

Floor Slab Isolation for Mitigating the Seismic Response of Building

Hussam Mahmoud¹, Akshat Chulahwat²

- 1 Assistant Professor, Dept. of Civil and Environmental Engineering, Colorado State University, Fort Collins, United States E-mail: hussam.mahmoud@colostate.edu
- 2 Ph.D. Candidate, Dept. of Civil and Environmental Engineering, Colorado State University, Fort Collins, United States. E-mail: akshat.chulahwat @ colostate.edu

ABSTRACT

This study extends previous research on isolated floor systems by introducing the analytical solution of buildings with slabs configured to act as pendulum and translational tuned mass dampers. Two structural isolation systems are introduced in this study. The first system comprises of a steel frame with selected floor slabs suspended from the beams above them using steel cables. Controlled energy dissipation and post-tensioning links are used to control the response of the slabs as desired. The links are installed between the bottom face of the suspended slabs and the floor beams underneath the slab. The second system is similar to first except the floor slabs are resting on curved supports to allow for self-centering of the slabs upon the conclusion of the seismic event. In addition, rubber bumpers are installed between the slab and the frame and their stiffness is relied upon to reduce the potential for impact between the slabs and the skeleton frame. The friction of the contact surface between the slab and the supports on which they rest can be used to dissipate energy, thereby providing some form of damping to the system. The proposed systems show superior performance over a conventional composite structure under dynamic excitations.

KEYWORDS: Suspended Slabs, Tuned Mass Damper, Dynamic Excitation, Isolated Floor Slabs

1. LITERATURE REVIEW

Structural systems capable of resisting large hazard demands while demonstrating superior performance by employing an easily replaceable energy dissipating elements are the way to future design against extreme earthquake hazards. Recently, a new type of isolation system has started to gain popularity referred to as 'Floor isolation systems'. The performance objectives of the mentioned system are to limit the global frame residual drift, reduce floor acceleration, and minimize repair cost and downtime associated with restoring the structural and nonstructural components of the system. Although floor isolation systems have been implemented in Japan for over 15 years [1,2] they have just recently started to gain popularity all over the world. Numerous researchers have developed innovative floor isolation systems and tested the efficiency of such systems. For example, in the US, the King County Emergency Center in Seattle has been equipped with a floor isolation system to protect its communications equipment. The floor system, developed by DIS (Dynamic Isolation systems), comprised of post-tensioned concrete floor isolated with a bidirectional spring unit that utilizes combination of coil springs and viscous fluid dampers. The floor isolation system has been studied under seismic effects and was found to be effective at reducing floor accelerations [3]. Lambrou and Constantinou demonstrated through experimental and analytical simulations that substantial reductions in the response of a computer cabinet could be achieved by isolating the floor with friction pendulum bearings with and without added viscous fluid dampers [4]. Takase N. et al. [1] and Kaneki M. et al. [2] developed and analyzed an interesting three dimensional floor isolated system that showed superior performance under large excitations. Liu and Warn [5] investigated the performance and sensitivity of floor isolation system numerically for upper levels of multi-story steel plate shear wall frames. The results of the study showed that the isolation system effectively limited the absolute acceleration demand at the cost of the displacement demand. Kasalanti et. al. developed a type of ball-in-cone floor isolation system which was shown to be very effective in mitigating seismic excitations [6]. In addition to floor isolation systems, another type of new isolation system – suspension systems, have also been investigated and implemented at numerous places. For example, the Westcoast Transmission Building in Vancouver (Canada), constructed in 1968-69 is suspended from its central concrete core using steel cables to isolate the main structure from seismic effects. Another interesting and innovative suspension system has been developed by Nakamura et al.- [7], which utilizes the concept of pendulum to suspend and isolate the entire structure from a reinforced concrete core such that the structure is independent of the vibrations of the core. Although suspension systems and floor isolation systems have been investigated independently, there are quite few studies on suspended floor isolation systems. Tatemichi et al. [8] investigated the behavior of floor slabs, suspended using hangar rods, in high-rise structures both analytically and experimentally for the purpose of seismic isolation and highlighted the effectiveness of the systems in high-rise structures. The concept has also been implemented practically in a museum in Japan - 'Ceramics park MINO (Gifu, Japan)'. The exhibition rooms of the museum have been built on the concept of suspended floor slab

system, in which the entire floor is suspended using hanger rods from the roof as shown in Figure 1.1. The system was tested before construction and showed promising behavior with respect to isolation [9,10]. These studies and developments clearly suggest the practicality and advantages of floor isolation and suspension systems for vibration mitigation of structures. Even though these systems can be complex and costly however, if properly implemented their superior performance can offset the cost of construction since only minimal retrofits will be required following a large seismic event. As an example, in the 1995 Hyogoken-Nanbu Earthquake it was reported that an existing floor isolated system performed quite effectively [4].



Figure 1.1 Floor suspension system adopted for exhibition rooms of 'Ceramic Park MINO' (a) Bird-eye view (b) Photograph of installation [9]

2. PROPOSED SYSTEMS

2.1 System description

In this study two types of floor isolation systems are discussed – suspension and sliding systems. The proposed suspension systems utilize the concept of MTMDs by isolating selected floor slabs from the beams above them using cables. The suspended slabs tend to act as a typical Pendulum Tuned Mass Damper (PTMD), thereby reducing the motion of the primary frame. In order to ensure controlled motion of the slabs two types of links are connected from the bottom face of the slab to the flange of the beam underneath the slab. The connected links are categorized into 2 types – Post-Tensioned (PT) links and Energy Dissipation (ED) links. The PT links are intended to provide the necessary stiffness to control the motion of slabs without yielding and to ensure zero residual drift through self-centering of the slab following the earthquake. The ED links are intended to act as energy dissipation elements. There are two models of suspension systems considered in this study, which differ in the type of connection used for suspending the slabs (cable-to-beam). In Model-I, roller connections are proposed to suspend the slabs, whereas in Model-II fixed connections are proposed, as shown in Figure 2.1.



Figure 2.1 Elevation view of proposed suspension systems and their components

For the sliding system only one model is considered in this study. The proposed system comprises of isolated floor slabs placed on curved supports so that they are free to move relative to the frame. The curvature in the supports allows for gravity to reposition the slab back to its original location, thereby provide a self-centering mechanism. To keep the motion of the slabs under check rubber bumpers or stiffness components are installed between the slab and the frame. The friction of the contact surface can be used to dissipate energy to some extent, which will provide some damping to the system. Hence, the sliding system tends to act as a typical Translational Tuned Mass Damper system. An elevation view of the proposed sliding system is shown in Figure 2.2. The sliding mechanism layout as shown would be pre casted as separate members before assembling the complete mechanism on the construction site. The curved support for the slab would be developed as pre cast concrete member and the sliding slab would be designed as a regular reinforced concrete slab.



Figure 2.2 Elevation view of proposed sliding system

2.2 Multi-isolated slab models

To analyze the proposed systems respective analytical models for both systems were developed as an analogue to a simultaneous, vertically and horizontally distributed Multi Tuned Mass Damper (MTMD) system. Moon [11] presented a similar analytical study on the vertical distribution of TMDs that showed an elaborate design procedure for tuning vertically distributed TMDs to different modes. In suspension Model-I, the slabs are suspended using roller connections (the rollers rest on the beam flange), which decouples the floor beam from the slab suspended from it and is thus coupled with only the slab above it. In case of Model-II, the fixed connections used for suspending the slabs from the beam result in coupling between the beam to the suspended slabs above and below it. The sliding model is quite similar to Model-I in principle, when the angle of rotation for the suspension systems is quite small, thus both models can be idealized as shown in Figure 2.3a while Model-II can be represented as shown in Figure 2.3b. The proposed analytical systems in this study not only accounts for vertical distribution, but also the horizontal distribution of slabs. The equations in this present study are able to incorporate the effect of variation in the placement of suspended slabs for optimal performance. In order to better understand the effect of isolating multiple slabs, the location and number of slabs to be isolated would be considered as a critical optimization parameter. From a practical perspective, all the slabs are assumed to have similar properties since it would be easier to implement practically and would also result in a simplified optimization procedure.



Figure 2.3 Analytical models of (a) Suspension Model-I and Sliding Model and (b) Suspension Model-II

2.3 Equations of motion

To understand the effect of suspending or isolating specific slabs a control parameter (C_n) is introduced in the equations of motion to indicate the presence or absence of slabs on a given floor. As shown in Eq. 2.1, C_n takes only binary values of 0 or 1 and allows for further generalization of the equations of motion. The proposed system configuration of slabs can be physically represented in a one-dimensional matrix as shown in Figure 2.4. Each value in the C_n matrix represents the pattern of the bays on the respected floor. It was assumed that for any case there would be no slab suspended at the bottom floor ($C_1 = 0$), since it would not make practical sense in terms of structural engineering application. The equations of motion for the systems takes into account the forces generated by all isolated slabs present in the system. The contribution of each floor's isolated slabs is accounted for based on modal ratios (q_n). The modal ratio is the ratio of mode shape for a particular floor (ϕ_n) to the mode shape of the top floor (ϕ_N). Eq. 2.2 and 2.3 are the equilibrium equations for the steel frame of Model-I and the sliding model, respectively.



Figure 2.4 Physical representation of variable C_n

Equations 2.2 and 2.3 were formulated by balancing the forces experienced by each story of the structure. The left hand side terms in Model-I equation (Eq. 2.2) represents the force generated in the frame due to its inertia, damping and stiffness, whereas in the sliding model (Eq. 2.3) the forces generated is only due to inertia and stiffness. The 1st term on the right is the force generated at the floor level due to dynamic excitation and the second term is the force generated by the neighboring isolated slabs. The term Γ_e is the modal participation factor, a_g is the input dynamic excitation, u_N , \dot{u}_N , \ddot{u}_N are the displacement, velocity and acceleration of N^{th} floor of the steel frame, u_{dnj} , \dot{u}_{dnj} are the displacement and velocity of isolated slab on n^{th} floor j^{th} bay, and m_{dnj} , c_{dnj} , k_{dnj} , μ_{dnj} are the mass, damping coefficient, stiffness and friction coefficient of n^{th} floor j^{th} bay isolated slab. The terms \tilde{m}_E , \tilde{k}_E and \tilde{c}_E are the effective mass, stiffness and damping of the frame, respectively, derived by assuming a pre-dominant 1st mode. It is recognized that the use of the 1st mode alone for analysis is not a comprehensive representation of the performance of the proposed systems. However, it is essential to first understand the behavior of the systems prior to introducing the effect of additional modes. Thus due to the limited scope of this study, the analysis was restricted to only the 1st mode and a comprehensive modal study using modified Newmark equations would be conducted as part of a future study. Similar to the frame, Eq. 2.4 and 2.5 are the equilibrium equations of the two models for the isolated slab on n^{th} floor and j^{th} bay, respectively, where 'n' represents the index for floor number (1 – N) and 'j' is the index for bay number (1 - M).

$$\widetilde{m}_{E} \ddot{\mathcal{U}}_{N} + \widetilde{c}_{E} \dot{\mathcal{U}}_{N} + \widetilde{k}_{E} \mathcal{U}_{N} = -a_{g} \Gamma_{e} + \sum_{n=1}^{N} \left[C_{n} \frac{\varphi_{1n}}{\varphi_{1N}} \left[\sum_{j=1}^{M} \left(k_{dnj} \mathcal{U}_{dnj} + c_{dnj} \dot{\mathcal{U}}_{dnj} \right) \right] \right]$$
(2.2)

$$\widetilde{m}_{E} \widetilde{u}_{N} + \widetilde{c}_{E} \dot{u}_{N} + \widetilde{k}_{E} u_{N} = -a_{g} \Gamma_{e} + \sum_{n=1}^{N} \left[C_{n} \frac{\varphi_{1n}}{\varphi_{1N}} \left[\sum_{j=1}^{M} \left(k_{dnj} u_{dnj} + \mu_{dnj} m_{dnj} g \right) \right] \right]$$
(2.3)

$$m_{dnj}\ddot{u}_{dnj} + c_{dnj}\dot{u}_{dnj} + k_{dnj}u_{dnj} + m_{dnj}\ddot{u}_n = -m_{dnj}a_g \tag{2.4}$$

$$m_{dnj}\ddot{u}_{dnj} + k_{dnj}u_{dnj} + m_{dnj}\ddot{u}_n = -m_{dnj}a_g - \mu_{dnj}m_{dnj}g$$
(2.5)

The equations for Model-II are also obtained in a similar manner. Eq. 2.6 is the equilibrium equation for the frame and Eq. 2.8 is the equation for the suspended slab on n^{th} floor and j^{th} bay. The left hand side in Eq. 2.6 is similar to Model-I, however the right side differs due to the difference in the coupling nature of the models. In Model-II the motion of isolated slabs is coupled to both its immediate upper and lower floors. Matrix A represents the forces generated by upper suspended slab on the floor and matrix B is the forces generated by the lower suspended slab, which are shown in Eq. 2.7. In Eq. 2.8 all the terms are similar to Eq. 2.4, except an additional force term is added to account for the coupling action generated in Model-II.

$$\widetilde{m}_{E} \widetilde{u}_{N} + \widetilde{c}_{E} \dot{u}_{N} + \widetilde{k}_{E} u_{N} = -a_{S} \Gamma_{e} + \sum_{n=1}^{N} \left[\frac{\varphi_{1n}}{\varphi_{1N}} [A_{n} + B_{n}] \right]$$
(2.6)

$$A = \begin{bmatrix} C_{2} \\ C_{n+1} \\ \cdots \\ C_{N} \\ 0 \end{bmatrix} \begin{bmatrix} \sum_{j=1}^{M} k_{d2j} u_{d2j} + c_{d2j} \dot{u}_{d2j} \\ \sum_{j=1}^{M} k_{dn+1j} u_{dn+1j} + c_{dn+1j} \dot{u}_{dn+1j} \\ \cdots \\ \sum_{j=1}^{M} k_{dNj} u_{dNj} + c_{dNj} \dot{u}_{dNj} \\ 0 \end{bmatrix} \begin{bmatrix} e^{\hat{e}} & 0 & u^{\hat{e}} \\ e^{\hat{e}} & C_{2} & u^{\hat{e}} \\ e^{\hat{e}} & C_{2} & u^{\hat{e}} \\ 0 & u^{\hat{e}} & A_{in} (u_{2} - u_{d2j} - u_{1}) \\ u^{\hat{e}} & A_{in} (u_{n} - u_{dnj} - u_{n-1}) \\ u^{\hat{e}} & u^{\hat{e}} \\ e^{\hat{e}} & \cdots & u^{\hat{e}} \\ e^{\hat{e}} & C_{N} & u^{\hat{e}} \\ e^{\hat{e}} & C_{N} & u^{\hat{e}} \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{n-1}) \\ u^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ u^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{dNj} - u_{N-1}) \\ e^{\hat{e}} & A_{in} (u_{N} - u_{in} - u_{in} \\ e^{\hat{e}} & A_{in} (u_{N} - u_{in} - u_{in} \\ e^{\hat{e}} & A_{in} (u_{N} - u_{in}$$

where,

$$m_{dnj}\ddot{u}_{dnj} + c_{dnj}\dot{u}_{dnj} + k_{dnj}u_{dnj} + m_{dnj}\ddot{u}_n + k_{in}(u_{dnj} + u_{n-1} - u_n) = -m_{dnj}a_g$$
(2.8)

3. SYSTEM PERFORMANCE AND OPTIMIZATION

3.1 Vertical Distribution of Isolated Slabs

The equations of motion for each system are solved to obtain the respective response functions. The performance of the suspended slab is seen to be a function of 3 parameters for the suspension systems; namely 1) the mass ratio (μ_d), which is ratio of the mass of a suspended slab to the mass of the remainder of the system (non-suspended slabs & frame skeleton), 2) the damping ratio of the energy dissipation links (ξ_d), and 3) the ratio of natural frequency of the slab to that of the frame (β_d). On the other hand, the performance of sliding system is seen to be a function of the same parameters as the suspension systems, except the damping ratio is replaced by the friction coefficient of sliding surface. The response functions were evaluated under dynamic excitations and the critical parameters were optimized to obtain the best performance of each system. Figure 3.1 shows the performance improvement of the three systems for a 10-story 5 bay structure in comparison to a composite floor slab counterpart, along with the optimum configuration of slabs to be suspended or isolated. The performance of the three systems was seen to significantly better compared to a composite system. The suspension systems showed nearly identical performance improvements in terms of the steel frame, however motion of slabs was seen to be more controlled for Model-II. The sliding system showed relatively less improvement than the suspension systems since it does not have an independent damping component. The damping in sliding system is derived from the friction, which cannot be substantially high otherwise it would adversely affect the motion of slab, thus would reduce the isolation capabilities of the system.



c) Performance and Optimum location for Sliding Model

Figure 3.1 Response improvement of proposed systems over composite frame system for 10 story 5 bay structure with their respective optimum configuration of isolated slabs

3.2 Horizontal Distribution of Isolated Slabs

The analysis results shown in the previous section assumed the same number of slabs to be suspended all across the bay in each case (i.e. no horizontal distribution of slabs). In this section, the effect of horizontal distribution of slabs is studied. Since the sliding model and suspension Model I are similar, the analysis is performed only on the suspension models. The results of the analysis for the 10-story 5-bay structure are shown in Table 3.1, where M represents the number of bays in which slabs can be suspended and H_s is the steel frame top floor displacement. From the results it is rather clear that as M decreases the suspended slabs pattern (C_n) tends to change such that the number of floors on which slabs need to be suspended increases to maintain an optimal performance. The performance parameters H_s and C_n indicates that varying the horizontal distribution of slabs (i.e. values of M), does not have an effect on the overall performance of the system (H_s is nearly the same

for both models as M varies) since the optimization algorithm compensates for reduced M values by distributing the slabs vertically. Therefore, it can be inferred that the systems require an optimal mass ratio value for best performance and the optimization algorithm derives a suitable C_n such that the system is able to achieve that mass ratio. In both models a similar trend of vertical distribution of suspended slabs is seen as M decreases. This could be explained mathematically from the response equations of the two suspended slab systems. The response equations, not shown in this papers, are a function of a term $\sum_{n=1}^{N} q_n \mu_{dnj}$, which is a product of n^{th} floor's modal ratio and mass ratio of suspended slab on n^{th} floor j^{th} bay summed over for all N floors. This term tends to drive the pattern of the suspended slabs i.e. the value of parameter C_n , and corresponds to a particular optimal mass ratio for the respected system. The optimization starts with the floor with the highest modal ratio (i.e. the top floor in this case due to mode 1) and sums $q_n \mu_{dnj}$ until it reaches a certain optimal value. In case a particular floor's $q_n \mu_{dnj}$ value cause the summation to exceed the optimal value, the optimization skips the particular floor i.e. it does not suspend any slabs on the floor and assigns value 0 to C_n . This highlights an interesting point that the pattern of suspended slabs correspond to the effective mass ratio of the system, which in turn is a function of the mode shape ratios of each floor. In this study, the 1^{st} mode is considered as dominant, hence the optimized pattern obtained corresponds to only if the 1st mode is dominant. In case of multi modal analysis the pattern would be a function of the modal participation factor and the floors' mode shape ratios of the first few dominant modes.

10-Story 5-Bay	Μ	C_n	β_d	ξ_d	H_S (in)
Model – I	5	[0110000111]	0.5525	0.2654	14.5435
Model – II	5	[0000000111]	0.4330	1.0460	15.4519
Model – I	4	[0111100111]	0.5767	0.2526	14.6153
Model – II	4	[0000000111]	0.4878	0.8413	15.6644
Model – I	3	[0111101111]	0.6602	0.2275	14.65
Model – II	3	[0000001111]	0.4904	0.8271	15.9741
Model – I	2	[011111111]	0.7007	0.2119	14.95
Model – II	2	[0000011111]	0.5219	0.7037	16.4739
Model – I	1	[011111111]	0.8370	0.1543	17.3
Model – II	1	[0011111111]	0.5934	0.46	18.1496

Table 3.1 Optimal Parameter values for horizontal distribution of suspended slabs in MSS models

4. CONCLUSIONS

In this study 3 types of floor isolation systems have been studied. The systems work on an intricate medley of 3 principles – vibration isolation of TMD systems, dissipation of energy from motion of slabs, and self–centering. The following observations and conclusions can be drawn from the present study.

- All the three systems performed substantially well against their composite counterpart. The sliding system performed relatively less than the suspension systems mainly due to the absence of an independent damping mechanism. Thus by adding a damping element the efficiency can be further improved.
- The optimal configuration for the three systems was seen to be quite similar and suggested that for the optimal performance of these systems all slabs do not need to be isolated.
- From the analysis of horizontal distribution of isolated slabs it was seen that the pattern of suspended slabs is related to the effective mass ratio of the system, which is the weighted sum of mass ratios of all isolated slabs weighted by the mode shape ratio of the floor on which it is suspended.
- The analysis further showed that as the number of slabs suspended on each floor are reduced, the optimization distributes the slabs vertically i.e. suspend slabs on more floors, to maintain the optimal mass ratio, and hence the same performance. Thus, pattern of suspended slabs can be altered to some extent while keeping the overall performance of the system constant.
- The analysis of the proposed systems suggests a significant potential for application in vibration isolation of structures.

REFERENCES

1. Takase, N., Arima, F., Tanaka, H., and Egasira, H. (1997). Development of Three-Dimensional

Floor-Isolated System. Part I: Dynamic Characteristics of Floor-Isolated Method by Suspended Structure. *Summaries of Technical Papers of Annual Meeting Architectural Institute of Japan*; B-2; 595–6 [in Japanese].

- Kaneko, M., Yasui, Y., and Okuda, Y. (1995). Simultaneous Horizontal and Vertical Vibration Tests of Three-Dimensional Floor Isolation System. *AIJ Journal of Technology and Design, Architectural Institute of Japan.* 1, 186 – 190 [In Japanese].
- 3. Cui S., Bruneau, M., and Kasalanati, A. (2005). Behavior of Bidirectional Spring Unit in Isolated Floor Systems. *ASCE Journal of Structural Engineering*. **136:8**, 944-952.
- 4. Lambrou, V., and Constantinou, M.C (1994). Study of Seismic Isolation Systems for Computer Floors. Technical report: NCEER-94-0020, National Center for Earthquake Engineering Research.
- 5. Liu, S., and Warn, G. P. (2012). Seismic Performance and Sensitivity of Floor Isolation Systems in Steel Plate Shear Wall Structures. *Engineering Structures*. **42**, 115-126.
- Kasalanati, A., Reinhorn, A., Constantinou, M. C., and Sanders, D. (1997). Experimental Study of Ball-in-Cone Isolation System. Building to Last: Proceedings of Structures Congress XV, American Society of Civil Engineering. 1191-1195.
- Nakamura, Y., Saruta, M., Wada, A., Takeuchi, T., Hikone, S., and Takahashi, T. (2011). Development of the Core-Suspended Isolation System. *Journal of Earthquake Engineering and Structural Dynamics*. 40, 429 – 447.
- 8. Tatemichi I., Kawaguchi, M., and Abe, M. (2004). A Study on Pendulum Seismic Isolators for High-Rise Buildings. *The 2004 Council on Tall Buildings and Urban Habitat (CTBUH) Conference*, Seoul, Korea.
- 9. Kawaguchi, M., Tatemichi I., Abe M., and Ide T. (2003). Development and Testing of a Seismically Isolated Floor System using Translational Pendulum Principle. *IASS-APCS 2003 Symposium*, Taipei, Taiwan.
- 10. Chen, Z. H., Kawaguchi, M., Tatemichi, I., and Abe, M. (2003). A Study on the Non-Parallel Swing System for Seismic Isolation. *IASS-APCS 2003 Symposium*, Taipei, Taiwan.
- 11. Moon, K. S. (2010). Vertically Distributed Multiple Tuned Mass Dampers in Tall Buildings: Performance Analysis and Preliminary Design. *Journal of Structural Design of Tall and Special Buildings*. **366**, 347–366.