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Magnetic Negative Stiffness Damper and Its Application to Stay Cables

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ABSTRACT

This paper introduces a novel design of magnetic negative stiffness dampers (MNSDs) and its application to vibration suppression for stay cables. The MNSDs are composed of a number of permanent magnets properly arranged in a conductive pipe, and such configurations efficiently integrate fully passive negative stiffness mechanism and eddy-current damping in a simple and compact design. When applied to vibration suppression of stay cables, MNSDs can achieve control performance comparable to semi-active or active control. The installation of MNSD can provide considerably high damping to cable structures. Negative stiffness springs may amplify local vibration amplitude at the damper location, through which the locking effect can be overcome and significant kinetic energy can be dissipated by viscous damping. Optimal damping performance corresponding to a certain value of negative stiffness can be obtained if the damper's damping coefficient is tuned properly.

KEYWORDS: *Magnetic stiffness damper, Vibration mitigation, Stay cable, Optimal damping ratio*

1. GENERAL INSTRUCTIONS

In order to protect primary structural systems from excessive vibrations, various dampers in passive, semi-active, or active modes have been invented. Stay cables, as a type of efficient load-carrying element in long bridges, are widely used around the world. Stay cables are generally flexible and have low damping level, so they are susceptible to excessive wind- or traffic-induced vibration (Yamaguchi and Fujino 1998). Excessive vibrations can result in undue stresses or fatigue failure in cables or connections and is a big threat to the serviceability and safety of bridges. Avoiding unexpectedly large oscillations in stay cables is of great interest in the field of civil engineering. Under normal circumstances, the damper is installed at a position close to one end of the cable, and acting in the transverse direction (Geurts et al. 1998).

Many researches have been carried out on vibration suppression of stay cables by using dampers. Vibration mitigation of cables with passive dampers has been systematically investigated. Carne (1981) and Kovacs (1982) studied the vibrations of a taut cable with an attached damper in the first place. Based on Kovacs's (1982) work, Pacheco et al. (1993) proposed a damping estimation curve for cable with viscous damper through detailed numerical calculations. The performance of both linear and nonlinear viscous dampers on stay-cable vibration mitigation was also analyzed through complex mode shapes (Main and Jones, 2002a, 2002b). Krenk and Høgsberg (2005) proposed an analytical solution to predict the vibration control performance of a taut cable with a transverse force. Fujino and Hoang (2008) derived the design formulas in consideration of cable sag and flexural rigidity of cable. The nonlinear vibration of cable-damper system was also studied by Yu & Xu (1999) and Xu & Yu (1999).

Active and semi-active control techniques usually provide better performance than passive dampers. Semi-active technology can provide much higher supplemental damping for stay cables, which has been proved both theoretically (Johnson et. al., 2003) and experimentally (Christenson et. al., 2006). Ni et al. (2002) and Wu and Cai (2006), among others, studied the semi-active control of stay cables using MR dampers and proved their control effectiveness by a set of tests. MR damper has also been used on real cables. Chen et al. (2003) installed MR dampers on cable-stayed bridge to suppress the rain-wind-induced cable vibration for the first time in the world. Li et. al. (2005) also designed and installed MR damper on the stay cables of Shandong Binzhou Yellow River Bridge.

The linear quadratic regulator (LQR) algorithm, a commonly adopted optimal control theory for active dampers, may produce a force-deformation relationship with significant negative-stiffness feature in some situations that

could benefit vibration control effect (Iemura and Pradono 2002). However, the feedback system and power supply system increases the design and application difficulties of semi-active NSD. A truly passive NSD is more attractive and practical. Several methods can realize passive negative stiffness. Passive negative stiffness springs can be made by the snap-through behavior of a pre-buckled beam (Dijkstra 1988; Lee et al. 2007). It can also be realized by a set of properly designed pre-compressed springs (Pasala et al. 2013). A friction pendulum sliding isolator with convex friction interface is another option (Iemura and Pradono, 2009). However, the existing designs of realizing passive negative stiffness have some limitations which may hinder the application on stay cable. For example, a pre-compressed spring exhibits negative-stiffness feature only when displaced beyond a certain limit. A pre-buckled beam exhibits asymmetrical negative stiffness about its initial position. A sliding isolator with convex friction interface is only effective in the horizontal direction with the presence of gravity.

Due to the above limitations, passive NSD has not been installed on stay cables, but semi-active NSDs can produce twice as much damping as common viscous dampers (Webber and Boston, 2011). This paper introduces a novel design of magnetic negative stiffness dampers (MNSDs) that is a truly passive NSD, and investigates its vibration control performance on stay cables. In general, MNSDs can achieve comparable performance with semi-active or active control. The installation of MNSD can provide considerable damping to stay cables. Negative stiffness springs may amplify local vibration amplitude at damper location, through which the locking effect can be overcome and significant kinetic energy can be dissipated by viscous damping or other types of damping. Optimal control performance corresponding to a certain value of negative stiffness can be obtained if the damper's damping coefficient is tuned properly.

2. MAGNETIC NEGATIVE STIFFNESS DAMPER

The new passive MNSD combines fully passive negative stiffness mechanism and eddy-current damping in a simple and compact design. As shown in Figure 1, the design configuration of MNSD mainly contains two static and one moving magnets, sliding bearings, fixing spacers, a conductive pipe, and a shaft. The moving magnet is fixed on a shaft by the fixing spacers, and the static magnets are fixed on conductive pipe. The three permanent magnets are aligned in the longitudinal direction and the movement is guided by the sliding bearing at two ends of the MNSD. The fixing spacers also protect the moving magnet from collisions with static magnets or boundaries.
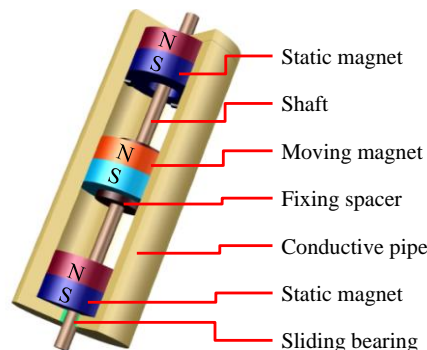

Figure 1 Configurations of MNSD

Figure 2 presents the design principle of MNSD. As shown in Figure 2(a), the negative stiffness is realized by magnetic force between moving magnet and static magnets. All the magnets in MNSD have same pole orientations and are arranged along the longitudinal axis of the damper. When the moving magnet is at the zero position that is equidistant to the two static magnets, the net magnetic force acting on the moving magnet equals to zero. The magnetic force between two magnets nonlinearly increases with decreasing separation distance. When the moving magnet moves upwards or downwards from zero displacement position, a force F in the opposite direction of the displacement X is required to keep the moving magnet in equilibrium, which is negative-stiffness force-displacement relationship (Figure 2(a)).

If a magnet is put inside a non-ferromagnetic conductive pipe, the movement of the magnet can produce eddy current within the pipe due to the variation in the magnetic flux. As a result, the generated eddy current will exert an electromagnetic force on the moving magnet (Figure 2(b)). Because the eddy-current damping force has a linear relationship with moving speed of magnets, it is equivalent to a passive viscous damping force.

As presented by Figure 2(c), the passive negative-stiffness produced by the magnetic interaction force and the

passive damping generated by eddy current can be integrated into a single MNSD. Compared with the existing design of negative-stiffness devices or systems, the proposed design presents more efficiency and compatibility. The MNSD has symmetrical negative stiffness about the equilibrium position, and is effective in all directions. This design can be applied to the vibration control of various mechanical and civil structures. The damper force of MNSD is controlled by magnet strength and magnet dimension. However, there exists a ceiling for single MNSD because both magnet strength and magnet dimension have an upper limit in manufacture. In practical applications, a larger damper force can also be obtained by repeating the unit shown in Figure 1 in the longitudinal direction. Adjacent units shall have enough separation space or be isolated magnetically in order to avoid interactions. The eddy-current damping in the proposed configurations can also be replaced by conventional viscous fluid damping if the latter is more suitable.

(a) Negative stiffness   (b) Eddy-current Damping   (c) Negative stiffness damper
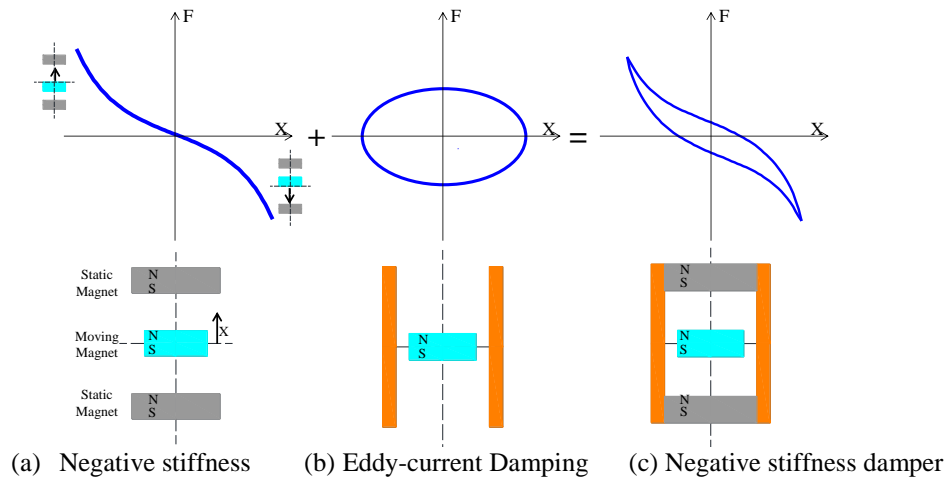
Figure 2 Design principle of MNSD

A MNSD was fabricated and tested in the laboratory. The magnets of MNSD are neodymium (NdFeB) permanent magnets with grades of N48 and the outer radius, inner radius and thickness of the magnets are 20mm, 5mm and 20mm, respectively. The conductive pipe is made of brass, and the material of shaft is stainless steel. The static and moving magnets are fixed on the conductive pipe and moving shaft, respectively. Sliding bearings were installed between the shaft and two end plates of the pipe to guide moving direction and minimize the friction. Figure 3(a) presents the MNSD on an MTS universal testing machine. Different loading frequencies, including a quasi-static loading rate, were applied to the MNSD. The damper force was measured by a load cell which connected the MNSD to the MTS machine, while the displacement was measured by the MTS machine.

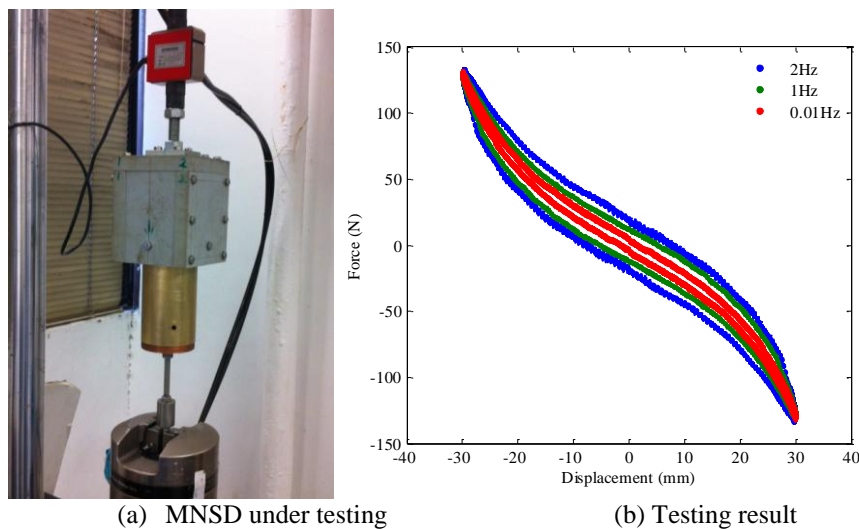(a) MNSD under testing   (b) Testing result

Figure 3 MNSD under cyclic testing

The experimental result is shown in Figure 3(b). The opposite directions of the force and displacement prove the concept of magnetic negative-stiffness mechanism. A symmetric but nonlinear negative stiffness can be observed in MNSD (Figure 3(b)). The moving magnet was excited with harmonic displacement. Given the same displacement amplitude, the eddy-current damping force of the MNSD increases with the vibration frequency.

3. VIBRATION SUPPRESSION OF STAY CABLE

Figure 4 shows a taut cable with length L, tension T, and mass per unit length m. A passive damper is installed in the transverse direction at a close distance a from the left end of the cable. The distance from the right end is equals to $a'=L-a$. Practically, a is often limited within the range of 2% to 5% of L, when the damper connects the cable and deck in a cable-stay bridge. The transverse free vibration of the cable with NSD is described by Eq. 3.1. (Krenk and Høgsberg ,2005).
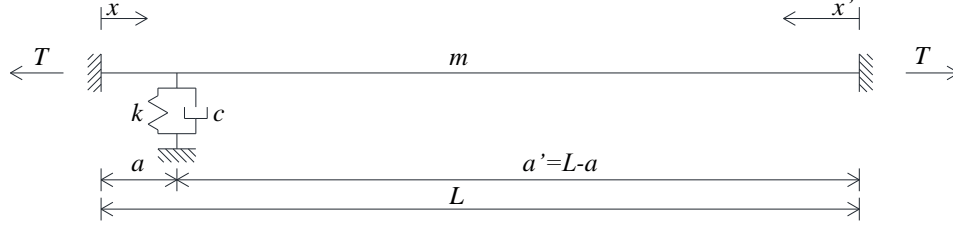

Figure 4 Taut cable with negative stiffness damper at x=a

$$T\frac{\partial^2 u}{\partial x^2} - m\frac{\partial^2 u}{\partial t^2} = f(t)\delta(x-a) \tag{3.1}$$

The variable $u(x,t)$ can be separated into

$$u(x,t) = w(x) \cdot q(t) \tag{3.2}$$

where $w(x)$ and $q(t)$ are the time-independent and time-dependent components of $u(x,t)$, respectively. $q(t)$ can also be expressed as:

$$q(t) = e^{pt} \tag{3.3}$$

where p is a complex number in the following form

$$p = -\delta\omega \pm i\omega\sqrt{1-\delta^2} \tag{3.4}$$

where δ is damping ratio and ω is the undamped natural frequency.

Eq. 3.1. can be discretized into 100 uniformly spaced segments with 99 internal nodes. The mass and stiffness matrices of the cable (Eq. 3.5.) are generated through finite difference method formulated by Mehrabi and Tabatabai (1998). Then Eq. 3.1 can be transformed into an eigenvalue problem. A full-scale stay cable with length of 122m is simulated. The prototype cable is one of the actual stay cables in a long-span cable-stayed bridge in China. The main parameters of the cable are: m=51.8kg/m, L=122m and T=3150kN.

$$\mathbf{M}p^2w + \mathbf{C}pw + \mathbf{K}w = 0 \tag{3.5}$$

3.1. Extreme damping

Once the complex number p is evaluated, the damping ratio of the cable with NSD can be evaluated according to Eq. 3.4. Figure 5(a) and Figure 5(b), respectively, show the variation of the damping ratio corresponding to the constant-k and constant-c cases. The peak of each curve in Figure 5(a) suggests the optimal damping ratio of the cable and the corresponding damping coefficient when k is kept constant. When the damper is installed at a 5% location, a linear viscous fluid damper can only achieve an optimal damping ratio around 2.5%, whereas an NSD can achieve a significantly higher optimal damper ratio. When $k=-0.9T/a$, the maximum achievable damping ratio of the cable is greater than 20%. Figure 5(b) shows the accurate variation of damping ratio with respect to stiffness coefficient when damping coefficient is kept constant. It should be noted that the constant-c

curves do not correspond to the optimal damping situations. For the first mode, the damping ratio of the cable increases with the level of negative stiffness of NSD. Even though the k and c coefficients in Figure 5(b) do not correspond to the optimal condition, an NSD can still achieve a damping performance superior to the optimal damping ratio achieved by conventional viscous fluid dampers. The results imply that adding negative stiffness spring to existing cables with conventional viscous fluid dampers can always improve the vibration mitigation performance, although the additional negative stiffness and the existing damping coefficient do not present an optimal combination.
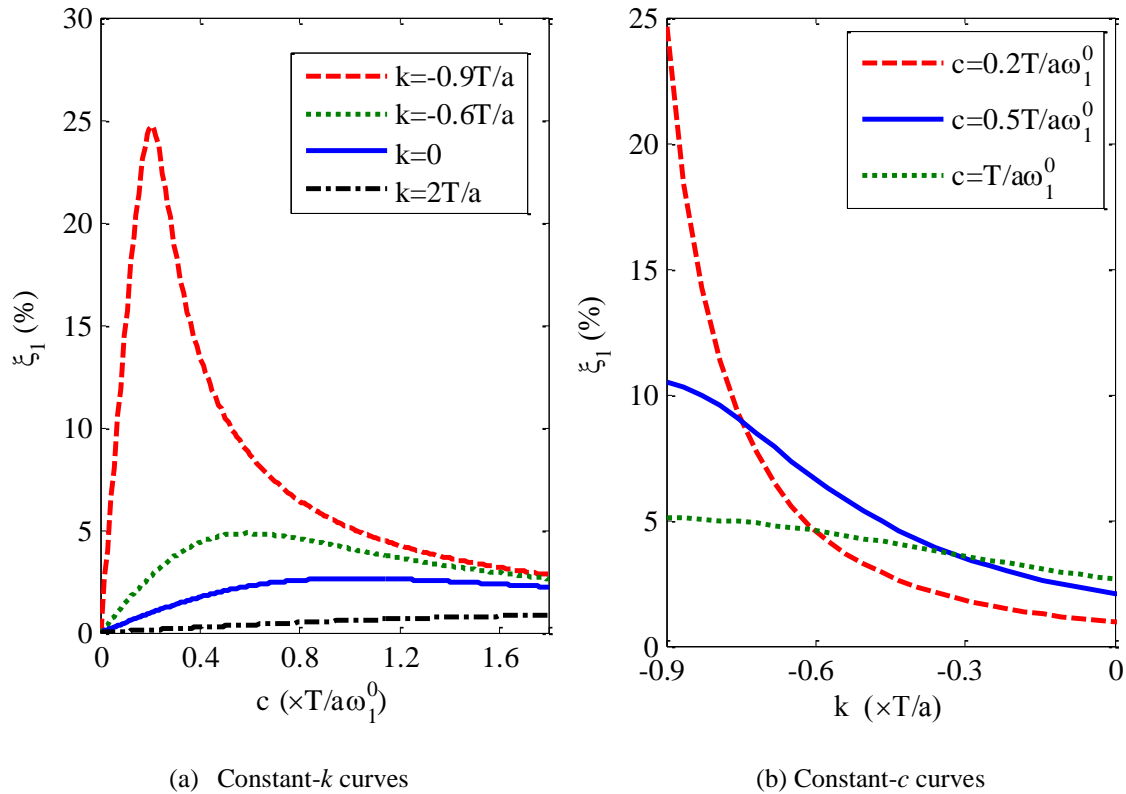

(a) Constant-k curves   (b) Constant-c curves

Figure 5 Damping ratio of cable with NSD

3.2. Overcome locking effect

The mode shapes of cable can be obtained from the eigenvector. Figure 6 shows the first model shapes of the cable with a damper installed at a 5% position. As shown in Figure 6, the presence of negative/positive stiffness springs causes the discontinuity in the slopes of the mode shapes at the damper location. Compared with the mode shape without control, the negative and positive stiffness springs tend to amplify and reduce the amplitudes of the mode shape at the damper location, respectively. If we focus on the right part of the cable, the negative and positive stiffness springs essentially prolong and shorten the effective cable lengths. As pointed out by Krenk and Høgsberg (2005), a longer distance a leads to a greater optimal damping ratio. Similar to positive stiffness damper, the addition of a viscous damper tends to reduce the amplitude at the damper location and essentially shorten the effective length of the cable. The reduced vibration amplitude reduces energy dissipation by the damper and consequently constrains the damping performance of any passive damper installed on a cable. When a significant damping coefficient c is added to the cable, it produces a "locking" effect that minimizes the damper's displacement and neutralizes the damping effect of the damper. When the negative stiffness spring and dashpot are simultaneously installed, the negative stiffness can successfully prevent the locking effect at the damper location, and the magnified damper vibration effectively enhances the energy dissipation via the dashpot. This implies that an NSD can achieve an improved global damping performance in bridge stay cables in comparison with traditional viscous fluid dampers, but it may also amplify the local vibration slightly at the damper location.
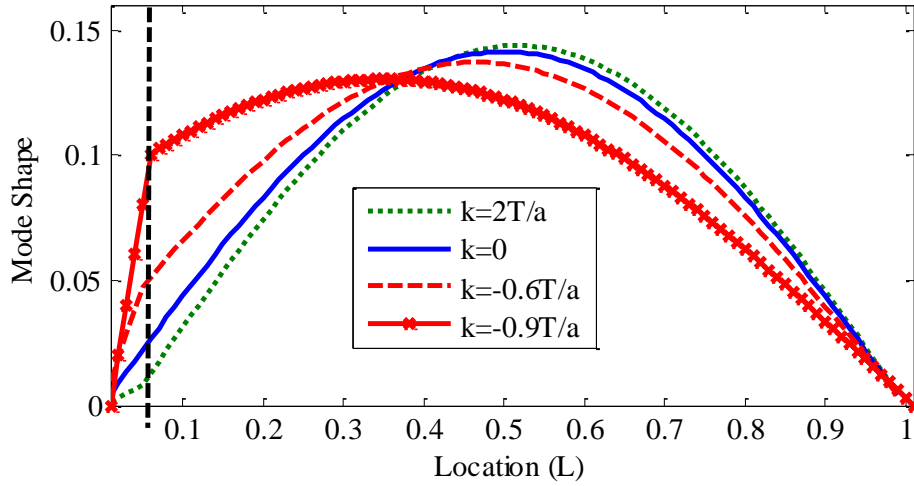
Figure 6 First mode shape of cable with negative stiffness damper

3.3. Optimal Design

As mentioned in section 3.1, for a specific value of negative stiffness k, there exists an optimal damping coefficient c to offer the highest damping ratio to the stay cable. According to Figure 7(a), the required damping coefficients decrease with the increasing negative stiffness. This finding implies that a higher level of negative stiffness is associated with a smaller optimal viscous coefficient. Figure 7(b) shows the optimal combination of the stiffness and damping coefficients and the associated optimal damping ratio of the cable. A conventional viscous fluid damper can achieve the maximum cable's damping ratio of 2.5%. when k=-0.3T/a, -0.6T/a and -0.9T/a, the optimal damping ratios of cable with NSDs are 3.7%, 6.3%, 24.7%, respectively, which apparently showcases the superior vibration control performance of NSDs.

(a) Optimal damping coefficient
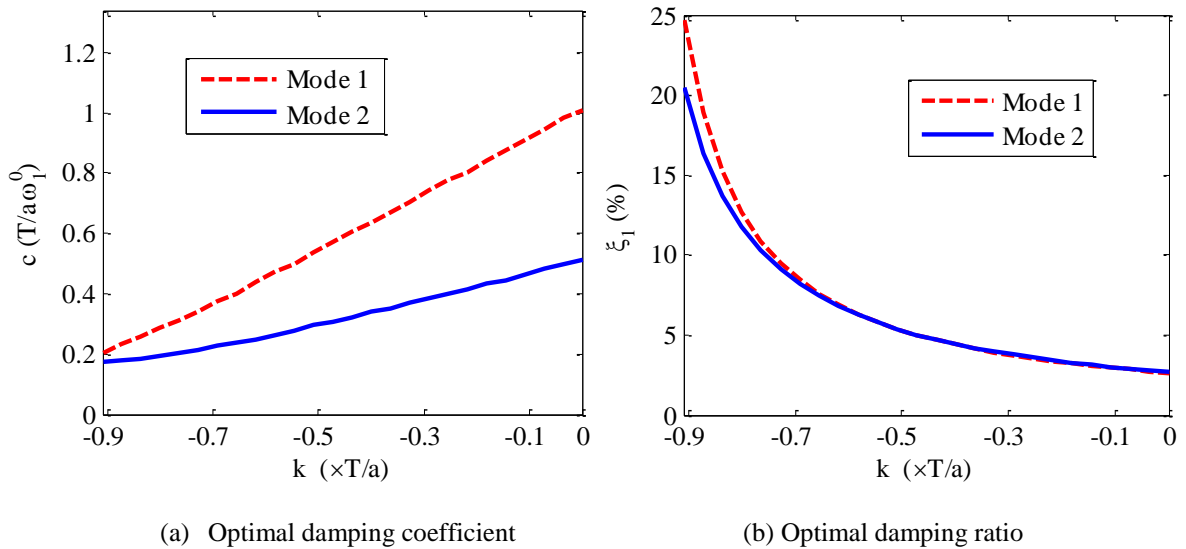
(b) Optimal damping ratio

Figure 7 Optimal tuning of NSD

4. CONCLUSION

This paper introduces MNSD and its vibration control performance when installed on a stay cable and close to a cable end. The damping performance is evaluated with varying negative stiffness and damping coefficients, and the optimal damping ratio is subsequently obtained. The superior vibration mitigation performance of NSDs installed on stay cables is illustrated through a series of comparisons with conventional viscous dampers. The main conclusions of this study are summarized as follows:

(1) An NSD can provide superior vibration mitigation performance to conventional passive dampers. The maximum achievable damping ratio increases with negative stiffness level, and a high damping ratio of cables can be achieved when a significant negative stiffness is applied. By contrast, a damper with positive stiffness is detrimental to cable vibration mitigation when installed close to a cable anchorage.

(2) An NSD can overcome the locking effect associated with conventional passive dampers when installed close to cable ends. Consequently, an NSD significantly enhances its energy dissipation by amplifying damper's displacement.

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