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$\mathbf{H}_\infty$ Static and Dynamic Output Feedback Control of the AMD Benchmark Problem

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Abstract

This study develops an active control methodology for the AMD benchmark problem of Spencer, et al. (1997) based on dynamic output feedback controllers designed using an $\mathbf{H}_\infty$ based approach. Kalman filter estimators of the states of a reduced order model of the benchmark structure are coupled to static state feedback controller gains to develop the dynamic feedback controllers. A method is outlined for designing $\mathbf{H}_\infty$ feedback controller gains, and a comparison is made between the effectiveness of $\mathbf{H}_\infty$ static output feedback and the dynamic acceleration feedback controllers. The results quantify the performance increase obtained with the additional complexity of the dynamic output feedback controllers compared to the static acceleration feedback controllers.

Introduction

For the AMD benchmark problem of Spencer, et al. (1997), controllers are designed using two $\mathbf{H}_\infty$ based approaches: one approach uses direct static output feedback of sensor measurements, and the second approach uses a dynamic output feedback controller that consists of static state feedback controller gains with a Kalman filter state estimator. The controllers considered in this study are designed by a continuous-time $\mathbf{H}_\infty$ controller approach, then discretized for simulation with the benchmark model.

The controllers are developed from a state space design model of the form:

$$\dot{\mathbf{x}}(\mathrm{t}) = \mathbf{A}\mathbf{x}(\mathrm{t}) + \mathbf{B}_{\mathrm{u}}\mathbf{u}(\mathrm{t}) + \mathbf{B}_{\mathrm{w}}\mathbf{w}(\mathrm{t}) \tag{1}$$

$$\mathbf{z}(\mathrm{t}) = \mathbf{C}_{\mathrm{z}}\mathbf{x}(\mathrm{t}) + \mathbf{D}_{\mathrm{zu}}\mathbf{u}(\mathrm{t}) + \mathbf{D}_{\mathrm{zw}}\mathbf{w}(\mathrm{t}) \tag{2}$$

$$\mathbf{y}(\mathrm{t}) = \mathbf{C}_{\mathrm{y}}\mathbf{x}(\mathrm{t}) + \mathbf{D}_{\mathrm{yu}}\mathbf{u}(\mathrm{t}) + \mathbf{D}_{\mathrm{yw}}\mathbf{w}(\mathrm{t}) \tag{3}$$

where $\mathbf{x}(\mathrm{t})$ is the state vector, $\mathbf{u}(\mathrm{t})$ is the vector of control inputs, $\mathbf{w}(\mathrm{t})$ is the vector of disturbance inputs, $\mathbf{y}(\mathrm{t})$ is the vector of sensor measurements, $\mathbf{z}(\mathrm{t})$ is the vector of regulated outputs, and $\mathbf{A}$, $\mathbf{B}_{\mathrm{u}}$, $\mathbf{B}_{\mathrm{w}}$, $\mathbf{C}_{\mathrm{z}}$, $\mathbf{D}_{\mathrm{zu}}$, $\mathbf{D}_{\mathrm{zw}}$, $\mathbf{C}_{\mathrm{y}}$, $\mathbf{D}_{\mathrm{yu}}$, and $\mathbf{D}_{\mathrm{yw}}$ are matrices of

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the appropriate dimension developed from the evaluation model. The static feedback controller gains define the controller command signal as $\mathbf{u}(\mathrm{t}) = -\mathbf{K}_{\mathrm{G}}\mathbf{y}(\mathrm{t})$, and the dynamic output feedback controllers are of the form:

$$\dot{\mathbf{x}}_{\mathrm{c}}(\mathrm{t}) = \mathbf{A}_{\mathrm{c}}\mathbf{x}_{\mathrm{c}}(\mathrm{t}) + \mathbf{B}_{\mathrm{c}}\mathbf{y}(\mathrm{t}) \tag{4}$$

$$\mathbf{u}(\mathrm{t}) = \mathbf{C}_{\mathrm{c}}\mathbf{x}_{\mathrm{c}}(\mathrm{t}) + \mathbf{D}_{\mathrm{c}}\mathbf{y}(\mathrm{t}) \tag{5}$$

where $\mathbf{x}_{\mathrm{c}}(\mathrm{t})$ is the state vector of the dynamic controller and $\mathbf{A}_{\mathrm{c}}$, $\mathbf{B}_{\mathrm{c}}$, $\mathbf{C}_{\mathrm{c}}$ and $\mathbf{D}_{\mathrm{c}}$ are the state description of the regulator. The idealized continuous closed loop system can then be described in the standard form by:

$$\dot{\hat{\mathbf{x}}}(\mathrm{t}) = \mathbf{A}_{\mathrm{cl}}\hat{\mathbf{x}}(\mathrm{t}) + \mathbf{B}_{\mathrm{cl}}\mathbf{w}(\mathrm{t}) \tag{6}$$

$$\mathbf{z}(\mathrm{t}) = \mathbf{C}_{\mathrm{cl}}\hat{\mathbf{x}}(\mathrm{t}) + \mathbf{D}_{\mathrm{cl}}\mathbf{w}(\mathrm{t}) \tag{7}$$

where $\hat{\mathbf{x}}(\mathrm{t})$ is the augmented state vector representing the dynamics of the system and controller. For this system, the $\mathbf{H}_\infty$ norm of the transfer function $\mathbf{T}_{\mathrm{zw}}$, from the excitation $\mathbf{w}(\mathrm{t})$ to the output $\mathbf{z}(\mathrm{t})$, is defined as:

$$\| \mathbf{T}_{\mathrm{zw}} \|_\infty \doteq \sup_{\mathbf{w}(\mathrm{t})} \frac{\| \mathbf{z}(\mathrm{t}) \|_2}{\| \mathbf{w}(\mathrm{t}) \|_2} \tag{8}$$

Controller Design

For the design of static output and state feedback controller gains, an iterative gradient search was implemented to find a locally optimal solution that met a prescribed attentuation of the $\mathbf{H}_\infty$ norm of the closed loop system. Defining the $\mathbf{H}_\infty$ norm of a closed loop system as:

$$\gamma_{\mathrm{CL}} \doteq \| \mathbf{T}_{\mathrm{zw}}(cl) \|_\infty = \| \mathbf{C}_{\mathrm{cl}}(sI - \mathbf{A}_{\mathrm{cl}})^{-1}\mathbf{B}_{\mathrm{cl}} + \mathbf{D}_{\mathrm{cl}} \|_\infty \tag{9}$$

and the open loop $\mathbf{H}_\infty$ norm as:

$$\gamma_{\mathrm{OL}} \doteq \| \mathbf{T}_{\mathrm{zw}}(ol) \|_\infty = \| \mathbf{C}_{\mathrm{z}}(sI - \mathbf{A})^{-1}\mathbf{B}_{\mathrm{w}} + \mathbf{D}_{\mathrm{zw}} \|_\infty \tag{10}$$

then the increase in performance attributed to the controller can be characterized by the $\mathbf{H}_\infty$ norm ratio $\bar{\gamma} \doteq \gamma_{\mathrm{CL}}/\gamma_{\mathrm{OL}}$. To characterize the control effort cost of this performance gain, we define the controller effort $\mathbf{H}_\infty$ norm as:

$$\gamma_{\mathrm{u}} \doteq \| \mathbf{T}_{\mathrm{uw}} \|_\infty = \| \mathbf{C}_{\mathrm{clu}}(sI - \mathbf{A}_{\mathrm{cl}})^{-1}\mathbf{B}_{\mathrm{cl}} + \mathbf{D}_{\mathrm{clu}} \|_\infty \tag{11}$$

where $\mathbf{C}_{\mathrm{clu}}$ and $\mathbf{D}_{\mathrm{clu}}$ are matrices mapping the augmented system states, $\hat{\mathbf{x}}(\mathrm{t})$, and excitation, $\mathbf{w}(\mathrm{t})$, to a measure of the controller effort. For this study, the measure of controller effort is taken as the controller command signal, $\mathbf{u}(\mathrm{t})$; however, other measures can be used, such as the actuator displacement or acceleration. To find a candidate controller, the optimization routine minimizes the control effort norm γ_{u} subject to a performance criteria constraint of $\bar{\gamma} < \bar{\gamma}_{\mathrm{des}}$, where $\bar{\gamma}_{\mathrm{des}}$ is a specified desired attenuation ratio. This procedure is used to find a sequence of controllers with varying performance and control effort characteristics by solving for controllers with sequentially lower values of $\bar{\gamma}_{\mathrm{des}}$, while using a previous controller solution as the

$\bar{\gamma}_{\text{des}}$	J1	J2	J3	J4	J5	σ_u	σ_{x_m}	$\sigma_{\ddot{x}_{am}}$
0.90	0.5507	0.9295	0.1087	0.1126	1.0105	0.0156	0.1424	1.8087
0.75	0.4823	0.7995	0.2310	0.2354	0.9469	0.0469	0.3026	1.6950
0.65	0.4302	0.7071	0.3341	0.3373	0.9302	0.0737	0.4377	1.6651
$\bar{\gamma}_{\text{des}}$	J6	J7	J8	J9	J10	$\max(u)$	$\max(x_m)$	$\max(\ddot{x}_{am})$
0.90	0.6012	1.1048	0.1109	0.1384	1.2048	0.0337	0.3571	5.6114
0.75	0.5547	1.0363	0.2468	0.2589	1.2281	0.1143	0.7339	5.8319
0.65	0.5300	1.0038	0.3727	0.3848	1.2647	0.1978	1.1139	5.9995

Table 1: Static Feedback Performance Indices

starting point for the optimization routine. The advantages of this solution method are apparent in the ease at which more complex $\mathbf{H}_\infty$ design modelling can be included in the solution process. The definition of the $\mathbf{H}_\infty$ norms can be extended to include non-linear, time-varying, or uncertain model descriptions without significant modification of the algorithms developed. Examples of this controller design approach including bounded uncertainty models and actuator saturation can be found in Breneman et al. (1997) and Chase et al. (1996).

The design of a dynamic output feedback controllers can be accomplished by formulating the optimization over the entire dynamic regulator, $\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$, $\mathbf{D}_c$, and including sensor noise in the excitation, $\mathbf{w}(t)$. This approach leads to optimization problems that can require extensive computational effort. A simpler approach is to couple an $\mathbf{H}_\infty$ state feedback controller gain with a state estimator designed by an $\mathbf{H}_\infty$ or alternate approach, such as a Kalman filter.

Example Controllers

The static and dynamic output controllers presented herein are designed for the AMD benchmark problem using a configuration where the sensor measurements $\mathbf{y}(t)$ are the absolute floor accelerations of the structure, the acceleration of the actuator piston, and the direct measurement of the ground acceleration. The external excitation, $\mathbf{w}(t)$, is the ground acceleration, and the regulated output is the full regulated output vector, $\mathbf{z}(t)$, defined by Spencer et al. (1997) which includes the structural displacements, velocities, and accelerations, and the actuator displacement, velocity and acceleration. A 12th order reduced dynamic model used for design of the example controllers was obtained by standard balanced model truncation techniques with tools available in the MATLAB controls toolbox. A frequency domain comparison of this reduced order model and the evaluation model shows that the dominant dynamic characteristics of the system are retained in the design model for the frequency ranges of most interest.

Static output feedback controller gains were generated for this configuration by the described optimization method for $\bar{\gamma}_{\text{des}}$ values of 0.95 to 0.10 in 0.05 increments and the performance measures J1 though J10 obtained from a SIMULINK analysis with the evaluation model. Below an attenuation ratio of $\bar{\gamma}_{\text{des}} = 0.65$, the controllers fail to meet the constraint that the maximum acceleration of the actuator piston is to be less than 6g's. Above $\bar{\gamma}_{\text{des}} = 0.65$ all the actuator effort contraints are met. The stochastic response performance indices, J1 through J5, are approximated by simulating the full

$\bar{\gamma}_{\text{des}}$	J1	J2	J3	J4	J5	σ_u	σ_{x_m}	$\sigma_{\ddot{x}_{am}}$
0.70	0.4931	0.8105	0.1215	0.1268	0.8209	0.0310	0.1592	1.4694
0.40	0.3700	0.5840	0.3068	0.3106	0.6008	0.0834	0.4019	1.0755
0.20	0.2562	0.4032	0.5543	0.5544	0.6374	0.1671	0.7261	1.1409
$\bar{\gamma}_{\text{des}}$	J6	J7	J8	J9	J10	$\max(u)$	$\max(x_m)$	$\max(\ddot{x}_{am})$
0.70	0.5493	1.0376	0.1347	0.1463	1.0986	0.0806	0.3920	5.1221
0.40	0.4952	0.9536	0.3586	0.3661	0.9836	0.2725	1.0881	4.4155
0.20	0.4055	0.7491	0.8353	0.8782	0.9668	0.6929	2.4975	4.8822

Table 2: Dynamic Feedback Performance Indices

evaluation model for 300 seconds using a dominant excitation frequency, ω_g, equal to the first dominant frequency of the idealized continuous closed loop system. The performance metrics are presented for the static output feedback controllers with $\bar{\gamma}_{\text{des}}$ values of 0.9, 0.75 and 0.65 in Table 1.

For the dynamic output feedback controllers, the state feedback controller gains are obtained using the same method as used for the static output feedback controller gains, assuming the controller has complete access to the states of the 12th-order reduced order model. A series of controller gains were again obtained, using $\bar{\gamma}_{\text{des}}$ values of 0.95 to 0.10 in increments of 0.05. A Kalman filter sensor error gain matrix, L, is obtained from the standard LQE method as implemented in the MATLAB Controls Toolbox command LQEW. For the Kalman filter design, the weighting matrices are equivalent to an RMS value of the ground excitation of 0.1g and the RMS values of the sensor noise at 0.01 volts, with the additional assumption that the noise terms are independent. Other weighting matrices can be used in the estimator implementation; however, simulation results did not show a significant improvement in the composite regulator's performance. The resulting continuous regulators are discretized via the Tustin method, and the performance indices determined with MATLAB Simulink analysis. The performance indices for the dynamic output feedback controllers for the 12th order model designed for $\bar{\gamma}_{\text{des}}$ of 0.7, 0.4, and 0.2 are presented in Table 2. The dynamic output controllers designed for $\bar{\gamma}_{\text{des}} < 0.15$ do not meet the maximum absolute actuator acceleration constraint of 6g's.

To compare the effectiveness of the two controller design methods described, Figure 1 plots the sum of J6 and J7 versus the sum of J8 and J9. In this comparison, the structural response is characterized by the normalized maximum structural drift, J6, and the normalized maximum structural acceleration, J7. The actuator effort is characterized by the normalized actuator displacement, J8, and the normalized actuator velocity, J9. Figure 1 shows data points for the controllers from the static output feedback (SOF) and the dynamic output feedback (DOF) methods that meet all the actuator constraints. As is apparent in Figure 1, the dynamic output controllers perform consistently more effectively than the static output controllers considered in this comparison. Additionally, much greater controller effort can be applied before the actuator contraints are not met. Although more efficient dynamic and static output controllers may be designed by more complex modelling, this study quantifies the improvements of controller performance gained by the additional complexity of using a $\mathbf{H}_\infty$ dynamic output feedback controller with the AMD benchmark problem.

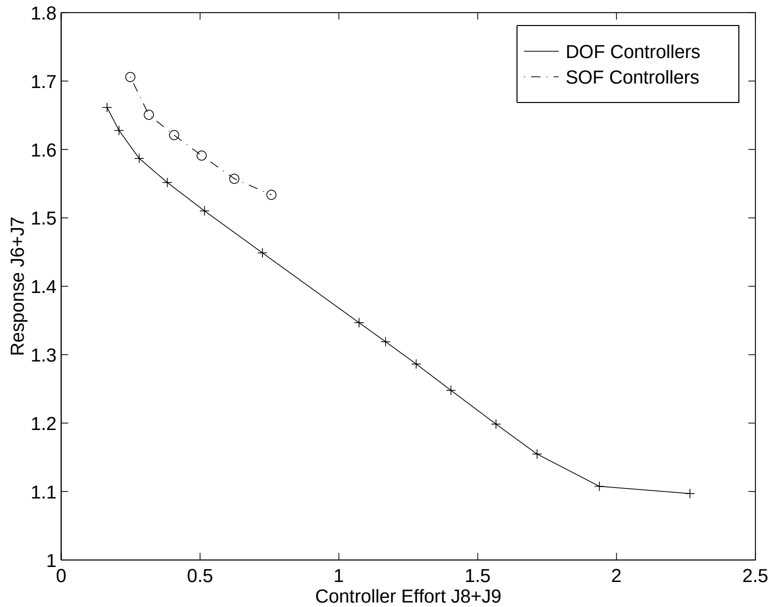

Figure 1: Static Versus Dynamic Output Feedback Efficiency

Conclusions

In summary, this research presented $\mathbf{H}_\infty$ based controller design approaches for the AMD benchmark problem, where both static and dynamic acceleration feedback controllers were studied and compared. The performance of example controllers, which met the contraints posed by the benchmark problem, were compared at various levels of actuator effort. Simulation results indicate that the $\mathbf{H}_\infty$ dynamic output feedback controllers were significantly more efficient than the static output feedback controllers and capable of attenuating the structural response to lower levels of dynamic response. Support for this research was provided in part by the National Science Foundation Presidential Young Investigator Award No. BCS-9058316. In addition, the authors wish to recognize the collaborative research efforts of Professor Rahmat Shoureshi at the Colorado School of Mines.

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