and compared to the results of the damage index method. It is found that the flexibility-based damage identification strategy is much more effective to locate damage than the damage index method. Damage loss estimation is also unaffected by the level of damping in the cable and it effectively resembles the exact damage loss.

KEYWORDS: *flexibility-based damage identification, cable stays, anchor zone, damage index, flexibility matrix.*

1. INTRODUCTION

The condition of transportation infrastructure in the United States is detrimental. Approximately 260 million trips are made over structurally deficient bridges [1]. Thus, the inspection, maintenance and repair of bridges is of paramount importance for the safety of the population. Significant effort in the development of more effective inspection and monitoring systems must be placed in order to catch up with the great number of damaged bridges in the country.

The structural integrity of bridges relies on the condition of critical load carrying elements. It follows then that the life expectancy of cable-supported bridges depends on the condition of the cables [2]. Damage in cables is primarily caused by corrosion and fatigue due to long-term service. It has been found that the exposure of cables to high levels of moisture can result in severe corrosion, reducing cable diameters up to 30% [3]. This translates into a loss in bending stiffness of up to 76%, and hence a significant reduction in loading capacity and vibration resistance.

Inspectors face a great challenge in evaluating the condition of main tension-resisting elements in cable-supported bridges. Visual inspections are still the most common method of assessing bridge structural integrity [4]. However, much can be missed during a visual inspection. The presence of steel protective pipes also restricts the number of nondestructive testing (NDT) methods that can be used to identify damage. Moreover, the most difficult region to evaluate is the critical anchorage zone since it is hidden from view and most NDT methods cannot be performed in this region [4]. This makes vibration-based methods more advantageous for these purposes.

This study investigates the effectives of a modified flexibility-based damage identification method to detect, locate and quantify damage in cable structures near the anchor zone. The theory of the method is explained and the effects of damage severity and cable damping are evaluated. Three steps in the method that represent a challenge in the automatic execution of the algorithm are also enhanced to reduce user interaction to a necessary minimum. The performance is also compared to that of a damage index method that has been used in the past to determine damage location.

2. THEORETICAL DEVELOPMENT

2.1 Modal flexibility matrix

The modal flexibility matrix can be calculated using [5]:

$$F = [\Phi]\begin{bmatrix} \ddots & & \\ & {}^{1}/_{\omega_j{}^2} & \\ & & \ddots \end{bmatrix}[\Phi]^T = [\Phi]\begin{bmatrix} \ddots & & \\ & {}^{1}/_{\lambda_j} & \\ & & \ddots \end{bmatrix}[\Phi]^T \tag{2.1}$$

where $[\Phi] = [\phi_1, \phi_2, \dots, \phi_n]$ is the mode shape matrix, ϕ_j is the *j*-th mode shape, ω_j are the natural frequencies per mode *j* in radians per second, and λ_j are the eigenvalues per mode *j*. After determining the mode shapes and natural frequencies of the structure from a power spectrum analysis, the mode shapes are mass-normalized. The differential flexibility matrix, ΔF can be determined by subtracting the undamaged modal flexibility matrix, F_u from the damaged modal flexibility matrix, F_d.

2.2 Eigenparameter decomposition of modal flexibility matrix method

The eigenparameter decomposition (ED) of modal flexibility matrix method is a modal flexibility-based damage identification method that was developed by Yang and Liu in 2008 [6]. An alternate way of determining the differential flexibility matrix proposed by the authors and the method's three damage identification components are shown below.

2.2.1 Approximated differential flexibility matrix

An approximated way of determining ΔF using the mode shapes and eigenvalues of the undamaged and damaged structure is proposed:

$$\Delta F \cong \sum_{j=1}^{NM} \frac{1}{\lambda_{dj}} \phi_{dj}\phi_{dj}^T - \sum_{j=1}^{NM} \frac{1}{\lambda_{uj}} \phi_{uj}\phi_{uj}^T \tag{2.2}$$

where *NM* is the number of modes.

2.2.2 Damage detection

The eigenvalue problem for the differential flexibility matrix is solved. The number of damaged elements in the system, q is given by the number of non-zero values in the diagonal of the eigenvalue matrix of the differential flexibility matrix, Λ.

2.2.3 Damage location

The eigenvalue problem can also be solved for each elemental stiffness matrix of the discretized structure. The stiffness connectivity matrix is calculated such that:

$$C = [c_1 \quad c_2 \quad \cdots \quad c_m \quad \cdots \quad c_{NE}] \tag{2.3}$$

$$c_m = \sqrt{\sigma_m} u_m \tag{2.4}$$

where *NE* is the total number of elements, c_m is the *m*-th elemental stiffness connectivity vector, and σ_m and u_m are the non-zero eigenvalue and eigenvector of the *m*-th elemental stiffness matrix, respectively.

Finally, the location matrix is generated by:

$$L = U^T F_u C \tag{2.5}$$

where *U* is the eigenvector matrix of ΔF. The location of the damaged elements is given by the columns of *L* with zero value entries.

2.2.4 Damage quantification

If the number of damaged elements is q, the columns in L corresponding to the damaged elements can be assembled into a matrix S as:

$$S = [l_1 \quad l_2 \quad \cdots \quad l_q] \tag{2.6}$$

It can be shown that there exists a relationship between the eigenvalue matrix of the differential flexibility matrix and S defined by:

$$\Lambda = S \Delta P S^T \tag{2.7}$$

The matrix ΔP is a diagonal matrix whose entries are the elemental stiffness parameters, α_m. Each elemental stiffness parameter represents the fraction of elemental stiffness loss due to damage, so that $0 \leq \alpha_m \leq 1.0$. The greater the value of the stiffness parameter, the more severe the damage.

The relationship in Equation 2.7 still remains valid when the rows containing zero entries in S, and the rows and columns containing zero diagonal entries in Λ are removed to form matrices S^* and Λ^*, respectively. The damage severity can then be determined by simply solving for ΔP:

$$\Delta P = (S^*)^{-1} \Lambda^* (S^{*T})^{-1} \tag{2.8}$$

2.3 Automation of decision-making components in the ED method

There are three instances during the execution of this method in which the correct identification of zero value entries is critical for proper damage quantification. Zeros must be identified: (1) in the location matrix (L) in order to locate damage and form matrix S according to Equation 2.6, (2) in the S matrix in order to form matrix S^*, which should be a square matrix, and (3) in the eigenvalue matrix Λ to form the square matrix Λ^*. The formation of S^* and Λ^* are essential to calculate the estimated elemental stiffness parameters according to Equation 2.8.

When attempting to run the algorithm automatically, the software can only identify a value as zero if it is exactly zero. In the great majority of cases, there will not be any exact zero entries in the diagonal of the eigenvalue matrix of the differential flexibility matrix or in the location matrix. This requires that a user initially interprets the number and location of damaged elements by inspecting the entries in the location and eigenvalue matrices. Once the damaged and undamaged elements are identified, quantification can be determined automatically. A set of formulas that turn the zero value entries into exact zeros in L, S and Λ is developed for programming purposes.

The columns of the location matrix that form matrix S are those that contain zeros. These columns correspond to the damaged elements. Therefore, the location matrix must be modified such that:

$$L = \frac{round(Lx)}{x} \tag{2.9a}$$

where $x_1 < x < x_2$ and

$$x_1 = \frac{0.5}{min(min|L(:,undamaged\ elements)|)} \tag{2.9b}$$

$$x_2 = \frac{0.5}{max(min|L(:,damaged\ elements)|)} \tag{2.9c}$$

Once matrix S is formed, the zero rows are removed to form S^*. In order to identify the zero rows, the matrix S must be modified such that:

$$S = \frac{round(Sy)}{y} \tag{2.10a}$$

where $y_1 < y < y_2$ and

$$y_1 = \frac{0.5}{min(min|S(damaged\ elements,:)|)} \tag{2.10b}$$

$$y_2 = \frac{0.5}{max(max|S(undamaged\ elements,:)|)} \tag{2.10c}$$

Finally, the rows and columns containing zero diagonal entries in Λ must be removed to form Λ^*. The eigenvalue matrix Λ must be modified as:

$$\Lambda = \frac{round(\Lambda z)}{z} \tag{2.11a}$$

where $z_1 < z < z_2$ and

$$z_1 = \frac{0.5}{min|diag(\Lambda(damaged\ elements))|} \tag{2.11b}$$

$$z_2 = \frac{0.5}{max|diag(\Lambda(undamaged\ elements))|} \tag{2.11c}$$

It must be noted that in a 2DOF system the lower bounds and upper bounds for y and z are interchanged, i. e. the lower bound for z is z_2 while the upper bound is z_1, and the lower bound for y is y_2 while the upper bound is y_1.

2.4 Damage index method

In the damage index (DI) method, damaged elements can be identified based on the relatively larger values of the damage indices [7, 8]. A damage index, β_j relates the deformation of the *j*-th element in the *i*-th mode (Δ_{ij}) with the deformation of the corresponding element and mode in the damaged structure (Δ_{ij}^*). For *NE* number of elements, f_{ij} is defined by:

$$f_{ij} = \frac{(\Delta_{ij})^2}{\sum_{j=1}^{NE}(\Delta_{ij})^2} \tag{2.12}$$

and f_{ij}* is the complex conjugate of f_{ij} such that:

$$f_{ij}^* = \frac{(\Delta_{ij}^*)^2}{\sum_{j=1}^{NE}(\Delta_{ij}^*)^2} \tag{2.13}$$

The damage index is determined by:

$$\beta_j = \frac{\frac{f_{ij}^*+1}{f_{ij}+1}+1}{2} \tag{2.14}$$

The damage index calculated by Equation 2.14 can only be represented for one mode at a time. To take multiple modes into account, the following relationship must be used:

$$\beta_j = \frac{\frac{(\sum_{i=1}^{NM} f_{ij}^*)+1}{(\sum_{i=1}^{NM} f_{ij})+1}+1}{2} \tag{2.15}$$

The normalized damage index, Z_j for each element *j* is determined assuming the standard form of the damage index:

$$Z_j = \frac{\beta_j - \mu_\beta}{\sigma_\beta} \tag{2.16}$$

where μ_β and σ_β are the mean and standard deviation of the damage indices per mode *i*. To classify an element as damaged or undamaged, a threshold λ is established so that if $Z_j \geq \lambda$ the element is damaged and if $Z_j < \lambda$ the element is undamaged.

3. NUMERICAL VALIDATION

3.1 Description of numerical simulation

A cable structure is modeled as a two-degree-of-freedom (2DOF) system simulating a real cable structure with two acceleration sensors uniformly distributed along the cable length, as shown in Figure 3.1. Damage is simulated as a loss in bending stiffness on the second element only (shown in red) to emulate damage near the anchorage. Stiffness loss is varied in increments of 10%. The structure is modeled in Simulink/MATLAB using state-space representation. Band-limited white noise (BLWN) is input into the second DOF and the acceleration responses of both DOFs are used to perform a power spectral analysis. The signal duration is 1,000 seconds, the sampling frequency is 128 Hz and the number of points used for the Fast Fourier Transform is 16,384. The mode shapes and frequencies of the damaged and undamaged structure are determined from the frequency response functions of each DOF. These in turn are used to determine the modal flexibility matrices, F_u and F_d.

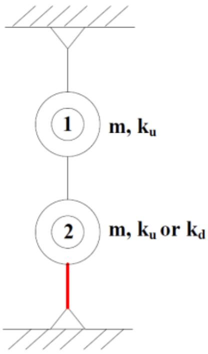

Figure 3.1 Two-degree-of-freedom cable model analyzed

The ED and DI methods for damage identification are then implemented and their performance evaluated. The effect of using the exact differential flexibility matrix versus the approximate in the ED method is also examined. The effect of damping in each method is investigated by comparing results of the cable with critical damping ratios of 3%, 2%, and 1%, and no damping.

It is desired to apply damage identification strategies with as little user interference as possible. Hence, a 3DOF model of the cable structure is also analyzed in order develop general equations that contribute to the automation of the quantification part of the ED method algorithm for multiple DOF systems, as explained in Section 2.3.

3.2 Eigenparameter decomposition of modal flexibility matrix method

This section illustrates the many variables that have a role in the performance of the ED method. The 2DOF system is evaluated for stiffness losses varying from 0-50% in increments of 10%. Damage detection and location are correctly established in all cases, independently of damage severity, damping ratio or the method of determining the differential flexibility matrix.

The stiffness loss estimated by the ED method using the exact differential flexibility matrix (as explained in Section 2.1) compared to the performance using the approximate differential flexibility matrix (Equation 2.2) with 3% damping is shown in Figure 3.2. It can be observed that the damage extent is consistently overestimated and is more conservative as the damage is more severe. When the approximate differential flexibility matrix is used to solve the initial eigenvalue problem, the overestimation is slightly larger than when the exact differential flexibility matrix is used.

However, it is found that the relation between the exact and estimated stiffness loss is very predictable, so a mathematical relation between them can be established. This relation has been found by means of an exponential regression with a relative predictive power of $R^2 = 0.9781$ as:

$$estimated\ loss = 7.57814e^{0.0538\ exact\ loss} \quad (3.1)$$

Therefore, the initial estimated stiffness loss can be converted to a final estimation that is very close to the exact loss, as shown in Figure 3.3. Equation 3.1 provides a reliable means of reducing the degree of overestimation intrinsic in this method.

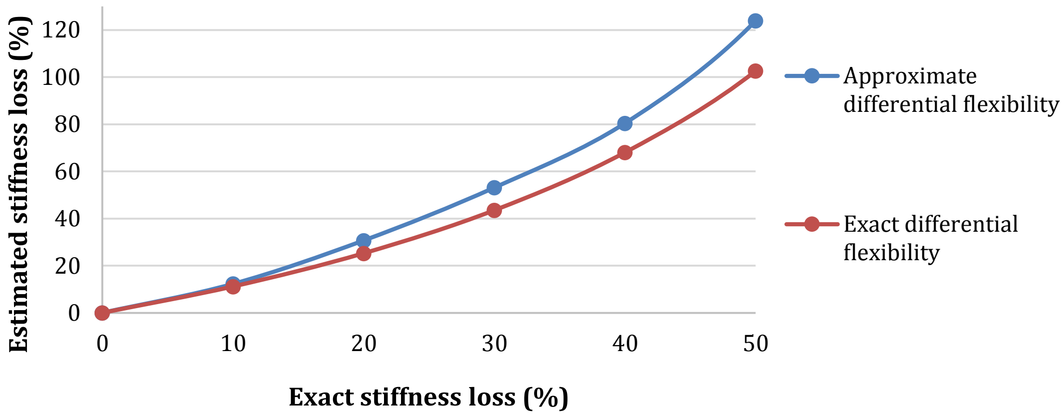

Figure 3.2 Estimated vs. exact cable bending stiffness loss using the approximate or exact differential flexibility matrix

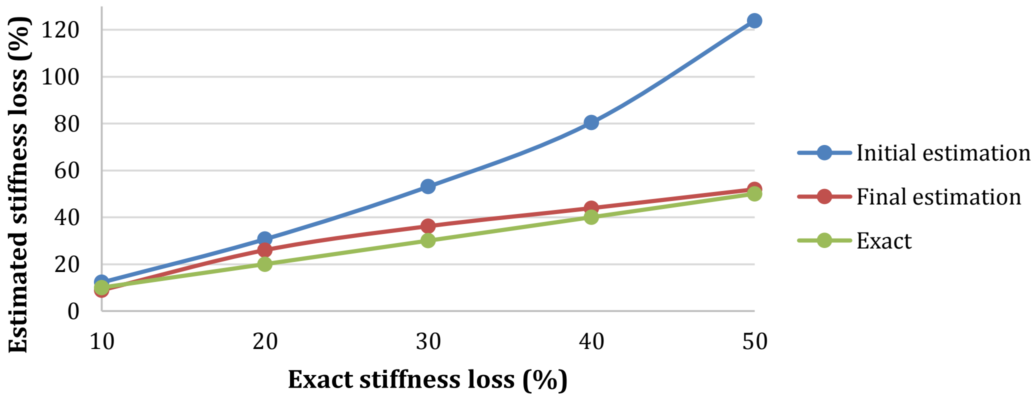

Figure 3.3 Initial and final estimations of the ED method

The final damage severity estimations for the undamped system and the system with critical damping ratios of 3%, 2%, and 1% are shown in Table 3.1. The estimation in the undamped structure is slightly smaller than that in any of the three damped cable cases. However, it can be seen by comparing the three cases where damping is included that the amount of damping has no impact on the estimation in stiffness loss.

Table 3.1 Final stiffness loss estimation of ED method for different damping ratios

Loss (%)	Final loss estimation by ED method (%)				Error (%)			
	$\xi = 0.03$	$\xi = 0.02$	$\xi = 0.01$	undamped	$\xi = 0.03$	$\xi = 0.02$	$\xi = 0.01$	undamped
0	-	-	-	-	-	-	-	-
10	8.93	8.90	8.82	8.56	10.7327	11.0364	11.7979	14.4104
20	25.99	26.00	25.99	25.85	29.9575	29.9878	29.9575	29.2582
30	36.19	36.19	36.19	36.16	20.6263	20.6380	20.6497	20.5329
40	43.90	43.90	43.89	43.88	9.7524	9.7581	9.7350	9.6945
50	51.93	51.95	51.95	51.89	3.8679	3.8919	3.9099	3.7837

3.3 Damage index method

The damage indices for the case with 10% stiffness loss and 3% critical damping are shown in Figure 3.4. The values of the normalized damage indices considering the first mode, second mode or all modes are all ±0.7071 for each element. This is also true for all damage severity cases with 2% and 1% critical damping, and no damping. The normalized damage indices in the DI method have shown to be ineffective in locating damage in the cable structure with 2 DOFs. It is expected, however, that as more DOFs are considered, as in more complex structures, normalized damage indices are more trustworthy in revealing damage location since the distribution of a random variable reveals the expected value when the population is larger. This application would be impractical for a cable structure since it would require a large number of acceleration sensors to be placed.

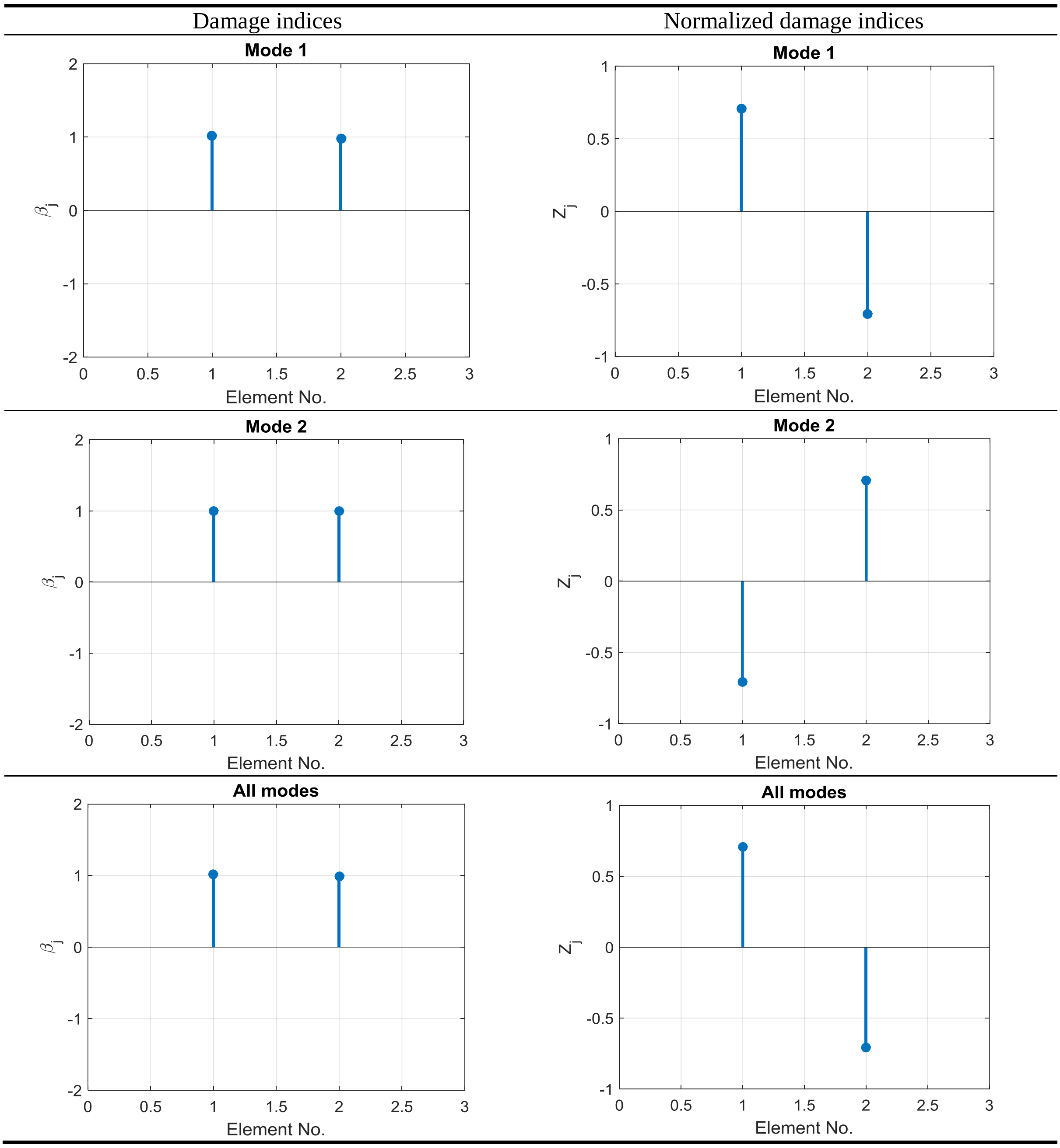

Figure 3.4 Damage indices for a 3% damped system with 10% stiffness loss in element 2

The unnormalized indices can indicate damage more appropriately if the first mode or all modes of vibration are considered simultaneously in the analysis. When only the first mode is considered, the damage indices for the 3% damping case with 10% stiffness loss are 1.0149 for the first element and 0.9801 for the second element. When all modes are considered, these are 1.0125 and 0.9877, respectively. These results are the most consistent with damage location, although the percentage difference between elements when all modes are considered is merely 2.52%. When only the first mode is considered, the percentage difference between elements is 3.55%. When only the second mode is considered in the analysis, the damage indices are 0.9994 for the first element and 1.0004 for the second element, which is not consistent with the location of damage. The difference in damage indices between elements is also very slight, making it impossible to determine an appropriate damage threshold, λ.

4. CONCLUSION

The automated ED method has shown to be effective in rapidly determining the existence, location and severity of damage near simulated cable anchor zones, with no false indications. Damage location is always correctly determined with the ED method, while the DI method does not clearly reveal damage location. Although the initial damage severity estimation with the ED method is very conservative, especially when damage is more severe, an exponential expression can be used to determine a final estimation that resembles the real damage severity quite effectively. The steps proposed for the automation of the algorithm make damage quantification much faster than the unaltered method. These findings and developments represent a step towards the implementation of the ED method for damage identification in real cable structures.

ACKNOWLEDGEMENT

This material is based upon work supported by the U.S. Department of Homeland Security under the UConn HS-STEM Program in Transportation Security for Cyber-Physical Systems, directed by Dr. Nicholas Lownes. The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the U.S. Department of Homeland Security.

REFERENCES

1. American Society of Civil Engineers – ASCE (2013). Bridges. 2013 Report Card for America's Infrastructure.
2. Wickramasinghe, W. R., Thambiratnam, D. P., and Chan, T. H. T. (2013). Damage Detection in Cable Structures using Vibration Characteristics. *4th International Conference on Structural Engineering and Construction Management*. Kandy, Sri Lanka.
3. Sloane, M. J. D., Betti, R., Marconi, G., Hong, A. L., and Khazem, D. (2012). An Experimental Analysis of a Nondestructive Corrosion Monitoring System for Main Cables of Suspension Bridges. *Journal of Bridge Engineering*. **18:4**, 653-662.
4. National Cooperative Highway Research Program – NCHRP (2005). Synthesis 353: Inspection and Maintenance of Bridge Stay Cable Systems, Transportation Research Board.
5. Pandey, A. K. and Biswas, M. (1994). Damage Detection in Structures using Changes in Flexibility. *Journal of Sound and Vibration*, **169: 1**, 3-17.
6. Yang, Q. W. and Liu, J. K. (2008). Damage Identification by the Eigenparameter Decomposition of Structural Flexibility Change. *International Journal for Numerical Methods in Engineering*, **78:4**, 444-459.
7. Park, S., Kim, Y., and Stubbs, N. (2002). Nondestructive Damage Detection in Large Structures via Vibration Monitoring. *Electronic Journal of Structural Engineering*, **2**, 59-75.
8. Park, S., Bolton, R., and Stubbs, N. (2006). Blind Test Results for Nondestructive Damage Detection in a Steel Frame. *Journal of Structural Engineering*, **132: 5**, 800-809.