Structural Control: A Benchmark Comparison

To appear as a special issue of
Earthquake Engineering and Structural Dynamics

Background

Many algorithms and devices have been proposed for control of civil engineering structures, each with its own advantages, depending on the specific application and desired effect. To date, a definitive study demonstrating the pros and cons of the different approaches is unavailable. Clearly, the ability to make direct comparisons between strategies employing various algorithms and devices is necessary to focus future efforts in the most promising directions and to effectively set performance goals and specifications.

The ASCE Committee on Structural Control has initiated this activity by sponsoring a benchmark structural control session at the 1997 ASCE Structures Congress, to be held in Portland, Oregon. Significant input from both the Committee on Structural Control and the session participants was sought in developing the definition of the benchmark problems to be studied (an active mass driver system and an active tendon system). The session participants will report on the effectiveness of various control strategies applied to common analytical models of a structural systems and discuss the ability of their approaches to meet prescribed performance specifications. The analytical models considered accurately reflect full-scale structures, control devices and sensors.

Call for Contributors

To obtain broader participation in the this benchmark study and wider distribution of the associated results than is possible through a single conference session, a special issue of *Earthquake Engineering and Structural Dynamics* has been arranged to report the results of the benchmark study. By focusing on issues of full-scale implementability and performance, the special issue is expected to be of interest to both practitioners wanting to learn about the emerging field of structural control, as well as researchers and students currently engaged in system development.

All researchers are welcome to study the structural control benchmark problem presented for this session. A tentative schedule for this project is as follows:

- May 31, 1996: Secure the contributors to the special journal issue
- January 1, 1997: Manuscripts due
- March 15, 1997: Completion of the regular review process
- Late 1997: Publication of the special journal issue

Researchers interested in contributing to this special issue of *Earthquake Engineering and Structural Dynamics* and can meet the prescribed schedule, please send an e-mail response at your earliest convenience (spencer.1@nd.edu). Of course, all papers will be reviewed following standard procedures of the Journal.

For more information, contact Prof. B.F. Spencer, Jr. via e-mail at: spencer.1@nd.edu

Benchmark Problems in Structural Control
Part I: Active Mass Driver System

B.F. Spencer Jr.,[1] S.J. Dyke[2] and H.S. Deoskar[2]

Abstract

This paper presents the overview and problem definition for a benchmark structural control problem. The structure considered — chosen because of the widespread interest in this class of systems (Soong 1990; Housner, *et al.* 1994b; Fujino, *et al.* 1996) — is a scale model of a three-story building employing an active mass driver. A model for this structural system, including the actuator and sensors, has been developed directly from experimentally obtained data and will form the basis for the benchmark study. Control constraints and evaluation criteria are presented for the design problem. A simulation program has been developed and made available to facilitate comparison of the efficiency and merit of various control strategies. A sample control design is given to illustrate some of the design challenges.

Introduction

Tremendous progress has been made over the last two decades toward making active structural control a viable technology for enhancing structural functionality and safety against natural hazards such as strong earthquakes and high winds. The success of the First World Conference on Structural Control, held in Pasadena, California in August 1994, demonstrated the world-wide interest in structural control (Housner, *et al.* 1994a). The Conference attracted over 300 participants from 35 countries. Over the years, many control algorithms and devices have been investigated, each with its own merits, depending on the particular application and desired effect. Clearly, the ability to make direct comparisons between systems employing these algorithms and devices is necessary to focus future efforts in the most promising directions and to effectively set performance goals and specifications. Indeed, the development of guide-

---

1. Prof., Dept. of Civil Engrg. and Geo. Sci., Univ. of Notre Dame, Notre Dame, IN 46556. M. ASCE.
2. Grad. Assist., Dept. of Civil Engrg. and Geo. Sci., Univ. of Notre Dame, Notre Dame, IN 46556. Stud. M. ASCE.

lines governing both performance and implementability is a meaningful and important task in itself.
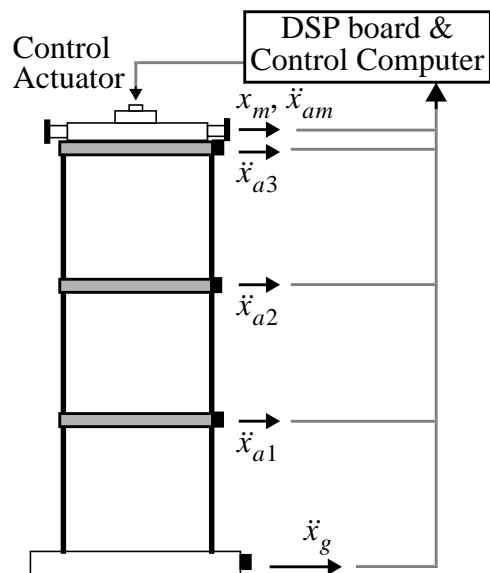
This paper presents a benchmark structural control problem that can be used to evaluate the relative effectiveness and implementability of various structural control algorithms and to provide an analytical *testbed* for evaluation of control design issues such as model order reduction, spillover, control-structure interaction, limited control authority, sensor noise, available measurements, computational delay, *etc.* To achieve a high level of realism, an *evaluation* model is presented in the problem definition which is derived directly from experimental data obtained at the Structural Dynamics and Control/Earthquake Engineering Laboratory (SDC/EEL) at the University of Notre Dame. This model accurately represents the behavior of the laboratory structure and fully incorporates actuator/sensor dynamics. Herein, the evaluation model will be considered as the real structural system. In general, controllers that are successfully implemented on the evaluation model can be expected to perform similarly in the laboratory setting. Several evaluation criteria are given, along with the associated control design constraints. A sample control design is presented to illustrate some of the design challenges. This benchmark problem can be viewed as an initial step toward development of standardized performance evaluation procedures.
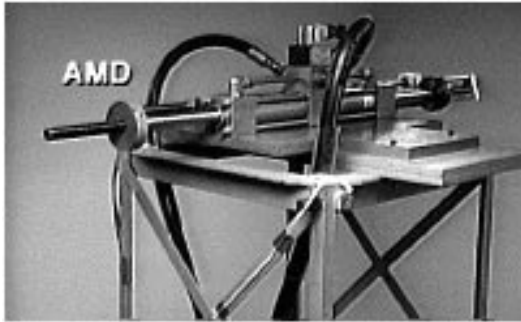
Experimental Structure

The structure on which the evaluation model is based is an actively controlled, three-story, single-bay, model building considered in Dyke, *et al.* (1994a, 1995a). The test structure, shown in Figs. 1 and 2, is designed to be a scale model of the prototype
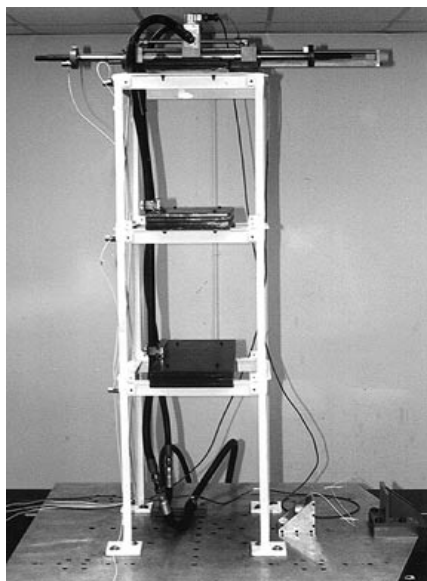
Figure 1. Three Degree-of-Freedom Test Structure with AMD System.

Figure 2. Schematic Diagram of Experimental Setup.

Figure 3. Active Mass Driver.

building discussed in Chung, *et al.* (1989) and is subject to one-dimensional ground motion. The building frame is constructed of steel, with a height of 158 cm. The floor masses of the model weigh a total of 227 kg, distributed evenly between the three floors. The time scale factor is 0.2, making the natural frequencies of the model approximately five times those of the proto-type. The first three modes of the model structural system are at 5.81 Hz, 17.68 Hz and 28.53 Hz, with associated damping ratios given, respectively, by 0.33%, 0.23%, and 0.30%. The ratio of model quantities to those corresponding to the prototype structure are: force = 1:60, mass = 1:206, time = 1:5, displacement = 4:29 and acceleration = 7:2.

For control purposes, a simple implementation of an active mass driver (AMD) was placed on the third floor of the structure. The AMD consists of a single hydraulic actuator with steel masses attached to the ends of the piston rod (see Fig. 3). The servo-actuated hydraulic cylinder has a 3.8 cm diameter and a 30.5 cm stroke. For this experiment, the moving mass for the AMD was 5.2 kg and consisted of the piston, piston rod and the steel disks bolted to the end of the piston rod. The total mass of the structure, including the frame and the AMD, was 309 kg. Thus, the moving mass of the AMD is 1.7% of the total mass of the structure. Because hydraulic actuators are inherently open loop unstable, position feedback was employed to stabilize the control actuator. The position of the actuator was obtained from an LVDT (linear variable differential transformer), rigidly mounted between the end of the piston rod and the third floor.

Structural displacements and velocities are difficult to obtain directly in full scale structures, because they must be measured relative to an inertial reference frame. Alternatively, acceleration measurements can readily be acquired at arbitrary locations on the structure. For this experiment, accelerometers were positioned on the ground, on each floor of the structure, and on the AMD, as shown in Fig. 2**l**. The displacement of the AMD relative to the third floor was also measured using the LVDT mentioned above. Thus, the measurements that are directly available for control force determination are the three floor acceleration measurements, the ground acceleration, and the displacement and acceleration of the AMD (see Fig. 2). Additionally, pseudo absolute velocities are available by passing the measured accelerations through a second order filter that is essentially a high-pass filter in series with an integrator.

Evaluation Model

A high-fidelity, linear time-invariant state space representation of the input-output model for the structure described in the previous section has been developed. The model has 28 states and is of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{E}\ddot{x}_g \tag{1}$$

$$\mathbf{y} = \mathbf{C}_y\mathbf{x} + \mathbf{D}_y u + \mathbf{F}_y\ddot{x}_g + \mathbf{v} \tag{2}$$

$$\mathbf{z} = \mathbf{C}_z\mathbf{x} + \mathbf{D}_z u + \mathbf{F}_z\ddot{x}_g \tag{3}$$

where $\mathbf{x}$ is the state vector, $\ddot{x}_g$ is the scalar ground acceleration, u is the scalar control input, $\mathbf{y} = [x_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}, \ddot{x}_g]'$ is the vector of responses that can be directly measured, $\mathbf{z} = [x_1, x_2, x_3, x_m, \dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}]'$ is the vector of responses that can be regulated. Here, x_i is the displacement of the ith floor relative to the ground, x_m is the displacement of the AMD relative to the third floor, $\ddot{x}_{ai}$ is the absolute acceleration of the ith floor, $\ddot{x}_{am}$ is the absolute acceleration of the AMD mass, $\mathbf{v}$ is the vector of measurement noises, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{E}$, $\mathbf{C}_y$, $\mathbf{D}_y$, $\mathbf{C}_z$, $\mathbf{D}_z$, $\mathbf{F}_y$, and $\mathbf{F}_z$ are matrices of appropriate dimension. The coefficient matrices in Eqs. (1–3) are determined from the data collected at the SDC/EEL using the identification methods presented in Dyke, *et al.* (1994a,b, 1995a,b). The resulting model represents the input-output behavior of the structural system up to 100 Hz and includes the effects of actuator/sensor dynamics and control-structure interaction. Figure 4 provides a representative comparison between the model from Eqs. (1–3) and the experimental data. Note that the experimental data was obtained using a 16-bit data acquisition board. Thus, the difference between the experimental data and the model in Fig. 4a at low frequencies is due to the finite precision of the data acquisition system.

The model given in Eqs. (1–3) is termed the *evaluation* model and will be used to assess the performance of candidate controllers; that is, the evaluation model is considered herein to be the true representation of the structural system.

Control Design Problem

The control design problem is to determine a discrete-time, feedback compensator of the form

$$\mathbf{x}^c_{k+1} = f_1(\mathbf{x}^c_k, \underset{\sim}{\mathbf{y}}_k, u_k, k) \tag{4}$$

$$u_k = f_2(\mathbf{x}^c_k, \underset{\sim}{\mathbf{y}}_k, k) \tag{5}$$

where $\mathbf{x}_k^c$, $\underset{\sim}{\mathbf{y}}_k$ and u_k are the state vector for the compensator, the output vector and the control command, respectively, at time $t = kT$. For this problem, $\dim(\mathbf{x}^c) \le 12$ is required, and the performance of all control designs must be assessed using the evaluation model described previously. For each proposed control design, performance and stability robustness should be discussed. As detailed in the following paragraphs, the merit of a controller will be based on criteria given in terms of both rms and peak response quantities. Normally, smaller values of the evaluation criteria indicate superior performance.

Evaluation Criteria: RMS Responses

Assume that the input excitation $\ddot{x}_g$ is a stationary random process with a spectral density defined by the Kanai-Tajimi spectrum

$$S_{\ddot{x}_g \ddot{x}_g}(\omega) = \frac{S_0(4\zeta_g^2 \omega_g^2 \omega^2 + \omega_g^4)}{(\omega^2 - \omega_g^2)^2 + 4\zeta_g^2 \omega_g^2 \omega^2} \tag{6}$$

where ω_g and ζ_g are unknown, but assumed to lie in the following ranges: $20 \text{ rad/sec} \le \omega_g \le 120 \text{ rad/sec}$, $0.3 \le \zeta_g \le 0.75$. To have a basis for comparison, the spectral intensity is chosen such that the rms value of the ground motion takes a constant value of $\sigma_{\ddot{x}_g} = 0.12$ g's, *i.e.,*

$$S_0 = \frac{0.03\zeta_g}{\pi\omega_g(4\zeta_g^2 + 1)} \; g^2 \cdot \sec \tag{7}$$
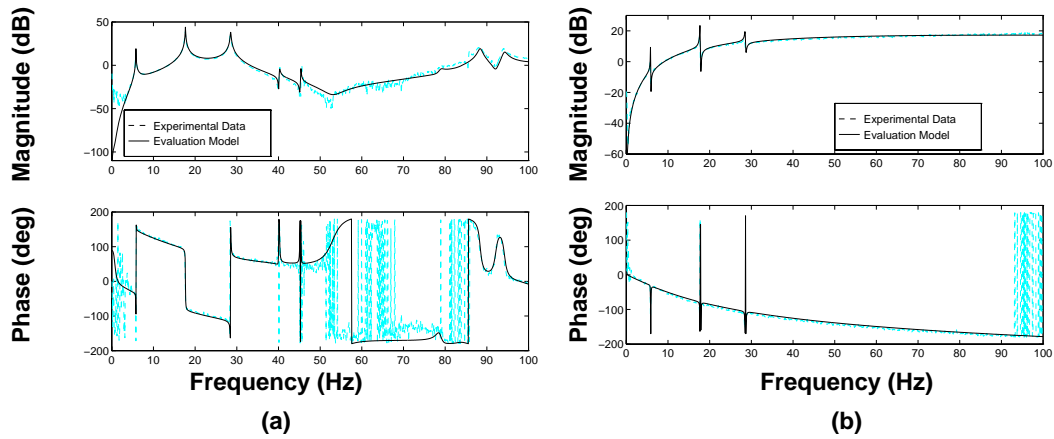
Figure 4. Representative Comparison of the Transfer Functions for the Test Structure and the Evaluation Model. (a) Actuator Command to 1st Floor Absolute Acceleration, (b) Actuator Command to AMD Absolute Acceleration.

The first criterion on which controllers will be evaluated is based on their ability to minimize the maximum rms interstory drift due to all admissible ground motions. Therefore, the nondimensionalized measure of performance is given by

$$
J_1 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{d_1}}{\sigma_{x_{3o}}}, \frac{\sigma_{d_2}}{\sigma_{x_{3o}}}, \frac{\sigma_{d_3}}{\sigma_{x_{3o}}} \right\}
\tag{8}
$$

where σ_{d_i} is the stationary rms interstory drift for the ith floor, and $\sigma_{x_{3o}} = 1.31$ cm is the worst-case stationary rms displacement of the third floor of the uncontrolled building over the class of excitations considered (occurring when $\omega_g = 37.3$ rad/sec, $\zeta_g = 0.3$). The interstory drifts are given respectively by $d_1(t) = x_1(t)$, $d_2(t) = x_2(t) - x_1(t)$ and $d_3(t) = x_3(t) - x_2(t)$.

A second evaluation criterion is given in terms of the maximum rms absolute acceleration, yielding a performance measure given by

$$
J_2 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{\ddot{x}_{a1}}}{\sigma_{\ddot{x}_{a3o}}}, \frac{\sigma_{\ddot{x}_{a2}}}{\sigma_{\ddot{x}_{a3o}}}, \frac{\sigma_{\ddot{x}_{a3}}}{\sigma_{\ddot{x}_{a3o}}} \right\}
\tag{9}
$$

where $\sigma_{\ddot{x}_{ai}}$ is the stationary rms acceleration for the ith floor, and $\sigma_{\ddot{x}_{a3o}} = 1.79$ g's is the worst-case stationary rms acceleration of the third floor of the uncontrolled building (occurring when $\omega_g = 37.3$ rad/sec, $\zeta_g = 0.3$).

The hard constraints for the control effort are given by $\sigma_u \leq 1$ volt, $\sigma_{\ddot{x}_{am}} \leq 2$ g's and $\sigma_{x_m} \leq 3$ cm. Additionally, candidate controllers are to be evaluated based on the required control resources. Three quantities, σ_{x_m}, $\sigma_{\dot{x}_m}$ and $\sigma_{\ddot{x}_{am}}$, should be examined to make the assessment. The rms actuator displacement, σ_{x_m}, provides a measure of the required physical size of the device. The rms actuator velocity, $\sigma_{\dot{x}_m}$, provides a measure of the control power required. The rms absolute acceleration $\sigma_{\ddot{x}_{am}}$ provides a measure of the magnitude of the forces that the actuator must generate to execute the commanded control action. Therefore, the nondimensionalized control resource evaluation criteria are

$$
J_3 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{x_m}}{\sigma_{x_{3o}}} \right\}
\tag{10}
$$

$$
J_4 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{\dot{x}_m}}{\sigma_{\dot{x}_{3o}}} \right\}
\tag{11}
$$

$$
J_5 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{\ddot{x}_{am}}}{\sigma_{\ddot{x}_{a3o}}} \right\}
\tag{12}
$$

where $\sigma_{\dot{x}_{3o}} = 47.9$ cm/sec is the worst-case stationary rms velocity of the third floor relative to the ground for the uncontrolled structure (occurring when $\omega_g = 37.3$ rad/sec, $\zeta_g = 0.3$).

Evaluation Criteria: Peak Responses

Here, the input excitation $\ddot{x}_g$ is assumed to be a historical earthquake record. Both the 1940 El Centro NS record and the NS record for the 1968 Hachinohe earthquake should be considered. Because the system under consideration is a scale model, the time scale should be increased by a factor of 5 (*i.e.*, the earthquakes occur in 1/5 the recorded time). The required scaling of the magnitude of the ground acceleration is 3.5. The evaluation criterion is based on minimization of the nondimensionalized peak interstory drifts due to both earthquake records. For each earthquake, the maximum drifts are nondimensionalized with respect to the uncontrolled peak third floor displacement, denoted x_{3o}, relative to the ground. Therefore, the performance measure is given by

$$J_6 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \left\{ \frac{|d_1(t)|}{x_{3o}}, \frac{|d_2(t)|}{x_{3o}}, \frac{|d_3(t)|}{x_{3o}} \right\} \right] \tag{13}$$

A second performance evaluation criterion is given in terms of the peak acceleration, yielding

$$J_7 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \left\{ \frac{|\ddot{x}_{a1}(t)|}{\ddot{x}_{a3o}}, \frac{|\ddot{x}_{a2}(t)|}{\ddot{x}_{a3o}}, \frac{|\ddot{x}_{a3}(t)|}{\ddot{x}_{a3o}} \right\} \right] \tag{14}$$

where the accelerations are nondimensionalized by the peak uncontrolled third floor acceleration, denoted $\ddot{x}_{a3o}$, corresponding respectively to each earthquake.

The control constraints are $\max_t |u(t)| \le 3$ volts, $\max_t |x_m(t)| \le 9$ cm, $\max_t |\ddot{x}_{am}(t)| \le 6$ g's, and both the El Centro and the Hachinohe earthquakes should again be considered. Additionally, the candidate controllers are to be evaluated in terms of the required control resources as follows

$$J_8 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|x_m(t)|}{x_{3o}} \right] \tag{15}$$

$$J_9 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|\dot{x}_m(t)|}{\dot{x}_{3o}} \right] \tag{16}$$

$$J_{10} = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|\ddot{x}_{am}(t)|}{\ddot{x}_{a3o}} \right] \tag{17}$$

where $\dot{x}_{3o}$ is the peak uncontrolled third floor relative velocity corresponding respectively to each earthquake.

For the El Centro earthquake, $x_{3o} = 3.37$ cm, $\dot{x}_{3o} = 131$ cm/sec and $\ddot{x}_{a3o} = 5.05$ g's. For the Hachinohe earthquake, $x_{3o} = 1.66$ cm, $\dot{x}_{3o} = 58.3$ cm/sec and $\ddot{x}_{a3o} = 2.58$ g's.

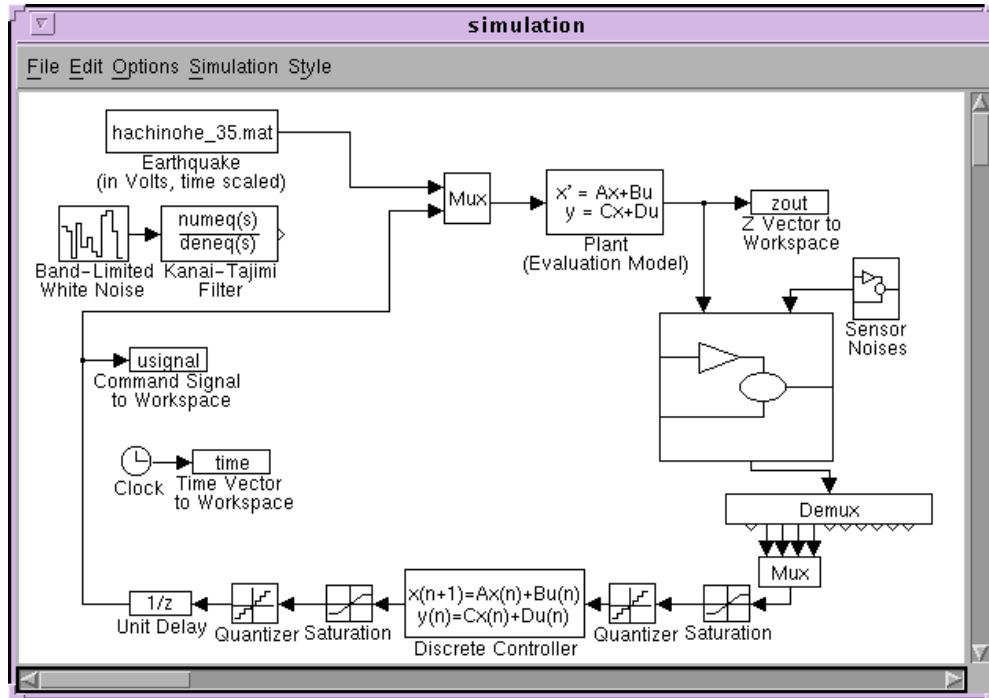## Control Implementation Constraints

To make the benchmark problem as realistic as possible, the following implementation constraints are placed on the system:

1. As indicated previously, the measurements that are directly available for use in determination of the control action are $\mathbf{y} = [x_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}, \ddot{x}_g]'$. Although absolute velocities are not available, they can be closely approximated by passing the measured accelerations through a second order filter with the following transfer function

$$H_{\dot{x}\ddot{x}}(s) = \frac{39.5s}{39.5s^2 + 8.89s + 1} \tag{18}$$

where $\dot{x}$ is the pseudo velocity response in that it will track the absolute velocity response above 1 Hz. Therefore, the pseudo velocities, $\dot{x}_{a1}, \dot{x}_{a2}, \dot{x}_{a3}, \dot{x}_{am}, \dot{x}_g$, are also available for determination of the control action, and the combined output vector is given by $\mathbf{y} = [x_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}, \ddot{x}_g, \dot{x}_{a1}, \dot{x}_{a2}, \dot{x}_{a3}, \dot{x}_{am}, \dot{x}_g]'$. For more information regarding practical issues associated with implementing the filter in Eq. (18), see Ivers and Miller (1991).

2. The controller for the structure is digitally implemented with a sampling time of $T = 0.001$ sec.

3. A computation delay of 200 μ sec is required to perform the D-matrix calculations in the control action determination and for the associated A/D and D/A conversions.

4. The A/D and D/A converters on the digital controller have 12-bit precision and a span of ± 3 V.

5. Each of the measured responses contains an rms noise of 0.01 Volts, which is approximately 0.3% of the full span of the A/D converters. The measurement noises

Figure 5. SIMULINK Model for the Benchmark Problem.

are modeled as Gaussian rectangular pulse processes with a pulse width of 0.001 seconds.

6. To account for limited computational resources in the digital controller, the controller given in Eqs. (4) and (5) is restricted to have no more than 12 states.

7. The performance of each control design should be evaluated using the 28 state evaluation model given in Eqs. (1)–(3).

8. The controller given in Eqs. (4) and (5) is required to be stable.

The SIMULINK (1994) model shown in Fig. 5 has been developed to simulate the features and limitations of this structural control problem. Note that, although the controller is digital, the structure is still modeled as a continuous system. To reduce integration errors, a time step of 0.0001 sec is used in the simulation.

Sample Control Design

To illustrate some of the constraints and challenges of this benchmark problem, a sample linear quadratic Gaussian (LQG) control design is presented. The first step in this process is to develop a reduced order model, designated the *design* model, for purposes of control design. The design model has the form

$$\dot{\mathbf{x}}_r = \mathbf{A}_r \mathbf{x}_r + \mathbf{B}_r u + \mathbf{E}_r \ddot{x}_g \tag{19}$$

$$\mathbf{y}_r = \mathbf{C}_{yr}\mathbf{x}_r + \mathbf{D}_{yr}u + \mathbf{F}_{yr}\ddot{x}_g + \mathbf{v}_r \tag{20}$$

where $\mathbf{x}_r$ is a 10-dimensional state vector, $\mathbf{y}_r = [\ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}]'$ and $\mathbf{A}_r$, $\mathbf{B}_r$, $\mathbf{E}_r$, $\mathbf{C}_{yr}$, $\mathbf{D}_{yr}$ and $\mathbf{F}_{yr}$ are the reduced order coefficient matrices. This design will not make use of the measurement of the ground excitation, the pseudo absolute velocities or the actuator displacement, although these measurements are available.

To simplify design of the controller, $\ddot{x}_g$ is taken to be a stationary white noise, and an infinite horizon performance index is chosen that weights the accelerations of the three floors, *i.e.*,

$$J = \lim_{\tau \to \infty} \frac{1}{\tau} \, \mathrm{E}\left[\int_0^\tau \{(\mathbf{C}_{yr}\mathbf{x}_r + \mathbf{D}_{yr}u)'\mathbf{Q}(\mathbf{C}_{yr}\mathbf{x}_r + \mathbf{D}_{yr}u) + ru^2\}dt\right] \tag{21}$$

where $r = 50$, and all of the elements of the weighting matrix $\mathbf{Q}$ are zero, except for $Q_{11} = Q_{22} = Q_{33} = 1$. Further, the measurement noise is assumed to be identically distributed, statistically independent Gaussian white noise processes, and $S_{\ddot{x}_g\ddot{x}_g}/S_{v_iv_i} = \gamma = 25$.

The separation principle allows the control and estimation problems to be considered separately, yielding a controller of the form (Stengel 1986; Skelton 1988)

$$u = -\mathbf{K}\hat{\mathbf{x}}_r \tag{22}$$

where $\hat{\mathbf{x}}_r$ is the Kalman Filter estimate of the state vector based on the reduced order model. By the certainty equivalence principle (Stengel 1986; Skelton 1988), $\mathbf{K}$ is the full state feedback gain matrix for the deterministic regulator problem given by

$$\mathbf{K} = (\tilde{\mathbf{N}}' + \mathbf{B}_r'\mathbf{P})/\tilde{r} \tag{23}$$

where $\mathbf{P}$ is the solution of the algebraic Riccati equation given by

$$\mathbf{0} = \mathbf{P}\tilde{\mathbf{A}} + \tilde{\mathbf{A}}'\mathbf{P} - \mathbf{P}\mathbf{B}_r\mathbf{B}_r'\mathbf{P}/\tilde{r} + \tilde{\mathbf{Q}} \tag{24}$$

and

$$\tilde{\mathbf{Q}} = \mathbf{C}_{yr}'\mathbf{Q}\mathbf{C}_{yr} - \tilde{\mathbf{N}}\tilde{\mathbf{N}}'/\tilde{r} \tag{25}$$

$$\tilde{\mathbf{N}} = \mathbf{C}_{yr}'\mathbf{Q}\mathbf{D}_{yr} \tag{26}$$

$$\tilde{r} = r + \mathbf{D}_{yr}'\mathbf{Q}\mathbf{D}_{yr} \tag{27}$$

$$\tilde{\mathbf{A}} = \mathbf{A}_r - \mathbf{B}_r\tilde{\mathbf{N}}'/\tilde{r} \tag{28}$$

Calculations to determine $\mathbf{K}$ were done using the MATLAB (1994) routine *lqry.m* within the control toolbox.

The Kalman Filter optimal estimator is given by

$$\dot{\hat{\mathbf{x}}}_r = \mathbf{A}_r \hat{\mathbf{x}}_r + \mathbf{B}_r u + \mathbf{L}(\mathbf{y} - \mathbf{C}_{yr} \hat{\mathbf{x}}_r - \mathbf{D}_{yr} u) \tag{29}$$

$$\mathbf{L} = [\underline{\mathbf{R}}^{-1}(\gamma \mathbf{F}_{yr} \mathbf{E}_r' + \mathbf{C}_{yr} \mathbf{S})]' \tag{30}$$

where $\mathbf{S}$ is the solution of the algebraic Riccati equation given by

$$\mathbf{0} = \mathbf{S}\underline{\mathbf{A}} + \underline{\mathbf{A}}'\mathbf{S} - \mathbf{S}\underline{\mathbf{G}}\mathbf{S} + \underline{\mathbf{H}} \tag{31}$$

and

$$\underline{\mathbf{A}} = \mathbf{A}_r' - \mathbf{C}_{yr}'\underline{\mathbf{R}}^{-1}(\gamma \mathbf{F}_{yr} \mathbf{E}_r') \tag{32}$$

$$\underline{\mathbf{G}} = \mathbf{C}_{yr}'\underline{\mathbf{R}}^{-1}\mathbf{C}_{yr} \tag{33}$$

$$\underline{\mathbf{H}} = \gamma \mathbf{E}_r \mathbf{E}_r' - \gamma^2 \mathbf{E}_r \mathbf{F}_{yr}'\underline{\mathbf{R}}^{-1}\mathbf{F}_{yr}\mathbf{E}_r' \tag{34}$$

$$\underline{\mathbf{R}} = \mathbf{I} + \gamma \mathbf{F}_{yr}\mathbf{F}_{yr}' \tag{35}$$

Calculations to determine $\mathbf{L}$ were done using the MATLAB routine *lqew.m* within the control toolbox.

Finally, the controller is put in the form of Eqs. (4–5) using the bilinear transformation (Antoniou, 1993) to yield the following compensator

$$\mathbf{x}_{k+1}^c = \mathbf{A}^c \mathbf{x}_k^c + \mathbf{B}^c \mathbf{y}_k \tag{36}$$

$$u_k = \mathbf{C}^c \mathbf{x}_k^c + \mathbf{D}^c \mathbf{y}_k \tag{37}$$

As required, the $\dim(\mathbf{x}_k^c) = 10 \leq 12$.

To assess the performance of the sample controller, it is implemented on the evaluation model discussed previously. Based on an eigenvalue analysis, both the controller and the closed-loop system are stable. The loop gain transfer function was used to provide an indication of the closed-loop stability of the system. Here, the loop gain transfer function is defined as the transfer function of the system formed by breaking the control loop at the input to the system (Dyke, *et al.*, 1994b, 1995a,b). The loop gain formed with this control design is provided in Fig. 6. A control design was considered to be robust if the magnitude of the loop gain was below –5 dB at all frequencies above 35 Hz.

For the first five evaluation criteria, the rms values of the constraint variables are $\sigma_{x_m} = 0.671$ cm, $\sigma_{\ddot{x}_{am}} = 1.12$ g and $\sigma_u = 0.143$ V. For evaluation criteria six through ten, the peak values of the constraint variables are $x_m = 2.00$ cm, $\ddot{x}_{am} = 4.83$ g and $u = 0.526$ V. Thus, all control design constraints were achieved with this control design. The associated evaluation criteria are given in Table 1. Note that the rms and peak responses were determined through simulation using the SIM-ULINK program shown in Figure 4. The rms responses were calculated assuming an ergodic response and averaging over a 300 second time period.
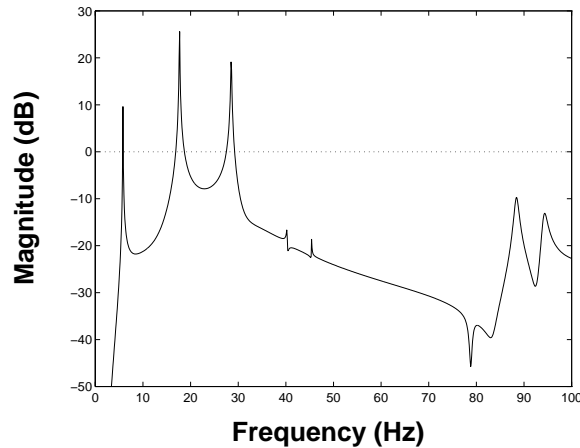
Figure 6. Loop Gain Transfer Function for Sample Control Design.

Table 1: Evaluation Criteria for the Sample Controller.

J_1	0.283	J_6	0.456
J_2	0.440	J_7	0.681
J_3	0.510	J_8	0.669
J_4	0.513	J_9	0.771
J_5	0.628	J_{10}	1.28

Closure

The 28-state evaluation model, as well as the 10-state control design model, the MATLAB m-file used to do the sample control design, the input data and the simulation model are available on the World Wide Web at:

http://www.nd.edu/~quake/

If you cannot access the World Wide Web or have questions regarding the benchmark problem, please contact the senior author via e-mail at: spencer.1@nd.edu.

Control designs can be submitted via e-mail to the senior author for inclusion in the benchmark control homepage. For each control design, please provide the SIMULINK program in which the controller is implemented, a complete write-up of the control design, and the values of the respective evaluation criteria found from the simulation.

Acknowledgments

Appendix I – References

Antoniou, A. (1993). *Digital Filters: Analysis, Design, and Applications*, McGraw-Hill, Inc., New York, pp. 444–446.

Chung, L.L., Lin, R.C., Soong, T.T. and Reinhorn, A.M. 1989. Experiments on Active Control for MDOF Seismic Structures," *J. of Engrg. Mech.*, ASCE, Vol. 115, No. 8, pp. 1609–27.

Dyke, S.J., Spencer Jr., B.F., Belknap, A.E., Ferrell, K.J., Quast, P., and Sain, M.K. (1994a). "Absolute Acceleration Feedback Control Strategies for the Active Mass Driver." *Proc. First World Conference on Structural Control*, Pasadena, California, August 3–5, 1994, Vol. 2, pp. TP1:51–TP1:60.

Dyke, S.J., Spencer Jr., B.F., Quast, P., Sain, M.K., Kaspari Jr., D.C. and Soong, T.T. (1994b). "Experimental Verification of Acceleration Feedback Control Strategies for An Active Tendon System," *National Center for Earthquake Engineering Research Technical Report NCEER–94–0024*, August 29.

Dyke, S.J., Spencer Jr., B.F., Quast, P., Kaspari Jr., D.C. and Sain, M.K. (1995a). "Implementation of an Active Mass Driver Using Acceleration Feedback Control." *Microcomputers in Civil Engrg.*, in press.

Dyke, S.J., Spencer Jr., B.F., Quast, P., Sain, M.K., Kaspari Jr., D.C. and Soong, T.T. (1995b). "Acceleration Feedback Control of MDOF Structures." *J. Engrg. Mech., ASCE*, in press.

Fujino, Y., Soong, T.T. and Spencer Jr., B.F. (1996). "Structural Control: Basic Concepts and Applications." *Proceedings of the ASCE Structures Congress XIV*, Chicago, Illinois, April 15–18.

Housner, G.W., Masri, S.F., and Chassiakos, A.G., Eds. (1994a). *Proceedings of the First World Conference on Structural Control*, International Association for Structural Control, Los Angeles.

Housner, G.W., Soong, T.T. and Masri, S. (1994b). "Second Generation of Active Structural Control in Civil Engineering." *Proc. First World Conference on Structural Control*, Pasadena, California, August 3–5, 1994, Vol. 1, pp. Panel:3–18.

Ivers, D.E. and Miller, L.R. (1991). "Semi-Active Suspension Technology: An Evolutionary View." *DE-Vol. 40, Advanced Automotive Technologies*, (S.A. Velinsky, R.H. Fries and D. Wang, Eds.), ASME Book No. H00719, pp. 327-346.

MATLAB (1994). The Math Works, Inc. Natick, Massachusetts.

SIMULINK (1994). The Math Works, Inc. Natick, Massachusetts.

Skelton, R.E. (1988). *Dynamic Systems Control: Linear Systems Analysis and Synthesis*. Wiley, New York.

Soong, T.T. *Active Structural Control: Theory and Practice*, Longman Scientific and Technical, Essex, England, 1990.

Stengel, R.F. (1986). *Stochastic Optimal Control: Theory and Application*. Wiley, New York.

Appendix II – Nomenclature

$$\left.\begin{array}{c} \mathbf{A}, \mathbf{B}, \mathbf{E} \\ \mathbf{C}_y, \mathbf{D}_y, \mathbf{F}_y \\ \mathbf{C}_z, \mathbf{D}_z, \mathbf{F}_z \end{array}\right\}$$ – state space matrices for the evaluation model

$$\left.\begin{array}{c} \mathbf{A}_r, \mathbf{B}_r, \mathbf{E}_r \\ \mathbf{C}_{yr}, \mathbf{D}_{yr}, \mathbf{F}_{yr} \end{array}\right\}$$ – state space matrices for the reduced order model

$$\left.\begin{array}{c} \mathbf{A}^c, \mathbf{B}^c \\ \mathbf{C}^c, \mathbf{D}^c \end{array}\right\}$$ – state space matrices for the discrete controller

d_i – interstory drift of the ith floor

$f_1(\cdot), f_2(\cdot)$ – feedback compensator functions

$H_{\dot{x}\ddot{x}}(s)$ – transfer function of the filters used to obtain the pseudo absolute velocities

J – performance function for LQG control design

J_i – ith evaluation criteria

$\mathbf{K}$ – full state feedback gain matrix

k – discrete time step index

$\mathbf{L}$ – state estimator gain matrix

$\mathbf{P}$ – algrebraic Riccati matrix solution for regulator

$\mathbf{Q}, r$ – weights in LQG control design

$\mathbf{S}$ – algrebraic Riccati matrix solution for state estimator

$S_{v_i v_i}$ – magnitude of the constant two-sided spectral density for the white
noises modeling the measurement noise in the LQG control design

$S_{\ddot{x}_g \ddot{x}_g}$ – magnitude of the constant two-sided spectral density for the white
noise used to model the ground excitation in the LQG control design

T – sampling time

u – scalar control input

u_k – scalar control input at time $t = kT$

$\mathbf{v}$ – measurement noise vector for the evaluation model

$\mathbf{v}_r$ – measurement noise vector for the control design model

$\mathbf{x}$ – state vector for the evaluation model

$\mathbf{x}_k^c$ – state vector for the discrete controller at time $t = kT$

$\mathbf{x}_r$ – state vector for the control design model

$\hat{\mathbf{x}}_r$ – estimated state vector for the control design model

x_i – displacement of the ith floor relative to the ground

x_m – displacement of the actuator piston relative to the third floor

x_{3o} – peak third floor displacement response relative to the ground of
the uncontrolled building for each respective historical earthquake

$\dot{x}_i$ – velocity of the ith floor relative to the ground

$\dot{x}_{3o}$ – peak third floor velocity response relative to the ground of
the uncontrolled building for each respective historical earthquake

$\dot{x}_{ai}$ – pseudo absolute velocity of the ith floor

$\dot{x}_m$ – velocity of the actuator piston relative to the third floor

$\dot{\tilde{x}}_{am}$ – pseudo absolute velocity of the actuator piston

$\dot{\tilde{x}}_g$ – pseudo absolute velocity of the ground

$\ddot{x}_{ai}$ – absolute acceleration of the *i*th floor

$\ddot{x}_{am}$ – absolute acceleration of the actuator piston

$\ddot{x}_g$ – absolute acceleration of the ground

$\ddot{x}_{a3o}$ – peak third floor absolute acceleration of the uncontrolled building
 for each respective historical earthquake

$\mathbf{y}$ – vector of directly measured responses

$\mathbf{y}_k$ – vector of directly measured responses sampled at time $t \ = \ kT$

$\mathbf{y}_r$ – output vector for the control design model

$\underset{\sim}{\mathbf{y}}$ – vector of responses available for calculation of the control

$\underset{\sim}{\mathbf{y}}_k$ – vector of responses available for calculation of the control sampled
 at time $t \ = \ kT$

$\mathbf{z}$ – vector of regulated responses

σ_{d_i} – rms interstory drift of the *i*th floor

σ_u – rms control signal

σ_{x_m} – rms displacement of the actuator piston relative to the third floor

$\sigma_{\ddot{x}_g}$ – rms ground acceleration

$\sigma_{x_{3o}}$ – worst-case stationary rms displacement of the third floor of the
 uncontrolled building relative to the ground

$\sigma_{\dot{x}_m}$ – rms velocity of the actuator piston relative to the third floor

$\sigma_{\dot{x}_{3o}}$ – worst-case stationary rms velocity of the third floor of the
 uncontrolled building relative to the ground

$\sigma_{\ddot{x}_{a3o}}$ – worst-case stationary rms absolute acceleration of the third floor of the
 uncontrolled building

Benchmark Problems in Structural Control
Part II: Active Tendon System

B.F. Spencer Jr.,[1] S.J. Dyke[2] and H.S. Deoskar[2]

Introduction

In a companion paper (Spencer, *et al.* 1997), an overview and problem defini-
tion was presented for a well-defined benchmark structural control problem for a
model building configured with an active mass driver (AMD). A second benchmark
problem is posed here based on a high-fidelity analytical model of three-story, tendon-
controlled structure at the National Center for Earthquake Engineering Research
(NCEER) (Chung, *et al.* 1989; Dyke, *et al.* 1996). The purpose of formulating this
problem is to provide another setting in which to evaluate the relative effectiveness
and implementability of various structural control algorithms. To achieve a high level
of realism, an *evaluation* model is presented in the problem definition which is de-
rived directly from experimental data obtained for the structure. This model accurate-
ly represents the behavior of the laboratory structure and fully incorporates actuator/
sensor dynamics. As in the companion paper, the evaluation model will be considered
as the real structural system. In general, controllers that are successfully implemented
on the evaluation model can be expected to perform similarly in the laboratory setting.
Several evaluation criteria are given, along with the associated control design con-
straints.

Experimental Structure

The structure on which the evaluation model is based is the actively controlled,
three-story, single-bay, model building considered in Chung, *et al.* (1989). The test
structure, shown in Figs. 1 and 2, has a mass of 2,950 kg, distributed among the three
floors, and is 254 cm in height. The ratio of model quantities to those corresponding to
the prototype structure are: force = 1:16, mass = 1:16, time = 1:2, displacement = 1:4
and acceleration = 1:1. Due to the time scaling, the natural frequencies of the model

---

1. Prof., Dept. of Civil Engrg. and Geo. Sci., Univ. of Notre Dame, Notre Dame, IN 46556. M. ASCE.
2. Grad. Assist., Dept. of Civil Engrg. and Geo. Sci., Univ. of Notre Dame, Notre Dame, IN 46556.
Stud. M. ASCE.
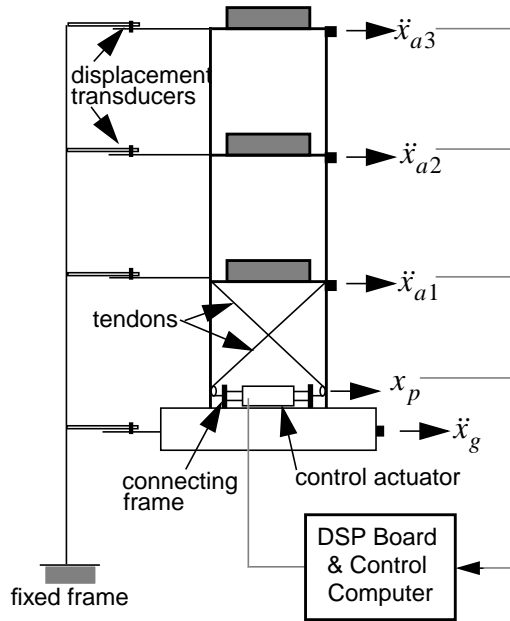
Figure 1. Schematic of Experimental Setup.
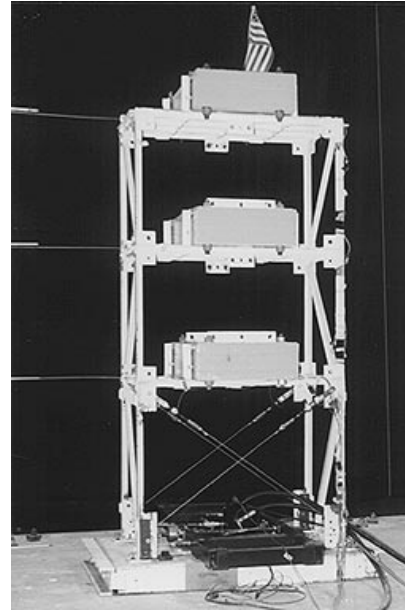
Figure 2. Three-Degree-of-Freedom Test Structure.

are approximately twice those of the prototype. The first three modes of the model system are at 2.27 Hz, 7.33 Hz, and 12.24 Hz, with associated damping ratios given, respectively, by 0.6%, 0.7%, and 0.3%.

A hydraulic control actuator, four pretensioned tendons, and a stiff steel frame connecting the actuator to the tendons are provided to apply control forces to the test structure. The four diagonal tendons transmit the force from the control actuator to the first floor of the structure, and the steel frame connects the actuator to the tendons. Because hydraulic actuators are inherently open-loop unstable, a feedback control system is employed to stabilize the control actuator and improve its performance. This feedback signal is a combination of the position, velocity and pressure measurements. For this actuator, an LVDT (linear variable differential transformer), rigidly mounted to the piston, provides the displacement measurement, which is the primary feedback signal. This measurement is also sent through an analog differentiator to provide a velocity measurement, and a pressure transducer across the actuator piston provides the pressure measurement.

The structure was fully instrumented to provide for a complete record of the motions undergone by the structure during testing. Accelerometers positioned on each floor of the structure measured the absolute accelerations, and an accelerometer located on the base measured the ground excitation, as shown in Fig. 1. The displacement of the actuator was measured using the LVDT mentioned above. Force transducers were placed in series with each of the four tendons and their individual outputs were

combined to determine the total force applied to the structure. Additional measurements were also available for model verification. Displacement transducers on the base and on each floor were attached to a fixed frame (*i.e.*, not attached to the earthquake simulator), as shown in Fig. 1, to measure the absolute displacements of the structure and of the base. The relative displacements were determined by subtracting the base displacement from the absolute displacement of each floor. Because a fixed frame was necessary to measure the relative displacements of the structure, these measurements would not be directly available in a full-scale implementation. Thus, the displacement measurements are used only for model verification and are not available for feedback in the control system.

Evaluation Model

A high-fidelity, linear time-invariant state space representation of the input-output model for the structure described in the previous section has been developed. The model has 20 states and is of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{E}\ddot{x}_g \tag{1}$$

$$\mathbf{y} = \mathbf{C}_y\mathbf{x} + \mathbf{D}_y u + \mathbf{F}_y\ddot{x}_g + \mathbf{v} \tag{2}$$

$$\mathbf{z} = \mathbf{C}_z\mathbf{x} + \mathbf{D}_z u + \mathbf{F}_z\ddot{x}_g \tag{3}$$

where $\mathbf{x}$ is the state vector, $\ddot{x}_g$ is the scalar ground acceleration, u is the scalar control input, $\mathbf{y} = [x_p, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, f, \ddot{x}_g]'$ is the vector of responses that can be directly measured, $\mathbf{z} = [x_1, x_2, x_3, x_p, \dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_p, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, f]'$ is the vector of responses that can be regulated. Here, x_i is the displacement of the ith floor relative to the ground, x_p is the displacement of the control actuator, $\ddot{x}_{ai}$ is the absolute acceleration of the ith floor, $\mathbf{v}$ is the vector of measurement noises, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{E}$, $\mathbf{C}_y$, $\mathbf{D}_y$, $\mathbf{C}_z$, $\mathbf{D}_z$, $\mathbf{F}_y$ and $\mathbf{F}_z$ are matrices of appropriate dimension. The coefficient matrices in Eqs. (1–3) are determined from the data collected at the NCEER using the identification methods presented in Dyke, *et al.* (1994a,b, 1996a,b). The resulting model represents the input-output behavior of the structural system up to 50 Hz and includes the effects of actuator/sensor dynamics and control-structure interaction. Figure 3 provides a representative comparison between selected model transfern functions and the experimental data.

The model given in Eqs. (1–3) is termed the *evaluation* model and will be used to assess the performance of candidate controllers; that is, the evaluation model is considered herein to be the true representation of the structural system.
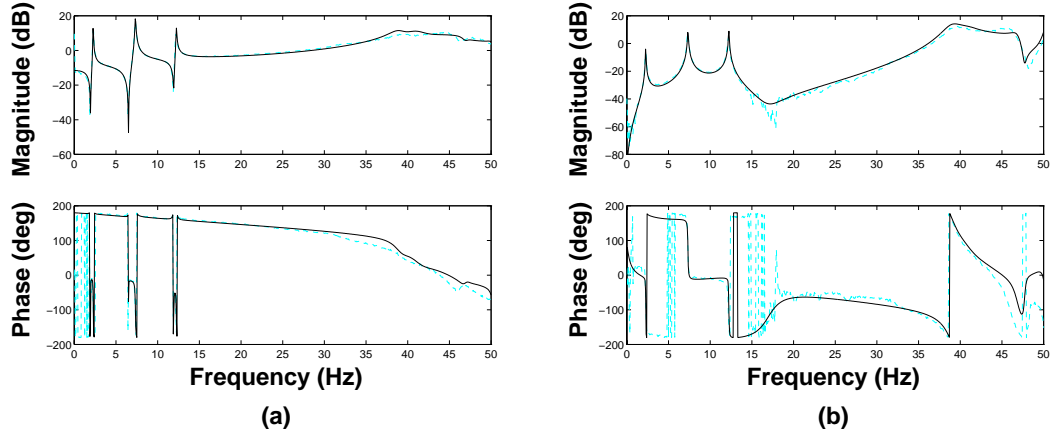
Figure 3. Representative Comparison of the Transfer Functions for the Test Structure and the Evaluation Model. (a) Actuator Command to the Force in the Tendons, (b) Actuator Command to the Third Floor Absolute Acceleration.

Control Design Problem

The control design problem stated here follows the companion paper (Spencer, *et al.* 1997) and is stated as: determine a discrete-time, feedback compensator of the form

$$\mathbf{x}^c_{k+1} = f_1(\mathbf{x}^c_k, \underset{\sim}{\mathbf{y}}_k, u_k, k) \qquad (4)$$

$$u_k = f_2(\mathbf{x}^c_k, \underset{\sim}{\mathbf{y}}_k, k) \qquad (5)$$

where $\mathbf{x}^c_k$, $\underset{\sim}{\mathbf{y}}_k$ and u_k are the state vector for the compensator, the output vector and the control command, respectively, at time $t = kT$. For this problem, $\dim(\mathbf{x}^c) \leq 12$ is required, and the performance of all control designs must be assessed using the evaluation model described previously. For each proposed control design, performance and stability robustness should be discussed. As detailed in the following paragraphs, the merit of a controller will be based on criteria given in terms of both rms and peak response quantities. Normally, smaller values of the evaluation criteria indicate superior performance.

Evaluation Criteria: RMS Responses

Assume that the input excitation $\ddot{x}_g$ is a stationary random process with a spectral density defined by the Kanai-Tajimi spectrum

$$S_{\ddot{x}_g \ddot{x}_g}(\omega) = \frac{S_0(4\zeta_g^2\omega_g^2\omega^2 + \omega_g^4)}{(\omega^2 - \omega_g^2)^2 + 4\zeta_g^2\omega_g^2\omega^2} \tag{6}$$

where ω_g and ζ_g are unknown, but assumed to lie in the following ranges: 8 rad/sec $\leq \omega_g \leq 50$ rad/sec, $0.3 \leq \zeta_g \leq 0.75$. To have a basis for comparison, the spectral intensity is chosen such that the rms value of the ground motion takes a constant value of $\sigma_{\ddot{x}_g} = 3.4 \cdot 10^{-2}$ g's, *i.e.,*

$$S_0 = \frac{2.35 \cdot 10^{-3}\zeta_g}{\pi\omega_g(4\zeta_g^2 + 1)} \ g^2 \cdot \sec \tag{7}$$

The first criterion on which controllers will be evaluated is based on their ability to minimize the maximum rms interstory drift due to all admissible ground motions. Therefore, the nondimensionalized measure of performance is given by

$$J_1 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{d_1}}{\sigma_{x_{3o}}}, \frac{\sigma_{d_2}}{\sigma_{x_{3o}}}, \frac{\sigma_{d_3}}{\sigma_{x_{3o}}} \right\} \tag{8}$$

where σ_{d_i} is the stationary rms interstory drift for the ith floor, and $\sigma_{x_{3o}} = 2.34$ cm is the worst-case stationary rms displacement of the third floor of the uncontrolled building over the class of excitations considered (occurring when $\omega_g = 14.5$ rad/sec, $\zeta_g = 0.3$). The interstory drifts are given respectively by $d_1(t) = x_1(t)$, $d_2(t) = x_2(t) - x_1(t)$ and $d_3(t) = x_3(t) - x_2(t)$.

A second evaluation criterion is given in terms of the maximum rms absolute acceleration, yielding a performance measure given by

$$J_2 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{\ddot{x}_{a1}}}{\sigma_{\ddot{x}_{a3o}}}, \frac{\sigma_{\ddot{x}_{a2}}}{\sigma_{\ddot{x}_{a3o}}}, \frac{\sigma_{\ddot{x}_{a3}}}{\sigma_{\ddot{x}_{a3o}}} \right\} \tag{9}$$

where $\sigma_{\ddot{x}_{ai}}$ is the stationary rms acceleration for the ith floor, and $\sigma_{\ddot{x}_{a3o}} = 0.485$ g's is the worst-case stationary rms acceleration of the third floor of the uncontrolled building (occurring when $\omega_g = 14.5$ rad/sec, $\zeta_g = 0.3$).

The hard constraints for the control effort are given by $\sigma_u \leq 1$ volt, $\sigma_f \leq 4$ kN and $\sigma_{x_p} \leq 1$ cm. Additionally, candidate controllers are to be evaluated based on the required control resources. Three quantities, σ_{x_p}, $\sigma_{\dot{x}_p}$ and σ_f, should be examined to make the assessment. The rms actuator displacement, σ_{x_p}, provides a measure of the required physical size of the device. The rms actuator velocity, $\sigma_{\dot{x}_p}$, provides a measure of the control power required. The rms absolute acceleration σ_f provides a measure of the magnitude of the forces that the actuator must generate to execute the

commanded control action. Therefore, the nondimensionalized control resource evaluation criteria are

$$J_3 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{x_p}}{\sigma_{x_{3o}}} \right\} \tag{10}$$

$$J_4 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{\dot{x}_p}}{\sigma_{\dot{x}_{3o}}} \right\} \tag{11}$$

$$J_5 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_f}{W} \right\} \tag{12}$$

where $\sigma_{\dot{x}_{3o}} = 33.3$ cm/sec is the worst-case stationary rms velocity of the third floor relative to the ground for the uncontrolled structure (occurring when $\omega_g = 14.5$ rad/ sec, $\zeta_g = 0.3$), and W is the weight of the building (289 kN).

Evaluation Criteria: Peak Responses

Here, the input excitation $\ddot{x}_g$ is assumed to be a historical earthquake record. Both the 1940 El Centro NS record and the NS record for the 1968 Hachinohe earthquake are considered. Because the system under consideration is a scale model, the time scale should be increased by a factor of 2 (*i.e.*, the earthquakes occur in 1/2 the recorded time). The required scaling of the magnitude of the ground acceleration is 1. The evaluation criterion is based on minimization of the nondimensionalized peak interstory drifts due to both earthquake records. For each earthquake, the maximum drifts are nondimensionalized with respect to the uncontrolled peak third floor displacement, denoted x_{3o}, relative to the ground. Therefore, the performance measure is given by

$$J_6 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \left\{ \frac{|d_1(t)|}{x_{3o}}, \frac{|d_2(t)|}{x_{3o}}, \frac{|d_3(t)|}{x_{3o}} \right\} \right] \tag{13}$$

A second performance evaluation criterion is given in terms of the peak acceleration, yielding

$$J_7 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \left\{ \frac{|\ddot{x}_{a1}(t)|}{\ddot{x}_{a3o}}, \frac{|\ddot{x}_{a2}(t)|}{\ddot{x}_{a3o}}, \frac{|\ddot{x}_{a3}(t)|}{\ddot{x}_{a3o}} \right\} \right] \tag{14}$$

where the accelerations are nondimensionalized by the peak uncontrolled third floor acceleration, denoted $\ddot{x}_{a3o}$, corresponding respectively to each earthquake.

The control constraints are $\max_t |u(t)| \le 3$ volts, $\max_t |x_p(t)| \le 3$ cm, $\max_t |f(t)| \le 12$ kN, and both the El Centro and the Hachinohe earthquakes should again be considered. Additionally, the candidate controllers are to be evaluated in terms of the required control resources as follows

$$J_8 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|x_p(t)|}{x_{3o}} \right] \tag{15}$$

$$J_9 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|\dot{x}_p(t)|}{\dot{x}_{3o}} \right] \tag{16}$$

$$J_{10} = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|f(t)|}{W} \right] \tag{17}$$

where $\dot{x}_{3o}$ is the peak uncontrolled third floor relative velocity corresponding respectively to each earthquake.

For the half-scale El Centro earthquake, $x_{3o} = 6.45$ cm, $\dot{x}_{3o} = 99.9$ cm/sec and $\ddot{x}_{a3o} = 1.57$ g's. For the half-scale Hachinohe earthquake, $x_{3o} = 3.78$ cm, $\dot{x}_{3o} = 56.1$ cm/sec and $\ddot{x}_{a3o} = 0.778$ g's.

Control Implementation Constraints

To make the benchmark problem as realistic as possible, the following implementation constraints are placed on the system:

1. As indicated previously, the measurements that are directly available for use in determination of the control action are $\mathbf{y} = [x_p, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, f, \ddot{x}_g]'$. Although absolute velocities are not available, they can be closely approximated by passing the measured accelerations through a second order filter with the following transfer function

$$H_{\dot{x}\ddot{x}}(s) = \frac{39.5s}{39.5s^2 + 8.89s + 1} \tag{18}$$

where $\dot{\underset{\sim}{x}}$ is the pseudo velocity response in that it will track the absolute velocity response above 1 Hz. Therefore, the pseudo velocities, $\dot{\underset{\sim}{x}}_{a1}, \dot{\underset{\sim}{x}}_{a2}, \dot{\underset{\sim}{x}}_{a3}, \dot{\underset{\sim}{x}}_g$, are also available for determination of the control action, and the combined output vector is given by $\underset{\sim}{\mathbf{y}} = [x_p, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, f, \ddot{x}_g, \dot{\underset{\sim}{x}}_{a1}, \dot{\underset{\sim}{x}}_{a2}, \dot{\underset{\sim}{x}}_{a3}, \dot{\underset{\sim}{x}}_g]'$. For more information regarding practical issues associated with implementing the filter in Eq. (18), see Ivers and Miller (1991).

2. The controller for the structure is digitally implemented with a sampling time of $T = 0.001$ sec.

3. A computation delay of 200 μ sec is required to perform the D-matrix calculations in the control action determination and for the associated A/D and D/A conversions.

4. The A/D and D/A converters on the digital controller have 12-bit precision and a span of ± 3 V.

5. Each of the measured responses contains an rms noise of 0.01 Volts, which is approximately 0.3% of the full span of the A/D converters. The measurement noises are modeled as Gaussian rectangular pulse processes with a pulse width of 0.001 seconds.

6. To account for limited computational resources in the digital controller, the controller given in Eqs. (4) and (5) is restricted to have no more than 12 states.

7. The performance of each control design should be evaluated using the 20 state evaluation model given in Eqs. (1)–(3).

8. The controller given in Eqs. (4) and (5) is required to be stable.

The SIMULINK (1994) model shown in Fig. 5 has been developed to simulate the features and limitations of this structural control problem. Note that, although the controller is digital, the structure is still modeled as a continuous system. To reduce integration errors, a time step of 0.0001 sec is used in the simulation.

Closure

The 20-state evaluation model, as well as the input data and the simulation model are available on the World Wide Web at:

http://www.nd.edu/~quake/

If you cannot access the World Wide Web or have questions regarding the benchmark problem, please contact the senior author via e-mail at: spencer.1@nd.edu.

Control designs can be submitted via e-mail to the senior author for inclusion in the benchmark control homepage. For each control design, please provide the SIMULINK program in which the controller is implemented, a complete write-up of the control design, and the values of the respective evaluation criteria found from the simulation.
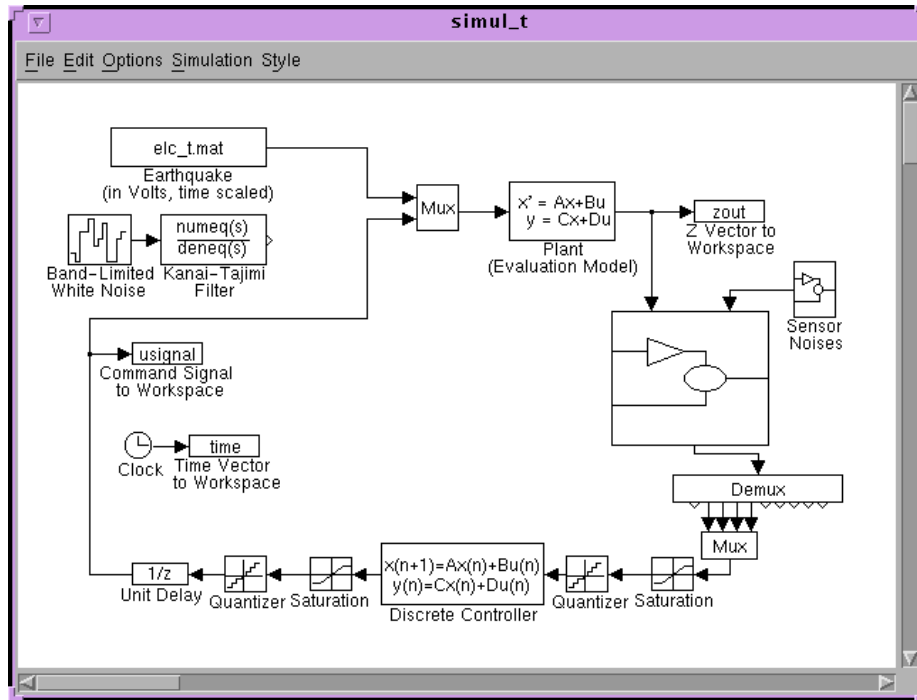
Figure 4. SIMULINK Model for the Benchmark Problem.

Acknowledgments

Appendix I – References

Chung, L.L., Lin, R.C., Soong, T.T. and Reinhorn, A.M. 1989. "Experiments on Active Control for MDOF Seismic Structures," *J. of Engrg. Mech.*, ASCE, Vol. 115, No. 8, pp. 1609–27.

Dyke, S.J., Spencer Jr., B.F., Belknap, A.E., Ferrell, K.J., Quast, P., and Sain, M.K. (1994a). "Absolute Acceleration Feedback Control Strategies for the Active Mass Driver." *Proc. First World Conference on Structural Control*, Pasadena, California, August 3–5, 1994, Vol. 2, pp. TP1:51–TP1:60.

Dyke, S.J., Spencer Jr., B.F., Quast, P., Sain, M.K., Kaspari Jr., D.C. and Soong, T.T. (1994b). "Experimental Verification of Acceleration Feedback Control Strategies for An Active Tendon System," *National Center for Earthquake Engineering Research Technical Report NCEER–94–0024*, August 29.

Dyke, S.J., Spencer Jr., B.F., Quast, P., Kaspari Jr., D.C. and Sain, M.K. (1996a). "Implementation of an Active Mass Driver Using Acceleration Feedback Control." *Microcomputers in Civil Engrg.*, in press.

Dyke, S.J., Spencer Jr., B.F., Quast, P., Sain, M.K., Kaspari Jr., D.C. and Soong, T.T. (1996b). "Acceleration Feedback Control of MDOF Structures." *J. Engrg. Mech., ASCE*, in press.

Fujino, Y., Soong, T.T. and Spencer Jr., B.F. (1996). "Structural Control: Basic Concepts and Applications." *Proceedings of the ASCE Structures Congress XIV*, Chicago, Illinois, April 15–18.

Housner, G.W., Masri, S.F., and Chassiakos, A.G., Eds. (1994a). *Proceedings of the First World Conference on Structural Control*, International Association for Structural Control, Los Angeles.

Housner, G.W., Soong, T.T. and Masri, S. (1994b). "Second Generation of Active Structural Control in Civil Engineering." *Proc. First World Conference on Structural Control*, Pasadena, California, August 3–5, 1994, Vol. 1, pp. Panel:3–18.

Ivers, D.E. and Miller, L.R. (1991). "Semi-Active Suspension Technology: An Evolutionary View." *DE-Vol. 40, Advanced Automotive Technologies*, (S.A. Velinsky, R.H. Fries and D. Wang, Eds.), ASME Book No. H00719, pp. 327-346.

MATLAB (1994). The Math Works, Inc. Natick, Massachusetts.

SIMULINK (1994). The Math Works, Inc. Natick, Massachusetts.

Soong, T.T. (1990). *Active Structural Control: Theory and Practice*, Longman Scientific and Technical, Essex, England.

Spencer Jr., B.F., Dyke, S. and Deoskar, H. (1997). "A Benchmark Problem in Structural Control." *Proceedings of the 1997 ASCE Structures Congress*, Portland, Oregon, April 13–16, 1997.

Appendix II – Nomenclature

$\mathbf{A}, \mathbf{B}, \mathbf{E}$

$\mathbf{C}_y, \mathbf{D}_y, \mathbf{F}_y$ – state space matrices for the evaluation model

$\mathbf{C}_z, \mathbf{D}_z, \mathbf{F}_z$

d_i – interstory drift of the ith floor

$f_1(\cdot), f_2(\cdot)$ – feedback compensator functions

$H_{\dot{x}\ddot{x}}(s)$ – transfer function of the filters used to obtain the pseudo absolute velocities

J_i – ith evaluation criteria

k – discrete time step index

T – sampling time

u – scalar control input

u_k – scalar control input at time $t = kT$

$\mathbf{v}$ – measurement noise vector for the evaluation model

$\mathbf{x}$ – state vector for the evaluation model

$\mathbf{x}_k^c$ – state vector for the discrete controller at time $t = kT$

x_i – displacement of the ith floor relative to the ground

x_p – displacement of the actuator piston relative to the ground

x_{3o} – peak third floor displacement response relative to the ground of
    the uncontrolled building for each respective historical earthquake

$\dot{x}_i$ – velocity of the ith floor relative to the ground

$\dot{x}_{3o}$ – peak third floor velocity response relative to the ground of
    the uncontrolled building for each respective historical earthquake

$\dot{\underset{\sim}{x}}_{ai}$ – pseudo absolute velocity of the ith floor

$\dot{x}_p$ – velocity of the actuator piston relative to the third floor

$\dot{\underset{\sim}{x}}_g$ – pseudo absolute velocity of the ground

$\ddot{x}_{ai}$ – absolute acceleration of the ith floor

f – net force in the tendons

$\ddot{x}_g$ – absolute acceleration of the ground

$\ddot{x}_{a3o}$ – peak third floor absolute acceleration of the uncontrolled building
    for each respective historical earthquake

$\mathbf{y}$ – vector of directly measured responses

$\mathbf{y}_k$ – vector of directly measured responses sampled at time $t = kT$

$\underset{\sim}{\mathbf{y}}$ – vector of responses available for calculation of the control

$\underset{\sim}{\mathbf{y}}_k$ – vector of responses available for calculation of the control sampled
    at time $t = kT$

$\mathbf{z}$ – vector of regulated responses

σ_{d_i} – rms interstory drift of the *i*th floor

σ_u – rms control signal

σ_{x_m} – rms displacement of the actuator piston relative to the third floor

$\sigma_{\ddot{x}_g}$ – rms ground acceleration

$\sigma_{x_{3o}}$ – worst-case stationary rms displacement of the third floor of the uncontrolled building relative to the ground

$\sigma_{\dot{x}_m}$ – rms velocity of the actuator piston relative to the third floor

$\sigma_{\dot{x}_{3o}}$ – worst-case stationary rms velocity of the third floor of the uncontrolled building relative to the ground

$\sigma_{\ddot{x}_{a3o}}$ – worst-case stationary rms absolute acceleration of the third floor of the uncontrolled building

σ_f – rms force in the tendons

$\sigma_{\ddot{x}_{ai}}$ – rms absolute acceleration of the *i*th floor

ω_g, ζ_g – parameters of the Kanai–Tajimi spectrum