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Synthesis of Controllers for the Active Mass Driver System in the Presence of Uncertainty

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Abstract

The structured singular value (μ) synthesis technique is used to design controllers for the Active Mass Damper (AMD) Benchmark problem. In addition to stated performance objectives, robustness of the controllers to high frequency unmodeled dynamics (the neglected high frequency modes of the evaluation model), modeling error in the actuator dynamics and variations in the first structural natural frequency and damping value are considered in the design. The resulting controller achieves similar performance levels on the nominal evaluation model and the evaluation model with significant changes in its first natural frequency and damping value.

Benchmark Problem: Active Mass Driver

The structured singular value (μ) framework is applied to the Active Mass Driver (AMD) benchmark problem describe in references [1, 2]. The objective is to actively control this three-story, single-bay, scale model of a building. A single active mass driver (AMD) actuator, located on the third floor of the structure, is used for control. The base of the structure is mounted to a shake table to simulate earthquake loadings. Six measurements are available for feedback: accelerometers at the base of the structure, on each story, and on the actuator mass and an LVDT displacement sensor attached to the actuator.

The objective is to design a discrete-time feedback compensator that minimizes ten performance objectives. Five of the performance objectives correspond to minimizing rms errors and the other five objectives correspond to minimizing maximum displacements, accelerations and voltages. Two linear time-invariant models of the AMD structure are provided. A 10 model is used for control design. A high-fidelity, 28-state evaluation model is provided for analysis and simulation. The AMD benchmark problem does not include any objectives or specifications on the robustness or insensitivity of the control design to modeling errors or model uncertainty. This is a central issue in the control of any real physical system. Therefore, the control design presented in this paper includes modeling errors to account for neglected high frequency unmodeled dynamics, uncertainty in the actuator, sensor noise and variations in the damping value and natural frequency of the first structural mode.

Control Problem Formulation

The structured singular value (μ) synthesis technique is used for controller design [3, 4, 5, 6, 7]. The AMD control problem is posed as a robust performance problem, with multiplicative plant uncertainty at the plant input, additive uncertainty around the plant, parametric uncertainty in the natural frequency and damping value of the first mode and minimization of weighted error transfer functions as the performance criterion [7]. The actuator voltage, displacement, velocity, and acceleration signals are weighted to insure that they do not exceed their physical capabilities. Sensor noise is included on the six measurements to mimic the experimental system. The performance objectives are included as minimizing weighted transfer functions associated with story velocities and accelerations and inter-story displacements. A diagram of the system interconnection structure used for control design and analysis is shown in in Figure 1.

The performance objective is to have the "true" structure, described by the control design and uncertainty models, achieve the desired performance objectives. Note that these models define a much richer set of structural systems than just the evaluation model. The evaluation model is included in this set of models to be controlled, but also included in this set are structures that have *different* natural frequency and damping values of the first structural mode, different actuator gain and phase characteristics and additional high frequency dynamics.

The Benchmark performance objectives are entirely driven by the ability of the controller to attenuate the response of the first mode of the AMD structure and the actuator acceleration limit. Therefore only the inter-story drift and the story accelerations need to be heavily penalized in the control design. The performance weighting functions in Figure 1 are defined as follows:

- $W_{\text{earthquake}}$ defined as $\frac{3.6e{-}4s^3+3.2e{+}1s^2+6.6e{+}4s+1.4e{+}4}{s^3+1.0e{+}3s^2+7.5e{+}4s+2.8e{+}5}$ is used to describe the square-root of the Kanai-Tajmi earthquake spectra.
- W_{act} weights to actuator control voltage input. It is selected to be 0.4.

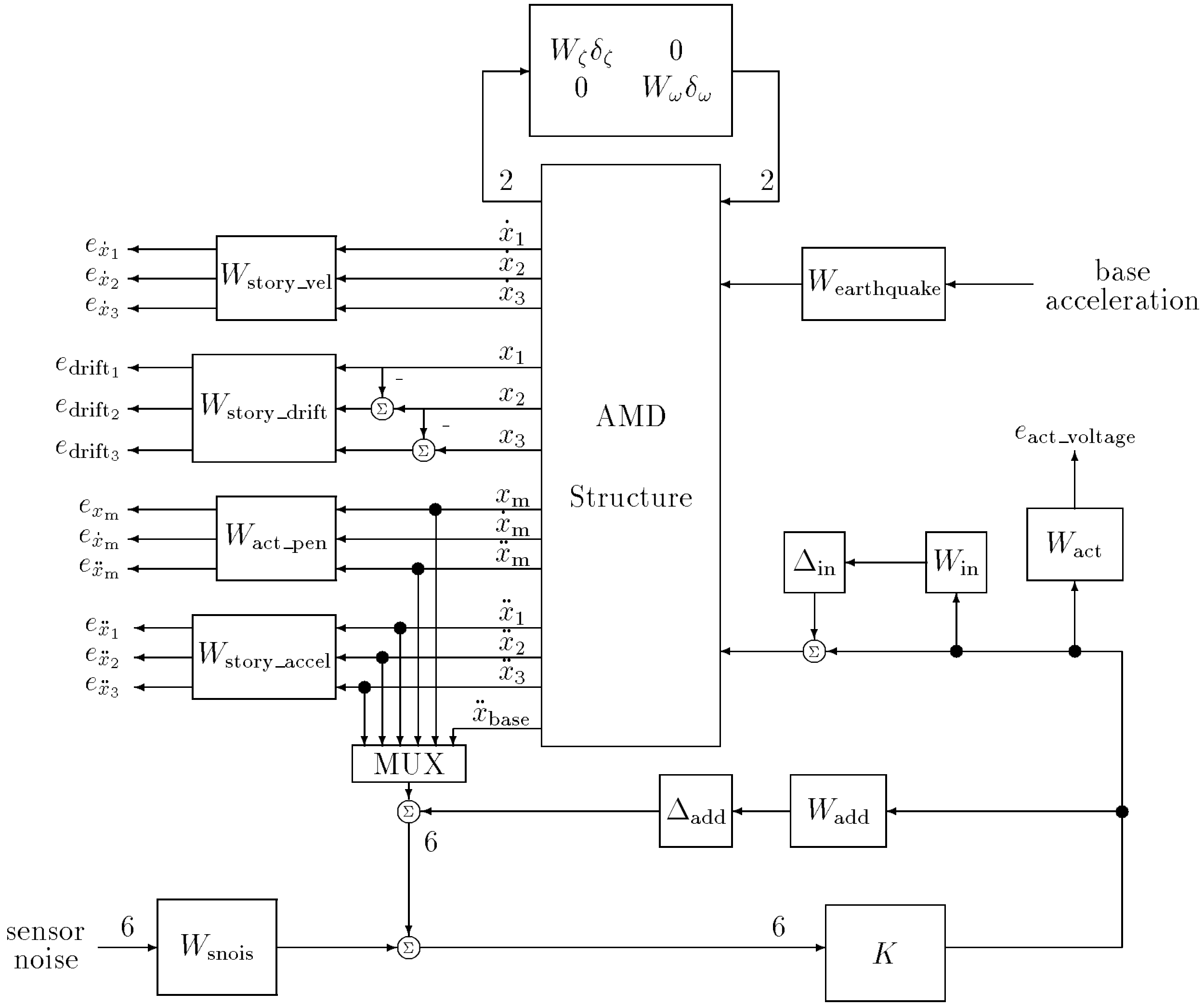

Figure 1: Interconnection Structure of the AMD Control Problem

This corresponds to a maximum of 2.5 volts being commanded to the actuator. W_{snois} is a 6×6 matrix with 0.001 in its diagonal entries. These are estimate of the sensor noise levels.

- $W_{\text{story\_vel}}$, $W_{\text{story\_drift}}$, and $W_{\text{story\_accel}}$ are 3×3 matrices with diagonal entries of (0.04, 0.025, 0.021), (0.06, 0.10, 0.23), and (0.011, 0.007, 0.006), respectively. These weights are selected to attenuate the response of the first mode in the control design.

- $W_{\text{act\_pen}}$ is a 3×3 matrix with diagonal entries of 0.04, 0.03, 0.002, respectively. Since the transfer functions between the actuator input and its displacement, velocity and acceleration are related and only the first mode plays a role in the performance objectives, only the actuator displacement output is heavily weighted.

- W_{in} is set to 0.1 corresponding to 10% uncertainty in the actuator response. W_{add} is defined as $\frac{.93s^2+167s+3225}{s^2+1251s+44429}$. A second order transfer transfer function that accounts for the high frequency dynamics in the evaluation model that were not included in the control design model.

- W_ζ is set to 0.5 or 50% error in the damping level of the first mode. A benefit of directly designing for uncertainty in the level of damping using $D-K$ iteration is it results in robustness to variations in the first structural mode. W_ω is zero in the control design and set to 0.25 (12%) error in the natural frequency of the first mode in the analysis.

Weighting functions serve two purposes in the $\mathcal{H}_\infty$ and μ framework: they allow the direct comparison of different performance objectives with the same norm and they allow frequency information to be incorporated into the analysis. All the weighted performance objectives are scaled to have an $\mathcal{H}_\infty$ less than 1 when they are achieved [6].

Results

A 28-state controller was synthesized using $D-K$ iteration μ synthesis approach. The balanced realization method was used to reduced the controller order to 12-states which is used in the analysis and simulations. The robust performance μ value was 0.85, note that the natural frequency and damping value were treated as real perturbations in the analysis, indicating that all performance and robustness objectives were simultaneously achieved. The μ-analysis test determined that the worst case variation from a performance and stability perspective was to perturb the first natural frequency from 36.48 rad/sec to 33.44 rad/sec and its damping value from 0.34% to 0.23%. Table 1 contains the J6 through J10 performance indices for the nominal and perturbed evaluation models. El Centro Earthquake:

The value of J1 through J5 with $w_g = 37.3$ rad/sec and $\zeta_g = 0.3$ for the nominal evaluation model are (0.146, 0.223, 0.617, 0.623, 0.600) respectively. Based on these results, μ synthesis was able to design a controller which achieved a significant level of performance on the AMD structure in the presence of modeling error.

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	El Centro		Hachinoche	
	Nominal Model	Perturbed	Nominal Model	Perturbed
J6	0.3122	0.2936	0.3778	0.3770
J7	0.4790	0.4889	0.6791	0.6984
J8	1.2670	1.2667	1.4289	1.5280
J9	1.2141	1.1200	1.5374	1.3974
J10	1.0951	1.2420	1.2368	1.3304
max act volt	1.191	1.187	0.661	0.708
max act disp	4.270	4.269	2.372	2.539
max act accel	5.530	6.272	3.191	3.432

Table 1: Nominal and Perturbed Performance Measures

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