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Application of Sliding Mode Control to a Benchmark Problem*

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Abstract

In this paper, both the methods of continuous sliding mode control (*CSMC*) and continuous sliding mode control with a compensator (*CSMC&C*) are applied to a benchmark problem; namely, an active mass driver system. For these control strategies, salient features of the controller design and their merit are described. Simulation results based on *CSMC* and *CSMC&C* are presented and compared with that of the *LQG* method. It is demonstrated that the control performances of *CSMC* and *CSMC&C* are quite comparable to that of *LQG*.

Introduction

The theory of sliding mode control (*SMC*) or variable structure system (*VS*S) was developed for robust control of uncertain nonlinear systems. Applications of continuous sliding mode control (*CSMC*) that does not have chattering effect to the following seismic-excited structures have been studied: (i) linear and nonlinear or hysteretic buildings [Yang et al 1994a, 1995a], (ii) sliding isolated buildings [Yang et al 1996a], and (iii) parametric control, such as the use of active variable dampers (AVD) on bridges [Yang et al 1995b] and active variable stiffness (AVS) systems [Yang et al 1996c]. In addition to full state feedback controllers, static output feedback controllers using only a limited number of sensors installed at strategic locations were also presented in the studies above. Shaking table experimental verifications of the *CSMC* methods for linear and sliding-isolated building models have been conducted [Yang et al 1996a, b]. Based on the simulations and experimental results, it was demonstrated that the continuous sliding mode control methods are robust and their performances are quite remarkable.

Recently, a technique for designing sliding mode controllers by introducing a fixed-order compensator using the linear quadratic optimal control theory (*LQR*) has been presented [Yang et al 1994b]. The main advantages of using a fixed-order compensator in sliding mode control are as follows: (i) the static output feedback

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controller can be designed systematically, and (ii) the modulation of the response quantities and control efforts can be made easily in a systematic manner for both the full-state and static output feedback controllers.

In this paper, both the methods of *CSMC* and *CSMC&C* are applied to a benchmark problem [Spencer et al 1997] for the evaluation of their performances. For the *CSMC* method, an observer described in Spencer et al (1997) is needed to estimate the state variables of the design model. For the *CSMC&C* method, on the other hand, an observer is not needed; however, a first order filter is implemented to each measurement (feedback) to facilitate the static output controller design. Simulation results based on *CSMC* and *CSMC&C* are presented and their performances are compared with that of the *LQG* method.

Formulation

The evaluation model representing the structure of benchmark problem is given in Spencer et al (1997). For controller design, the design˝model is expressed as

$$\dot{x}_r = A_r x_r + B_r u + E_r \ddot{x}_g \tag{1}$$

$$z_r = C_{zr} x_r + D_{zr} u + F_{zr} \ddot{x}_g \tag{2}$$

$$y_r = C_{yr} x_r + D_{yr} u + F_{yr} \ddot{x}_g \tag{3}$$

in which x_r is a 10-state vector, z_r is a 12-control output vector, and y_r is a *m* measured output vector.

Continuous Sliding Mode Control (CSMC): The *CSMC* controller is given by [Yang et al 1994a]

$$u = K_b \, \hat{x}_r + K_f \, \ddot{x}_g \tag{4}$$

in which K_b and K_f are feedback and feedforward gain matrices, respectively,

$$K_b = -\,(P\,B_r)^{-1} P\,A_r - \delta\,B_r'\,P'\,P \;; \qquad K_f = -\,(P\,B_r)^{-1} P\,E_r \tag{5}$$

In Eqs.(5), $\delta > 0$ is the gain margin and *P* is the (1x10) sliding surface coefficient matrix that can be determined by minimizing the objective function *J*,

$$J = \int_0^\infty x_r'\,C_{zr}'\,Q\,C_{zr}\,x_r\,\mathrm{dt} \tag{6}$$

where *Q* is a (12x12) weighting matrix [Yang et al 1994a].

The estimated state vector $\hat{x}_r$ for the design model is obtained from the Kalman-Bucy filter as follows [Spencer et al 1997]

$$\dot{\hat{x}}_r = A_r \hat{x}_r + B_r u + L(y_r - C_{yr}\hat{x}_r - D_{yr} u) \tag{7}$$

in which *L* is the observer gain.

Continuous Sliding Mode Control With Compensator (CSMC&C)*:* For *CSMC&C*, a first order filter is introduced to the output feedback vector y_r

$$\dot{\eta} = A_\eta \eta + B_\eta \, y_r \tag{8}$$

where η is a m-vector representing the new output feedback vector. Hence, combining with Eqs.(1) and (8), the augmented design model becomes a system of (10+*m*) state equations with $\tilde{x}_r = [x_r', \eta']'$ as the (10+*m*) augmented state vector.

The control output z_r and the new output feedback η for the augmented system become

$$z_r = \tilde{C}_{zr}\,\tilde{x}_r + D_{zr}\,u + F_{zr}\,\ddot{x}_g \;; \qquad \eta = \tilde{C}_{yr}\,\tilde{x}_r \tag{9}$$

where $\tilde{C}_{zr} = [C_{zr}\ ,\ 0]$ and $\tilde{C}_{yr} = [0\ ,\ I_m\ ]$ with I_m being a (*m*x*m*) identity matrix.

A compensator with a 2-dimensional state vector $q = [q_1\ ,\ q_2]'$ is introduced as

$$\dot{q}_1 = L_{11}\,q_1 + L_{12}\,q_2 + N_1\,\eta \tag{10}$$

$$\dot{q}_2 = L_{21}\,q_1 + L_{22}\,q_2 + N_2\,\eta + D_2\,u \tag{11}$$

and the sliding surface is expressed in terms of the compensator˝variables

$$S = P_1\,q_1 + P_2\,q_2 \tag{12}$$

In Eqs.(10)-(12), P_1, P_2, L_{11}, L_{21}, L_{12}, L_{22}, N_1, N_2 and D_2 are determined by minimizing the following objective function [Yang et al 1994b]

$$\hat{J} = \mathrm{E}\left[\int_0^\infty \bar{z}_r'\,Q_z\,\bar{z}_r + \eta'\,Q_\eta\,\eta + q_1'\,Q_{q_1}\,q_1 + u_{eq}'\,R_u\,u_{eq} + \dot{q}_1'\,R_{\dot{q}_1}\,\dot{q}_1\,\mathrm{dt}\right] \tag{13}$$

in which $\bar{z}_r = C_{zr}\,x_r + D_{zr}\,u_{eq}$ and u_{eq} is the equivalent control force given by $u_{eq} = G\,\eta + H\,q_1$, $G = -(P_2\,D_2)^{-1}\ (P_1\,N_1 + P_2\,N_2)$, $H = -(P_2\,D_2)^{-1}\ [P_1(L_{11} - L_{12}\,P_2^{-1}P_1) + P_2(L_{21} - L_{22}\,P_2^{-1}P_1)]$. The minimization procedures result in the *LQR* static output feedback in which iterative procedures are needed to solve nonlinear equations. The resulting *CSMC&C* controller is given by [Yang et al 1994b]

$$u = u_{eq} - [M_{c1} + (P_2\,D_2)^{-1}\delta\,P_1]\,q_1 - [M_{c1} + (P_2\,D_2)^{-1}\delta\,P_2]\,q_2 - D_2^{-1}\,E_{12} \tag{14}$$

in which $M_{c1} = (P_2\,D_2)^{-1}\,(P_1\,L_{12}\,P_2^{-1}\,P_1 + P_2\,L_{22}\,P_2^{-1}P_1)$, $M_{c2} = (P_2\,D_2)^{-1}\,(P_1\,L_{12} + P_2\,L_{22})$ and $\delta > 0$ is the gain margin.

Simulation Results

Numerical simulations were conducted using the MATLAB SIMULINK program for the evaluation model. Only the simulation results for the El Centro earthquake excitation are presented. For each control strategy, three different design cases are considered; namely, 5-sensor, 3-sensor and 1-sensor. The output feedback quantities for the three cases are as follows: (i) for 5-sensor case, $\mathrm{y_r} = [x_m,\ \ddot{x}_{a1},\ \ddot{x}_{a2},\ \ddot{x}_{a3},\ \ddot{x}_{am}]'$, (ii) for 3-sensor case, $y_r = [\ddot{x}_{a1},\ \ddot{x}_{a2},\ \ddot{x}_{a3}]'$, and (iii) for 1-sensor case, $\mathrm{y_r} = \ddot{\mathrm{x}}_{a3}$. Further, for the fairness of comparison, the feedforward compensation of the *CSMC* and *CSMC&C* was ignored. The design model, Eqs.(1)-(3), constructed by Spencer et al (1997) was used for the *LQG* controllers. The design model constructed by the 'balreal' and 'modred' function in MATLAB CONTROL SYSTEM TOOLBOX was used for the *CSMC* and *CSMC&C* controllers.

For *CSMC* controllers, control parameters are as follows; (i) 5-sensor case: *Q*=diag[1600, 1100, 1100, 0, 0, 0, 0, 70, 10, 15, 15, 1], δ=40, (ii) 3-sensor case: *Q*=diag[1100, 1100, 1100, 0, 0, 0, 0, 110, 10, 15, 15, 0], δ=40, and (iii) 1-sensor case: *Q* = diag[1500, 1100, 1100, 0, 10, 0, 0, 70, 10, 15, 15, 20], δ=40. For the observer, we consider γ = 25 as used in Spencer et al (1997).

For *CSMC&C* controllers, the control design parameters are as follows; (i) 5-sensor case: Q_z = diag[5500, 5500, 5500, 0, 0, 0, 0, 100, 10, 10, 10, 1500], Q_{q_1} =1, Q_η = 0, R_u = 0.1, $R_{\dot{q}_1}$ = 0.1, L_{12} = -1, L_{22} = -0.001, P_1 = 1, P_2 = 1000, D_2 = 1 and $\delta = 10^7$; (ii) 3-sensor case: Q_z = diag[8000, $8\,\mathrm{x}10^4$, 8000, 0, 0, 0, 0, 200, 10, 10, 10, 2700] and all other parameters are identical to case (i), and (iii) 1-sensor case : choose G = -1.38 and all other parameters are identical to case (i). For all the *CSMC&C* controllers above, the filter dynamics is constructed based on $A_\eta = -10\, I_m$ and $B_\eta = I_m$, where I_m is an (mxm) identity matrix.

We also conducted the *LQG* designs tuning to the control output z_r for comparisons as follows; (i) 5-sensor case: Q = diag[130, 100, 100, 0, 0, 0, 0, 0, 1, 1, 10, 68], R = 0.1, (ii) 3-sensor case: Q = diag[32, 10, 10, 0, 0, 0, 0, 0, 1, 1, 1, 5], R = 10, and (iii) 1-sensor case: Q = diag[50, 43, 43, 0, 0, 0, 0, 0, 1, 1, 10, 76], R = 0.1.

Within 10 seconds of the El Centro earthquake episode, peak response quantities of the evaluation model are presented in Table 1 for different controllers. In Table 1, J_6 and J_7 are max $|d_i| / x_{3o}$ and max $|\ddot{x}_{ai}| / \ddot{x}_{a3o}$ for i = 1, 2, 3, respectively. As observed from Table 1, the control performances for three control methods, i.e., *LQG*, *CSMC* and *CSMC&C* are quite comparable.

Conclusion and Discussion

The methods of continuous sliding mode control (*CSMC*) and continuous sliding mode control with a compensator (*CSMC&C*) have been applied to the benchmark active mass driver system. Simulation results indicate that the control performances of *LQG*, *CSMC* and *CSMC&C* are quite comparable. Due to the specific identification scheme used in the benchmark problem such that the state variables are fictitious and the output measurement $\mathrm{y_r}$ involves both the control signal $u(t)$ and the earthquake excitation $\ddot{x}_g$, the design of *CSMC* and *CSMC&C* controllers becomes more involved. The performances of *CSMC* and *CSMC&C* controllers may have been compromised because of the particular identification scheme used to construct the evaluation model.

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Table 1: Peak Response Quantities Subject to the El Centro Earthquake.

Quantities	*LQG*			*CSMC*			*CSMC&C*		
	Story			Story			Story		
	1	2	3	1	2	3	1	2	3
Five-Sensor Case, $y_r = [x_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}]'$									
J_6	0.29	0.17	0.14	0.30	0.17	0.13	0.32	0.19	0.13
J_7	0.27	0.43	0.45	0.28	0.44	0.46	0.47	0.50	0.63
J_8 (x_m in cm)	1.20 (4.127)			1.20 (4.148)			1.23 (4.229)		
J_9 ($\dot{x}_m$ in cm/sec.)	1.24 (162.4)			1.22 (160.2)			1.26 (165.8)		
J_{10} ($\ddot{x}_m$ in g)	1.11 (5.61)			1.17 (5.89)			1.16 (5.83)		
max $\lvert u(t) \rvert$ (Volt)	1.147			1.151			1.223		
Three-Sensor Case, $y_r = [\ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}]'$									
J_6	0.29	0.17	0.15	0.29	0.16	0.14	0.30	0.18	0.14
J_7	0.26	0.41	0.45	0.27	0.42	0.45	0.42	0.47	0.59
J_8 (x_m in cm)	1.28 (4.397)			1.27 (4.368)			1.16 (4.000)		
J_9 ($\dot{x}_m$ in cm/sec.)	1.28 (168.15)			1.29 (168.83)			1.22 (159.43)		
J_{10} ($\ddot{x}_m$ in g)	1.16 (5.85)			1.15 (5.83)			1.19 (5.99)		
max $\lvert u(t) \rvert$ (Volt)	1.214			1.213			1.144		
One-Sensor Case, $y_r = [\ddot{x}_{a3}]$									
J_6	0.30	0.18	0.13	0.30	0.17	0.13	0.32	0.21	0.12
J_7	0.28	0.44	0.50	0.28	0.42	0.49	0.41	0.48	0.67
J_8 (x_m in cm)	1.16 (4.000)			1.18 (4.047)			0.90 (3.105)		
J_9 ($\dot{x}_m$ in cm/sec.)	1.18 (154.21)			1.20 (157.65)			1.03 (135.57)		
J_{10} ($\ddot{x}_m$ in g)	1.18 (5.95)			1.18 (5.94)			1.18 (5.94)		
max $\lvert u(t) \rvert$ (Volt)	1.106			1.121			0.911		