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Benchmark Problems in Structural Control
Part I: Active Mass Driver System

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Introduction

Tremendous progress has been made over the last two decades toward making active structural control a viable technology for enhancing structural functionality and safety against natural hazards such as strong earthquakes and high winds. Over the years, many control algorithms and devices have been investigated, each with its own merits, depending on the particular application and desired effect. Clearly, the ability to make direct comparisons between systems employing these algorithms and devices is necessary to focus future efforts in the most promising directions and to effectively set performance goals and specifications.

This paper presents the overview and problem definition for a benchmark structural control problem that can be used to evaluate the relative effectiveness and implementability of structural control algorithms and to provide an analytical *testbed* for evaluation of various control design issues. The structure considered — chosen because of the widespread interest in this class of systems (Soong 1990; Housner, *et al.* 1994; Fujino, *et al.* 1996) — is a scale model of a three-story building employing an active mass driver. A model for this structural system, including the actuator and sensors, has been developed directly from experimentally obtained data and will form the basis for the benchmark study. Control constraints and evaluation criteria are presented for the design problem. A simulation program has been developed and made available to facilitate comparison of the efficiency and merit of various control strategies. This benchmark problem can be viewed as an initial step toward development of standardized performance evaluation procedures.

Experimental Structure

The structure on which the evaluation model is based is an actively controlled, three-story, single-bay, model building considered in Dyke, *et al.* (1996). The test structure, shown in Fig. 1, is designed to be a scale-model of the prototype building discussed in Chung, *et al.* (1989). The steel building frame is 158 cm tall, with floor masses weighing a total of 227 kg that are distributed evenly between the three floors. The time scale factor is 0.2, making the natural frequencies of the model approximately five times those of the prototype. The first three modes of the model structural system are at 5.81 Hz, 17.68 Hz and 28.53 Hz, with associated damping ratios given, respectively, by 0.33%, 0.23%, and 0.30%. The ratio of model quantities to those corresponding to the prototype structure are: force = 1:60, mass = 1:206, time = 1:5, displacement = 4:29 and acceleration = 7:2.

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For control purposes, a simple implementation of an active mass driver (AMD) was placed on the third floor of the structure. The AMD consists of a single hydraulic actuator with steel masses attached to the ends of the piston rod. For this experiment, the moving mass for the AMD was 5.2 kg. The total mass of the structure, including the frame and the AMD, was 309 kg. Thus, the moving mass of the AMD is 1.7% of the total mass of the structure.

While structural displacements and velocities are difficult to obtain directly in full-scale structures, acceleration measurements can readily be acquired at arbitrary locations on the structure. For this experiment, the measurements directly available for control force determination are the three floor accelerations, the ground acceleration, and the displacement and acceleration of the AMD. Additionally, pseudo absolute velocities are available by passing the measured accelerations through a second order filter that is essentially a high-pass filter in series with an integrator.

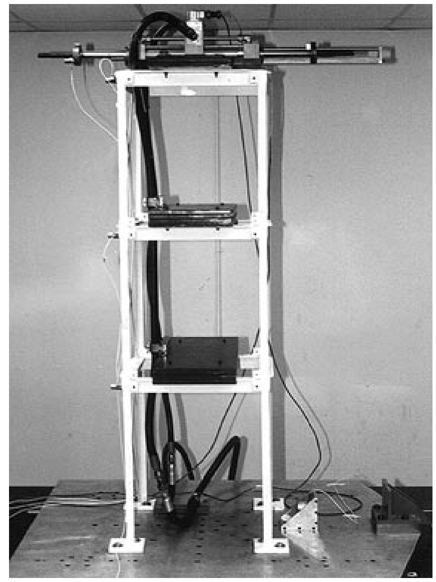

Figure 1. Three Degree-of-Freedom Test Structure with AMD System.

Evaluation Model

A high-fidelity, linear time-invariant state space representation of the input-output model for the structure described in the previous section has been developed. The model has 28 states and is of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{E}\ddot{x}_g \tag{1}$$

$$\mathbf{y} = \mathbf{C}_y\mathbf{x} + \mathbf{D}_y u + \mathbf{F}_y\ddot{x}_g + \mathbf{v}\,, \quad \mathbf{z} = \mathbf{C}_z\mathbf{x} + \mathbf{D}_z u + \mathbf{F}_z\ddot{x}_g \tag{2}$$

where $\mathbf{x}$ is the state vector, $\ddot{x}_g$ is the scalar ground acceleration, u is the scalar control input, $\mathbf{y} = [x_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}, \ddot{x}_g]'$ is the vector of responses that can be directly measured, $\mathbf{z} = [x_1, x_2, x_3, x_m, \dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}]'$ is the vector of responses that can be regulated. Here, x_i is the displacement of the *i*th floor relative to the ground, x_m is the displacement of the AMD relative to the third floor, $\ddot{x}_{ai}$ is the absolute acceleration of the *i*th floor, $\ddot{x}_{am}$ is the absolute acceleration of the AMD mass, and $\mathbf{v}$ is the vector of measurement noises. The coefficient matrices in Eqs. (1–2) are determined from the data collected at the SDC/EEL using the identification methods presented in Dyke, *et al*. (1996). The resulting model represents the input-output behavior of the structural system up to 100 Hz and includes the effects of actuator/sensor dynamics and control-structure interaction. The model given in Eqs. (1–2) is termed the *evaluation* model and will be used to assess the performance of candidate controllers; that is, the evaluation model is considered herein to be the true representation of the structural system.

Control Design Problem

The design problem is to determine a discrete-time, feedback compensator of the form

$$\mathbf{x}^c_{k+1} = f_1(\mathbf{x}^c_k, \underset{\sim}{\mathbf{y}}_k, u_k, k)\,, \quad u_k = f_2(\mathbf{x}^c_k, \underset{\sim}{\mathbf{y}}_k, k) \tag{3}$$

where $\mathbf{x}^c_k$, $\underset{\sim}{\mathbf{y}}_k$ and u_k are the state vector for the compensator, the output vector and the control command, respectively, at time $t = kT$. For this problem, $\dim(\mathbf{x}^c) \le 12$ is required, and the performance of all control designs must be assessed using the evaluation model described previously. For each proposed control design, performance and stability robustness should be discussed. As detailed in the following paragraphs, the merit of a controller will be based on criteria given in terms of both rms and peak response quantities. Normally, smaller values of the evaluation criteria indicate superior performance.

Evaluation Criteria: RMS Responses

Assume that the input excitation $\ddot{x}_g$ is a stationary random process with a spectral density defined by the Kanai-Tajimi spectrum

$$S_{\ddot{x}_g\ddot{x}_g}(\omega) = S_0(4\zeta_g^2\omega_g^2\omega^2 + \omega_g^4)/[(\omega^2 - \omega_g^2)^2 + 4\zeta_g^2\omega_g^2\omega^2] \tag{4}$$

where ω_g and ζ_g are unknown, but assumed to lie in the following ranges: $20 \text{ rad/sec} \le \omega_g \le 120 \text{ rad/sec}$, $0.3 \le \zeta_g \le 0.75$. To have a basis for comparison, the spectral intensity is chosen such that the rms value of the ground motion takes a constant value of $\sigma_{\ddot{x}_g} = 0.12$ g's, *i.e.*, $S_0 = 0.03\zeta_g/[\pi\omega_g(4\zeta_g^2 + 1)]$ g^2·sec.

The first criterion on which controllers will be evaluated is based on their ability to minimize the maximum rms interstory drift due to all admissible ground motions. Therefore, the nondimensionalized measure of performance is given by

$$J_1 = \max_{\omega_g,\, \zeta_g}\left\{\frac{\sigma_{d_1}}{\sigma_{x_{3o}}}, \frac{\sigma_{d_2}}{\sigma_{x_{3o}}}, \frac{\sigma_{d_3}}{\sigma_{x_{3o}}}\right\} \tag{5}$$

where σ_{d_i} is the stationary rms interstory drift for the ith floor, and $\sigma_{x_{3o}} = 1.31$ cm is the worst-case stationary rms displacement of the third floor of the uncontrolled building over the class of excitations considered (occurring when $\omega_g = 37.3$ rad/sec, $\zeta_g = 0.3$). The interstory drifts are given by $d_1(t) = x_1(t)$, $d_2(t) = x_2(t) - x_1(t)$ and $d_3(t) = x_3(t) - x_2(t)$.

A second evaluation criterion is given in terms of the maximum rms absolute acceleration, yielding a performance measure given by

$$J_2 = \max_{\omega_g,\, \zeta_g}\left\{\frac{\sigma_{\ddot{x}_{a1}}}{\sigma_{\ddot{x}_{a3o}}}, \frac{\sigma_{\ddot{x}_{a2}}}{\sigma_{\ddot{x}_{a3o}}}, \frac{\sigma_{\ddot{x}_{a3}}}{\sigma_{\ddot{x}_{a3o}}}\right\} \tag{6}$$

where $\sigma_{\ddot{x}_{ai}}$ is the stationary rms acceleration for the ith floor, and $\sigma_{\ddot{x}_{a3o}} = 1.79$ g's is the worst-case stationary rms acceleration of the third floor of the uncontrolled building (occurring when $\omega_g = 37.3$ rad/sec, $\zeta_g = 0.3$).

The hard constraints for the control effort are given by $\sigma_u \le 1$ volt, $\sigma_{\ddot{x}_{am}} \le 2$ g's and $\sigma_{x_m} \le 3$ cm. Additionally, three quantities, σ_{x_m}, $\sigma_{\dot{x}_m}$ and $\sigma_{\ddot{x}_{am}}$, should be examined to make the assessment of the required control resources. The rms actuator displacement, σ_{x_m}, provides a measure of the required physical size of the device. The rms actuator velocity, $\sigma_{\dot{x}_m}$, provides a measure of the control power required. The rms absolute acceleration $\sigma_{\ddot{x}_{am}}$ provides a measure of the magnitude of the forces that the actuator must generate to execute the commanded control action. Therefore, the nondimensionalized control resource evaluation criteria are

$$J_3 = \max_{\omega_g,\, \zeta_g}\left\{\frac{\sigma_{x_m}}{\sigma_{x_{3o}}}\right\}, \quad J_4 = \max_{\omega_g,\, \zeta_g}\left\{\frac{\sigma_{\dot{x}_m}}{\sigma_{\dot{x}_{3o}}}\right\}, \quad J_5 = \max_{\omega_g,\, \zeta_g}\left\{\frac{\sigma_{\ddot{x}_{am}}}{\sigma_{\ddot{x}_{a3o}}}\right\} \tag{7}$$

where $\sigma_{\dot{x}_{3o}} = 47.9$ cm/sec is the worst-case stationary rms velocity of the third floor relative to the ground for the uncontrolled structure (occurring when $\omega_g = 37.3$ rad/sec, $\zeta_g = 0.3$).

Evaluation Criteria: Peak Responses

Here, the input excitation $\ddot{x}_g$ is assumed to be a historical earthquake record. Both the 1940 El Centro NS record and the NS record for the 1968 Hachinohe earthquake should be considered. Because the system under consideration is a scale model, the time scale should be increased by a factor of 5. The required scaling of the magnitude of the ground acceleration is 3.5. The evaluation criterion is based on minimization of the nondimensionalized peak interstory drifts due to both earthquake records. For each earthquake, the maximum drifts are nondimensionalized with respect to the uncontrolled peak third floor displacement, denoted x_{3o}, relative to the ground. Therefore, the performance measure is given by

$$J_6 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \left\{ \frac{|d_1(t)|}{x_{3o}}, \frac{|d_2(t)|}{x_{3o}}, \frac{|d_3(t)|}{x_{3o}} \right\} \right] \tag{8}$$

A second performance evaluation criterion is given in terms of the peak acceleration, yielding

$$J_7 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \left\{ \frac{|\ddot{x}_{a1}(t)|}{\ddot{x}_{a3o}}, \frac{|\ddot{x}_{a2}(t)|}{\ddot{x}_{a3o}}, \frac{|\ddot{x}_{a3}(t)|}{\ddot{x}_{a3o}} \right\} \right] \tag{9}$$

where the accelerations are nondimensionalized by the peak uncontrolled third floor acceleration, denoted $\ddot{x}_{a3o}$, corresponding respectively to each earthquake.

The control constraints are $\max_t |u(t)| \le 3$ volts, $\max_t |x_m(t)| \le 9$ cm, $\max_t |\ddot{x}_{am}(t)| \le 6$ g's, and both the El Centro and the Hachinohe earthquakes should again be considered. Additionally, the candidate controllers are to be evaluated in terms of the required control resources as follows

$$J_8 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|x_m(t)|}{x_{3o}} \right], \quad J_9 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|\dot{x}_m(t)|}{\dot{x}_{3o}} \right], \tag{10}$$

$$J_{10} = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|\ddot{x}_{am}(t)|}{\ddot{x}_{a3o}} \right] \tag{11}$$

where $\dot{x}_{3o}$ is the peak uncontrolled third floor relative velocity corresponding respectively to each earthquake.

For the El Centro earthquake, $x_{3o} = 3.37$ cm, $\dot{x}_{3o} = 131$ cm/sec and $\ddot{x}_{a3o} = 5.05$ g's. For the Hachinohe earthquake, $x_{3o} = 1.66$ cm, $\dot{x}_{3o} = 58.3$ cm/sec and $\ddot{x}_{a3o} = 2.58$ g's.

Control Implementation Constraints

To make the benchmark problem as realistic as possible, the following implementation constraints are placed on the system:

1. As indicated previously, the measurements that are directly available for use in determination of the control action are $\mathbf{y} = [x_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}, \ddot{x}_g]'$. Although absolute velocities are not available, they can be closely approximated by passing the measured accelerations through a second order filter with the following transfer function

$$H_{\tilde{\dot{x}}\ddot{x}}(s) = 39.5s/(39.5s^2 + 8.89s + 1) \tag{12}$$

where $\tilde{\dot{x}}$ is the pseudo velocity response in that it will track the absolute velocity response above 1 Hz. Therefore, the pseudo velocities, $\tilde{\dot{x}}_{a1}, \tilde{\dot{x}}_{a2}, \tilde{\dot{x}}_{a3}, \tilde{\dot{x}}_{am}, \tilde{\dot{x}}_g$, are also available for determination of the control action, and the combined output vector is given by $\tilde{\mathbf{y}} = [x_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}, \ddot{x}_g, \tilde{\dot{x}}_{a1}, \tilde{\dot{x}}_{a2}, \tilde{\dot{x}}_{a3}, \tilde{\dot{x}}_{am}, \tilde{\dot{x}}_g]'$.

2. The controller for the structure is digitally implemented with a sampling time of $T = 0.001$ sec.
3. A computation delay of 200 μsec is required to perform the *D*-matrix calculations in the control action determination and for the associated A/D and D/A conversions.
4. The controller A/D and D/A converters have 12-bit precision and a span of ± 3 V.
5. Each of the measured responses contains an rms noise of 0.01 Volts, which is approximately 0.3% of the full span of the A/D converters. The measurement noises are modeled as Gaussian rectangular pulse processes with a pulse width of 0.001 seconds.
6. To account for limited computational resources in the digital controller, the controller given in Eq. (3) is restricted to have no more than 12 states.
7. The performance of each control design should be evaluated using the 28 state evaluation model given in Eqs. (1–2).
8. The controller given in Eq. (3) is required to be stable.

A SIMULINK (1994) model has been developed to simulate the features and limitations of this structural control problem. Note that, although the controller is digital, the structure is still modeled as a continuous system. To reduce integration errors, a time step of 0.0001 sec is used in the simulation.

Closure

The numerical models, the input data, the simulation model and an extended version of this paper are available on the World Wide Web at: *http://www.nd.edu/~quake/*
Questions regarding the benchmark problem can be directed to the senior author via e-mail at: spencer.1@nd.edu.

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