APPLICATION OF ENSEMBLE TRAINING OF A STRUCTURAL CONTROLLER TO THE AMD BENCHMARK PROBLEM

G.F. Panariello * R. Betti † R.W. Longman ‡

June 28, 1997

SUMMARY

A benchmark structural control problem has been proposed [1] in an attempt to evaluate the effectiveness of various control algorithms. The problem encompasses the design of an active mass damper (AMD) control system for a multi-degree-of-freedom (MDOF) building type structure subjected to earthquake-type excitation.

In vibration control for civil structures, linear quadratic optimal control is among the most popular techniques. Normally, this approach ignores the external excitation in the time-domain design process. In addition, this technique requires a full-order dynamic observer which is often unattainable. This paper focuses on the development of a new optimal control algorithm which

*Grad. Student Dept. of Civ. Engrg. and Engrg. Mech., Columbia University, New York, NY 10027.
†Assoc. Prof. Dept. of Civ. Engrg. and Engrg. Mech., Columbia University, New York, NY 10027.
‡Prof. Dept. of Mech. Engrg., Columbia University, New York, NY 10027.

includes the earthquake-type excitation explicitly in the design of control systems and the use of prescribed-order, output feedback controllers. In addition, this approach allows the inclusion of open-loop (feedforward) as well as closed-loop (feedback) control terms in the controller design.

The authors have previously designed an algorithm for full state feedback controllers trained on an ensemble of earthquakes.[2] A cost functional is minimized on an ensemble of 'known' earthquakes, using analytic gradient information, in order to determine constant control gains. The gradients are obtained in explicit form. The control system is then validated by testing on 'unknown' earthquakes.

The algorithm is now modified to develop a prescribed-order, output feedback controller for a specific MDOF system model.

INTRODUCTION

Substantial research efforts have focused on the analysis and design of active control systems for civil structures subjected to seismic excitation. The works of Soong [3,4], Yang [5], and Kobori [6], present a compilation of recent progress in active control research. One of the most popular design techniques, linear quadratic optimal control, usually ignores the effect of the earthquake excitation on the time-domain design of the controller. By neglecting the external disturbance, linear quadratic optimal control theory reduces to the solution of the classic Riccati matrix equation. Techniques to solve this equation are quite well developed[7]. The results are closed-loop controllers optimized without regard to the anticipated excitation. Retaining the external excitation in the optimization process requires that the time-history of such an excitation be known a priori, since the corresponding matrix equation must be solved backwards from some final time. In seismic analysis of structures, it is problematic to include the earthquake excitation since the next earthquake is not known in advance. However, time-histories of ground accelerations from prior earthquakes are known. Hence, these could be used to train the control system by explicitly including such ground motion representations in the control design algorithm. In addition open-loop control terms designed to cancel part of the incoming excitation can be included in the optimization process, yielding closed-open loop controllers.

The authors have previously presented [2] an efficient formulation to include the effects of the forcing function directly in the development of control systems. This design process yields controllers with feedback (closed-loop) and feedforward (open-loop) terms, whose gains are optimized by training on an ensemble of earthquakes. The formulation was based on a full state feedback approach.

The present work represents an extension of the previous approach [2] to include the effects of the earthquake-type external excitation in the design of the control systems for MDOF structural models in which full order state feedback is unachievable.

Structural control problems present many difficulties in the areas of modelling and measurements. The proposed benchmark study [1] attempts to address implementation issues by requiring the use of a prescribed (reduced)-order, output feedback controller in the design of an AMD control system for a three story model building subjected to earthquake-type excitation. This is directed towards two problems in particular. First, it is problematic to measure the entire state for a large system subjected to earthquake excitation. The lack of an inertial frame makes displacement and velocity measurements difficult. The benchmark study instead restricts the measurements to the absolute accelerations of the structural masses, the position of the control actuator piston, and the absolute acceleration of the control mass. This requires the use of output feedback controllers. In addition, since it can be easily measured, the absolute ground acceleration is included in the output measurements. Second, the benchmark study presents a 28 state structural model but requires the use of a prescribed order-controller (compensator) that has at most 12 states. This is an attempt to limit on-line computational requirements.

Linear quadratic optimal control generally requires the use of a full-order dynamic observer. An estimator may be used to reconstruct the state from measurements. This estimator is typically of the same order as the state, and for large scale problems this quickly becomes troublesome. The most popular approach to this difficulty is the use of truncation or model-reduction methods. An attempt is made to produce a reduced-order model which accurately represents the system. Then an estimator and a controller are designed to control the reduced-order model. One can only hope that this controller, when applied to the full-order system, results in good structural behavior. If not, as often happens, additional steps must be taken to improve the controlled systems behavior. There is no a priori reason to separate the model and the control problem. Kabamba and Longman [8,9] have shown an integrated formalism to optimize the closed-loop behavior of such systems. This study attempts to synthesize the inclusion of the excitation and open-loop controllers with this integrated formalism.

The current approach is to include the effects of a prescribed-order controller, and the use of output feedback, as opposed to full state feedback, in a linear quadratic optimal control design including the external excitation. Both feedback and feedforward terms are included in the controllers, whose gains are optimized by training on a set of known earthquakes. The proposed method will use known earthquake records to design an optimal controller whose control gains are constant and can be precalculated. The controller, having been designed with the knowledge of previous earthquakes, should be able to provide superior performance during an arbitrary future earthquake.

PROBLEM DEFINITION AND STATE SPACE EQUATIONS

The physical structure for the benchmark problem is an actively controlled, three story, single-bay, model building. The model has an active mass damper attached to the third floor. An evaluation model is presented by Spencer et al.[1] as a high-fidelity, linear, time-invariant 28 state space representation of the input-output model for the physical structure. The model is presented in continuous time as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{E}\ddot{x}_g(t) \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}_y\mathbf{x}(t) + \mathbf{D}_y u(t) + \mathbf{F}_y\ddot{x}_g(t) + \mathbf{v}(t) \tag{2}$$

$$\mathbf{z}(t) = \mathbf{C}_z\mathbf{x}(t) + \mathbf{D}_z u(t) + \mathbf{F}_z\ddot{x}_g(t) \tag{3}$$

where $\mathbf{x}$ is the state vector, u is the scalar control input, $\ddot{x}_g$ is the scalar ground acceleration, $\mathbf{y} = [x_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}, \ddot{x}_g]^T$, is the vector of responses that can be directly measured, and $\mathbf{z} = [x_1, x_2, x_3, x_m, \dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_p, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}]^T$, is the vector of responses that can be regulated. The matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{E}$, $\mathbf{C}_y$, $\mathbf{D}_y$, $\mathbf{F}_y$, $\mathbf{C}_z$, $\mathbf{D}_z$, and $\mathbf{F}_z$, are given for the benchmark problem. The vector $\mathbf{v}$ represents noise in the measurements. Eq. (1) represents the state space equations of motion, while eq. (2) and eq. (3) are the output equation and the controlled response equation, respectively.

For this benchmark study, the earthquake excitation $\ddot{x}_g(t)$, is modeled in both the time domain and the frequency domain. In the time domain, two acceleration time histories from previous earthquakes are used, the 1940 El Centro NS record and the 1968 Hachinohe NS record . In the frequency domain the excitation is modeled as a stationary random process with a spectral density defined by the Kanai-Tajimi spectrum, with a constant rms value of the ground motion.

The control problem is to design a discrete-time feedback compensator of the form:

$$\mathbf{x}^c_{k+1} = f_1(\mathbf{x}^c_k, \mathbf{y}_k, u_k, k) \tag{4}$$

$$u_k = f_2(\mathbf{x}^c_k, \mathbf{y}_k, k) \tag{5}$$

where $\mathbf{x}^c_k$, $\mathbf{y}_k$, and u_k are the discrete state vector for the compensator, the discrete output vector, and the discrete control command respectively, at the time $t = k\Delta t$. In this problem, the dimension of the vector $\mathbf{x}$ is limited up to 12 ($dim(\mathbf{x}^c) \leq 12$).

DISCRETE STATE SPACE EQUATIONS

However, since all the earthquake data is in discrete form and controller must be implemented in discrete form, it is more appropriate to formalize the proposed approach directly in discrete time. In a discrete time formulation, if the sampling time is relatively small, a zero order hold on the earthquake input and the control command can be satisfactorily assumed. In this case, the equations of motion eq. (1), the output equation eq. (2), and the controlled response vector eq. (3), are now expressed as:

$$\mathbf{x}_{k+1} = \mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d u_k + \mathbf{E}_d \ddot{x}_{gk}, \quad \mathbf{x}(k=0) = \mathbf{0} \tag{6}$$

$$\mathbf{y}_k = \mathbf{C}_y \mathbf{x}_k + \mathbf{D}_y u_k + \mathbf{F}_y \ddot{x}_{gk} + \mathbf{v}_k \tag{7}$$

$$\mathbf{z}_k = \mathbf{C}_z \mathbf{x}_k + \mathbf{D}_z u_k + \mathbf{F}_z \ddot{x}_{gk} \tag{8}$$

where $\mathbf{x}_k$, u_k, and $\ddot{x}_{gk}$ represent the discrete state vector, the discrete scalar control force, and the discrete scalar ground acceleration, respectively, at the time $t = k\Delta t$. The structure is assumed to be at equilibrium before the earthquake starts.

The authors have previously shown in detail the well–known parallelism between the continuous and discrete time formulations[2]. For details, the reader is referred to the previous work of the authors[2] or to the work of Meirovitch[10]. In summary, the discrete system matrix $\mathbf{A}_d$, the discrete controller location matrix $\mathbf{B}_d$, and the discrete external excitation location matrix $\mathbf{E}_d$ can be obtained from their continuous time counterparts by the following transformation:[11]

$$\mathbf{A}_d = e^{\mathbf{A}\Delta t} \tag{9}$$

$$\mathbf{B}_d = \int_0^{\Delta t} e^{\mathbf{A}\tau} d\tau \, \mathbf{B} \tag{10}$$

$$\mathbf{E}_d = \int_0^{\Delta t} e^{\mathbf{A}\tau} d\tau \, \mathbf{E} \tag{11}$$

These transformation relations correspond to the zero-order hold assumption for the control force and for the external excitation. In this case, equations (6), (7) and (8) become the exact discrete time representation of the equations (1), (2) and (3). The assumption of zero-order hold is certainly valid for digital controllers since their action is implemented at discrete time only. For the external excitation term, if different interpolation schemes are used (linear, quadratic, etc.), different expressions for eq. (11) are obtained.

PRESCRIBED ORDER AND OUTPUT FEEDBACK CONTROLLERS

To begin the design process, a discrete time controller which includes output feedback and acceleration feedforward terms is chosen. A size for the compensator is prescribed ($dim(\mathbf{x}^c) = 10$) to meet the benchmark limits, $dim(\mathbf{x}^c) \leq 12$. In this case the deterministic controller is represented as follows ($\mathbf{v}_k = \mathbf{0}$):

$$\mathbf{x}^c_{k+1} = \mathbf{D}^c \mathbf{x}^c_k + \mathbf{F}^c \mathbf{y}_k, \quad \mathbf{x}^c(k=0) = \mathbf{0} \tag{12}$$

$$u_k = \mathbf{G}^c \mathbf{x}^c_k + \mathbf{L}^c \ddot{x}_{gk} \tag{13}$$

Note that no attempt will be made to have the prescribed-order compensator accurately model the full-order system behavior. Instead, only good behavior from the controlled system as a whole is sought. All the matrices defining the controller ($\mathbf{D}^c$, $\mathbf{F}^c$, $\mathbf{G}^c$, and $\mathbf{L}^c$) are treated as unknown; their values are determined by the optimization process.

Since a deterministic approach is followed in the design process, the controller is designed considering no sensor noise ($\mathbf{v}_k = 0$). However, when testing the proposed controller for the benchmark problem using the given SIMULINK model, measurement noise has been included (rms noise of 0.01 Volts, as required). Indeed, all the benchmark requirements are met by using the discrete controller (designed with the discrete state space model) in simulations with the continuous state space SIMULINK model given in the benchmark problem. This includes A/D and D/A conversions (12-bit precision, $\pm$ 3 V.), the sampling period (0.001 sec.), and the time delay (200 μ sec.) as described by the benchmark literature. The controller is directly substituted for the reference controller in the benchmark simulations with the single exception that the new controller includes feedback from the entire output vector $\mathbf{y}(t)$.

AUGMENTED DISCRETE STATE SPACE

In order to derive an optimal solution for the controller, a new augmented discrete state vector, which is composed of the discrete system state vector $\mathbf{x}_k$ and the discrete compensator state vector $\mathbf{x}_k^c$, is defined. Such a new state vector will include the dynamics of both the structural system and the controller. Using eqs. (6), (12), and (13) the augmented discrete state can be written as:

$$\begin{bmatrix} \mathbf{x}_{k+1} \\ \mathbf{x}_{k+1}^c \end{bmatrix} = \begin{bmatrix} \mathbf{A}_d & \mathbf{B}_d\mathbf{G}^c \\ \mathbf{F}^c\mathbf{C}_y & \mathbf{D}^c + \mathbf{F}^c\mathbf{D}_y\mathbf{G}^c \end{bmatrix} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{x}_k^c \end{bmatrix} + \begin{bmatrix} \mathbf{B}_d\mathbf{L}^c + \mathbf{E}_d \\ \mathbf{F}^c\mathbf{D}_y\mathbf{L}^c + \mathbf{F}^c\mathbf{F}_y \end{bmatrix} \ddot{x}_{gk} \tag{14}$$

The augmented discrete state vector $\hat{\mathbf{x}}_k$ is defined and the unknown matrices $\mathbf{D}^c$, $\mathbf{F}^c$, $\mathbf{G}^c$, and $\mathbf{L}^c$ are collected as follows:

$$\hat{\mathbf{x}}_k = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{x}_k^c \end{bmatrix}, \qquad \mathbf{P} = \begin{bmatrix} \mathbf{0}_{1y} & \mathbf{G}^c \\ \mathbf{F}^c & \mathbf{D}^c \end{bmatrix}, \qquad \mathbf{L} = \begin{bmatrix} \mathbf{L}^c \\ \mathbf{0}_{x^c1} \end{bmatrix} \tag{15}$$

in which the matrix $\mathbf{0}_{ij}$ is the zero matrix of dimensions $i \times j$. It is important to point out that the matrix $\mathbf{P}$ contains all the information about the prescribed-order compensator and the closed-loop control gains, while the matrix $\mathbf{L}$ includes the open-loop control gains.

In order to simplify the lengthy mathematical derivations, let's define the additional known matrices, $\hat{\mathbf{A}}_d$, $\hat{\mathbf{B}}_d$, $\hat{\mathbf{C}}_y$, $\hat{\mathbf{D}}_y$, $\hat{\mathbf{F}}_y$, $\hat{\mathbf{E}}_d$, Φ_1, and Φ_2, as follows:

$$\hat{\mathbf{A}}_d = \begin{bmatrix} \mathbf{A}_d & \mathbf{0}_{xx^c} \\ \mathbf{0}_{x^cx} & \mathbf{0}_{x^cx^c} \end{bmatrix}, \hat{\mathbf{B}}_d = \begin{bmatrix} \mathbf{B}_d & \mathbf{0}_{xx^c} \\ \mathbf{0}_{x^c1} & \mathbf{I}_{x^c} \end{bmatrix}, \hat{\mathbf{C}}_y = \begin{bmatrix} \mathbf{C}_y & \mathbf{0}_{yx^c} \\ \mathbf{0}_{x^cx} & \mathbf{I}_{x^c} \end{bmatrix}, \hat{\mathbf{D}}_y = \begin{bmatrix} \mathbf{D}_y & \mathbf{0}_{yx^c} \\ \mathbf{0}_{x^c1} & \mathbf{0}_{x^cx^c} \end{bmatrix} \tag{16}$$

$$\hat{\mathbf{F}}_y = \begin{bmatrix} \mathbf{F}_y \\ \mathbf{0}_{x^c1} \end{bmatrix}, \hat{\mathbf{E}}_d = \begin{bmatrix} \mathbf{E}_d \\ \mathbf{0}_{x^c1} \end{bmatrix}, \Phi_1 = \begin{bmatrix} \mathbf{0}_{x1} & \mathbf{0}_{xx^c} \\ \mathbf{0}_{x^c1} & \mathbf{I}_{x^c} \end{bmatrix} \Phi_2 = \begin{bmatrix} \mathbf{0}_{yx} & \mathbf{0}_{yx^c} \\ \mathbf{0}_{x^cx} & \mathbf{I}_{x^c} \end{bmatrix} \tag{17}$$

in which the matrix $\mathbf{I}_i$ is the identity matrix of dimensions $i \times i$. This allows one to collect terms and rewrite the augmented discrete state space equations of motion, using eqs. (14), (15), (16), and (17), as:

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A}_d^* \hat{\mathbf{x}}_k + \mathbf{E}_d^* \ddot{x}_{gk} \tag{18}$$

where the two new matrices, $\mathbf{A}_d^*$ and $\mathbf{E}_d^*$, are defined as:

$$\mathbf{A}_d^* = [\hat{\mathbf{A}}_d + \hat{\mathbf{B}}_d \mathbf{P} \hat{\mathbf{C}}_y + \Phi_1 \mathbf{P} \hat{\mathbf{D}}_y \mathbf{P} \Phi_2], \quad \mathbf{E}_d^* = [\hat{\mathbf{E}}_d + \Phi_1 \mathbf{P} \hat{\mathbf{F}}_y + [\Phi_1 \mathbf{P} \hat{\mathbf{D}}_y + \hat{\mathbf{B}}_d] \mathbf{L}] \tag{19}$$

LINEAR QUADRATIC OPTIMAL CONTROL

In order to use an optimal control approach, it is necessary to define a discrete cost functional that includes the appropriately weighted variables which are critical to the problem. For this problem, such variables are the vector of responses that can be regulated $\mathbf{z}_k$, and the applied control force u_k are chosen. Minimizing this discrete cost functional with respect to the control and the state, constrained by the appropriate equations of motion, defines an optimal solution [12, 13]. For this problem, the form of the controller is known a priori, as presented in eqs. (12) and (13). Hence, it is necessary to define a quadratic discrete cost functional J_d, which will be minimized with respect to the control matrices $\mathbf{P}$ and $\mathbf{L}$, constrained by the augmented discrete state equations of motion, eq. (18). Since the form of the controller and the external excitation are known a priori, this optimization problem reduces to a parametric optimization for the control matrices. The formalism will first be derived for a single known external excitation, then extended to include an ensemble of earthquake records.

DISCRETE COST FUNCTIONAL

Let's define a discrete quadratic cost functional as:

$$J_d = \sum_{k=0}^{N} \{\mathbf{z}_k^T \mathbf{Q} \mathbf{z}_k + u_k^T \mathbf{R} u_k\} \tag{20}$$

where $\mathbf{Q}$ and $\mathbf{R}$ represent appropriate weighting matrices. The optimization problem can be stated as follows:

$$\min_{\mathbf{P},\mathbf{L}}(J_d) \tag{21}$$

$$\text{subject to: } \hat{\mathbf{x}}_{k+1} = \mathbf{A}_d^* \hat{\mathbf{x}}_k + \mathbf{E}_d^* \ddot{x}_{gk} \tag{22}$$

$$k = 0, 1, \ldots, N; \qquad N = t_f/\Delta t \tag{23}$$

where t_f represents the final duration of the dynamic analysis. Such a final time should be sufficiently longer than the duration of the earthquake excitation so that, if any instability effect is present, the optimization process will be able to recognize it and to take corrective action.

However, the cost functional must now be written only in terms of the augmented discrete state vector $\hat{\mathbf{x}}_k$ and the external excitation $\ddot{x}_{gk}$. By inserting the equation for the controller, eq. (13), into the equation for the discrete regulated response vector, eq. (8), and defining two new known matrices, $\hat{\mathbf{C}}_z$ and $\hat{\mathbf{D}}_z$, the regulated response vector can be expressed in terms of the augmented discrete state vector and the external excitation as follows:

$$\hat{\mathbf{C}}_z = [\,\mathbf{C}_z \quad \mathbf{0}_{zc}\,], \quad \hat{\mathbf{D}}_z = [\,\mathbf{D}_z \quad \mathbf{0}_{zc}\,] \tag{24}$$

$$\mathbf{z}_k = [\hat{\mathbf{C}}_z + \hat{\mathbf{D}}_z \mathbf{P} \Phi_2] \hat{\mathbf{x}}_k + [\hat{\mathbf{D}}_z \mathbf{L} + \mathbf{F}_z] \ddot{x}_{gk} \tag{25}$$

In addition, after some mathematical manipulations, the second term in the discrete cost functional, $u_k^T \mathbf{R} u_k$, can also be represented in terms of the augmented discrete state vector and the external excitation as:

$$u_k^T \mathbf{R} u_k = \hat{\mathbf{x}}_k^T \Phi_2^T \mathbf{P}^T \hat{\mathbf{R}} \mathbf{P} \Phi_2 \hat{\mathbf{x}}_k + 2\hat{\mathbf{x}}_k^T \Phi_2^T \mathbf{P}^T \hat{\mathbf{R}} \mathbf{L} \ddot{x}_{gk} + \ddot{x}_{gk} \mathbf{L}^T \hat{\mathbf{R}} \mathbf{L} \ddot{x}_{gk} \tag{26}$$

where:

$$\hat{\mathbf{R}} = \begin{bmatrix} \mathbf{R} & \mathbf{0}_{1x^c} \\ \mathbf{0}_{x^c1} & \mathbf{0}_{x^cx^c} \end{bmatrix} \tag{27}$$

Using eqs. (25) and (26) the discrete cost functional J_d, (eq. (20)), becomes:

$$J_d = \sum_{k=0}^{N} [\hat{\mathbf{x}}_k^T \mathbf{Q}_1 \hat{\mathbf{x}}_k + 2\hat{\mathbf{x}}_k^T \mathbf{Q}_2 \ddot{x}_{gk} + \ddot{x}_{gk}^T \mathbf{Q}_3 \ddot{x}_{gk}] \tag{28}$$

in which:

$$\mathbf{Q}_1 = \hat{\mathbf{C}}_z^T \mathbf{Q} \hat{\mathbf{C}}_z + 2\hat{\mathbf{C}}_z^T \mathbf{Q} \hat{\mathbf{D}}_z \mathbf{P} \Phi_2 + \Phi_2^T \mathbf{P}^T \hat{\mathbf{D}}_z^T \mathbf{Q} \hat{\mathbf{D}}_z \mathbf{P} \Phi_2 + \Phi_2^T \mathbf{P}^T \hat{\mathbf{R}} \mathbf{P} \Phi_2 \tag{29}$$

$$\mathbf{Q}_2 = \hat{\mathbf{C}}_z^T \mathbf{Q} \hat{\mathbf{D}}_z \mathbf{L} + \hat{\mathbf{C}}_z^T \mathbf{Q} \mathbf{F}_z + \Phi_2^T \mathbf{P}^T \hat{\mathbf{D}}_z^T \mathbf{Q} \hat{\mathbf{D}}_z \mathbf{L} + \Phi_2^T \mathbf{P}^T \hat{\mathbf{D}}_z^T \mathbf{Q} \mathbf{F}_z + \Phi_2^T \mathbf{P}^T \hat{\mathbf{R}} \mathbf{L} \tag{30}$$

$$\mathbf{Q}_3 = \mathbf{L}^T \hat{\mathbf{D}}_z^T \mathbf{Q} \hat{\mathbf{D}}_z \mathbf{L} + 2\mathbf{L}^T \hat{\mathbf{D}}_z^T \mathbf{Q} \mathbf{F}_z + \mathbf{F}_z^T \mathbf{Q} \mathbf{F}_z + \mathbf{L}^T \hat{\mathbf{R}} \mathbf{L} \tag{31}$$

From equation (28) it is clear that the discrete functional J_d is now a functionof the unknown control matrices P and L and of the discrete augmented state $\hat{\mathbf{x}}_k$.

AUGMENTED DISCRETE COST FUNCTIONAL AND CONDITIONS FOR OPTIMALITY

In the proposed optimal control approach, it is necessary to constrain the discrete cost functional J_d (eq. (28)) by the augmented discrete equations of motion, eq. (18). This is accomplished by the use of Lagrange undetermined multipliers λ_k. These multipliers, often called costate vectors, allow the construction of an adjoint discrete cost functional J_{da}, defined as:

$$J_{da} = \sum_{k=0}^{N} \left\{ \hat{\mathbf{x}}_k^T \mathbf{Q}_1 \hat{\mathbf{x}}_k + 2\hat{\mathbf{x}}_k^T \mathbf{Q}_2 \ddot{x}_{gk} + \ddot{x}_{gk}^T \mathbf{Q}_3 \ddot{x}_{gk} + \lambda_k^T [\mathbf{A}_d^* \hat{\mathbf{x}}_k + \mathbf{E}_d^* \ddot{x}_{gk} - \hat{\mathbf{x}}_{k+1}] \right\} \tag{32}$$

where λ_k, the discrete costate vector, has the same dimensions as the augmented state vector $\hat{\mathbf{x}}_k$. The term multiplied by λ_k in J_{da} is identically equal to zero at all times so that the value of the adjoint discrete cost functional is identical to that of the original functional. However, the value of the first variation of the functional is changed.

A properly defined adjoint discrete cost functional in terms of the augmented discrete state and the external excitation is now obtained and the problem expressed in eqs. (21), (22), and (23) is now reduced to a standard form:

$$\min_{\mathbf{P},\mathbf{L}}(J_{da}) \tag{33}$$

Since the cost functional has been reduced to the above compact form, the necessary conditions for optimality are equally compact. These conditions can be derived by setting the first variation of the adjoint discrete cost functional J_{da} with respect to the control and state equal to zero [8, 9]. In a discrete time formulation, the augmented cost functional can also be considered as a function of discrete variables (λ_j, $\hat{\mathbf{x}}_j$, $\mathbf{P}$ and $\mathbf{L}$), which are assumed to be uncorrelated, and the optimality conditions are simply reduced to:

$$\frac{\partial J_{da}}{\partial \lambda_j} = \mathbf{0} \quad j = 0, 1, \ldots, N \tag{34}$$

$$\frac{\partial J_{da}}{\partial \hat{\mathbf{x}}_j} = \mathbf{0} \quad j = 1, \ldots, N \tag{35}$$

$$\frac{\partial J_{da}}{\partial \hat{\mathbf{x}}_{N+1}} = \mathbf{0} \tag{36}$$

$$\frac{\partial J_{da}}{\partial \mathbf{P}} = \mathbf{0} \tag{37}$$

$$\frac{\partial J_{da}}{\partial \mathbf{L}} = \mathbf{0} \tag{38}$$

GOVERNING EQUATIONS

Discrete State and Costate Equations

In a discrete time formulation, the necessary conditions for optimality generate a series of difference equations. From the first necessary condition, eq. (34), the augmented state space equations of motion are recovered. Since the initial conditions for these equations are given, they can be solved forward in time to obtain the augmented state time history:

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A}_d^* \hat{\mathbf{x}}_k + \mathbf{E}_d^* \ddot{x}_{gk} \tag{39}$$

$$k = 0, 1, \ldots, N \tag{40}$$

$$\hat{\mathbf{x}}_0 = \mathbf{0} \tag{41}$$

From the necessary conditions expressed in eqs. (35) and (36), a series of difference equations for the costate vector λ_k and final conditions at $k = N$ are obtained, so that the following equations can now be solved backwards in time to obtain the costate time history:

$$\lambda_{k-1} = \mathbf{A}_d^{*T} \lambda_k + [\mathbf{Q}_1 + \mathbf{Q}_1^T]\hat{\mathbf{x}}_k + 2\mathbf{Q}_2 \ddot{x}_{gk} \tag{42}$$

$$k = 1, \ldots, N \tag{43}$$

$$\lambda_N = \mathbf{0} \tag{44}$$

It is important to note that these equations can now be solved backwards in time because the external excitation is recorded from a prior earthquake.

Explicit Gradient Equations

The final two necessary conditions, eqs. (37) and (38), lead to explicit gradient equations for the adjoint discrete cost functional J_{da} with respect to the control matrices $\mathbf{P}$ and $\mathbf{L}$. The structure of these equations encourage the use of numerical solution techniques that use explicit gradient information to obtain a minimum of J_{da}:

$$\begin{aligned}\frac{\partial J_{da}}{\partial \mathbf{P}} = \sum_{k=0}^{N} [2\{\hat{\mathbf{D}}_z^T \mathbf{Q} \mathbf{z}_k + \hat{\mathbf{R}}[\mathbf{P}\Phi_2 \hat{\mathbf{x}}_k + \mathbf{L}\ddot{x}_{gk}]\}\hat{\mathbf{x}}_k^T \Phi_2^T + \hat{\mathbf{B}}_d^T \lambda_k \hat{\mathbf{x}}_k^T \hat{\mathbf{C}}_y^T \\ + \hat{\mathbf{D}}_y^T \mathbf{P}^T \Phi_1^T \lambda_k \hat{\mathbf{x}}_k^T \Phi_2^T + \Phi_1^T \lambda_k \hat{\mathbf{x}}_k^T \Phi_2^T \mathbf{P}^T \hat{\mathbf{D}}_y^T + \Phi_1^T \lambda_k \ddot{x}_{gk}^T [\hat{\mathbf{F}}_y^T + \mathbf{L}^T \hat{\mathbf{D}}_y^T]] = \mathbf{0}\end{aligned} \tag{45}$$

$$\frac{\partial J_{da}}{\partial \mathbf{L}} = \sum_{k=0}^{N} [2\{\hat{\mathbf{D}}_z^T \mathbf{Q} \mathbf{z}_k + \hat{\mathbf{R}}[\mathbf{P}\Phi_2 \hat{\mathbf{x}}_k + \mathbf{L}\ddot{x}_{gk}]\}\ddot{x}_{gk}^T + [\hat{\mathbf{D}}_y^T \mathbf{P} \Phi_1^T + \hat{\mathbf{B}}_d^T]\lambda_k \ddot{x}_{gk}^T] = \mathbf{0} \tag{46}$$

The governing equations (eqs. (39)–(44)) define a two-point boundary value problem since initial conditions for $\hat{\mathbf{x}}_k$ are prescribed at $k = 0$, while the final conditions for λ_k are given at $k = N$. The discrete state equations for $\hat{\mathbf{x}}_k$ are independent of the discrete costate vectors, while the costate equations for λ_k contain the discrete state vectors explicitly. Therefore, it is possible to solve the discrete state equations forward in time first and then solve the discrete costate equations backwards in time to obtain the complete time histories for both.

Solving the previous system of equations, i.e. minimizing J_{da}, generates a solution for the unknown matrices $\mathbf{P}$ and $\mathbf{L}$, representing the solution of the problem. This development has designed a controller optimized over a single known earthquake record. However, since the controller must work well for any future earthquake, we will train the controller over an ensemble of earthquakes. This is done in an attempt to capture the essential characteristics necessary for the controller to perform well during any future earthquake. Such a controller can be considered as a "global" optimal control system for a set of known earthquakes.

MULTIPLE EARTHQUAKES

The previous solution is now extended to include an ensemble of earthquake records. The idea is to choose one set of values for the control matrices $\mathbf{P}$ and $\mathbf{L}$ that is optimized over an entire ensemble of known earthquakes. If such an ensemble contains p records, then a new problem can be formulated by defining a new augmented discrete cost functional J'_{da} which is the summation of the augmented discrete cost functionals for each of the p earthquakes. The multiple earthquake augmented discrete cost functional J'_{da} is now written as:

$$J'_{da} = \sum_{i=1}^{p} J^i_{da} \tag{47}$$

in which the augmented discrete cost functional for each earthquake is now expressed as:

$$J^i_{da} = \sum_{k=0}^{N} \left\{ \hat{\mathbf{x}}_k^{i^T} \mathbf{Q}_1 \hat{\mathbf{x}}_k^i + 2\hat{\mathbf{x}}_k^{i^T} \mathbf{Q}_2 \ddot{x}^i_{gk} + \ddot{x}^{i^T}_{gk} \mathbf{Q}_3 \ddot{x}^i_{gk} + \lambda_k^{i^T} \left[\mathbf{A}_d^* \hat{\mathbf{x}}_k^i + \mathbf{E}_d^* \ddot{x}^i_{gk} - \hat{\mathbf{x}}^i_{k+1} \right] \right\} \tag{48}$$

where the index i denotes the individual earthquake and runs from 1 to p.

The new problem statement can now be formulated as:

$$\min_{\mathbf{P},\mathbf{L}}(J'_{da}) \tag{49}$$

It is then necessary to find the minimum of this newly defined quadratic cost functional with respect to a single set of control gains. In this way, one can obtain one set of values for $\mathbf{P}$ and $\mathbf{L}$ that have been optimized over an ensemble of earthquakes and shoud assure good structural behavior for an arbitrary future earthquake.

By considering the augmented cost functional as a function of independent discrete variables, the discrete time necessary conditions for optimality are derived by setting the first variation of the multiple earthquake discrete augmented cost functional J'_{da} with respect to the control and the state equal to zero [8, 9, 14]:

$$\frac{\partial J'_{da}}{\partial \lambda^i_j} = \mathbf{0} \quad j = 0, 1, \ldots, N \tag{50}$$

$$\frac{\partial J'_{da}}{\partial \hat{\mathbf{x}}^i_j} = \mathbf{0} \quad j = 1, \ldots, N \tag{51}$$

$$\frac{\partial J'_{da}}{\partial \hat{\mathbf{x}}^i_{N+1}} = \mathbf{0} \tag{52}$$

$$\frac{\partial J'_{da}}{\partial \mathbf{P}} = \mathbf{0} \tag{53}$$

$$\frac{\partial J'_{da}}{\partial \mathbf{L}} = \mathbf{0} \tag{54}$$

with $i = 1, \ldots, p$.

GOVERNING EQUATIONS

Discrete State and Costate Equations

The necessary conditions for optimality generate p sets of equations, one set for each of the p earthquakes in the ensemble. From eq. (50) discrete state space equations of motion for each earthquake record are recovered. The initial conditions for each of these equations are identical and given. These equations can then be solved forwards in time to obtain all the discrete state time histories:

$$\hat{\mathbf{x}}_{k+1}^i = \mathbf{A}_d^* \hat{\mathbf{x}}_k^i + \mathbf{E}_d^* \ddot{x}_{gk}^i \tag{55}$$

$$k = 0, 1, \ldots N, \quad i = 1 \ldots p \tag{56}$$

$$\hat{\mathbf{x}}_0^i = \mathbf{0} \tag{57}$$

Similarly to the single earthquake case, difference equations for the discrete costate vectors λ_k^i and final conditions at k=N are obtained for each earthquake time history by using eqs(51) and (52):

$$\lambda_{k-1}^i = \mathbf{A}_d^{*T} \lambda_k^i + [\mathbf{Q}_1 + \mathbf{Q}_1^T] \hat{\mathbf{x}}_k^i + 2\mathbf{Q}_2 \ddot{x}_{gk}^i \tag{58}$$

$$k = 1, \ldots, N, \quad i = 1 \ldots p \tag{59}$$

$$\lambda_N^i = \mathbf{0} \tag{60}$$

It is noteworthy that, in the training process, all the earthquake time histories included in the ensemble are known a priori. Hence, eqs. (58) can be solved backwards in time to obtain all the discrete costate time histories.

Explicit Gradient Equations

The final necessary conditions, eqs. (53) and (54), lead to explicit gradient equations for the multiple earthquake augmented discrete cost functional J_{da}' with respect to the control matrices $\mathbf{P}$ and $\mathbf{L}$. Note that only a single gradient equation is generated for each control matrix, independetly from the number of earthquakes in the ensemble. This is a direct result of asking for a single set of control gains optimized across the entire ensemble; therefore, for any number of different earthquakes in the ensemble, the solution should converge to a single optimal solution associated with a single set of control gains.

The form of these equations encourages the use of numerical solution techniques that use explicit gradient information to obtain a minimum of J'_{da}:

$$\frac{\partial J'_{da}}{\partial \mathbf{P}} = \sum_{i=1}^{p}\sum_{k=0}^{N}[2\{\hat{\mathbf{D}}_z^T\mathbf{Q}\mathbf{z}_k^i + \hat{\mathbf{R}}[\mathbf{P}\Phi_2\hat{\mathbf{x}}_k^i + \mathbf{L}\ddot{x}_{gk}^i]\}\hat{\mathbf{x}}_k^{iT}\Phi_2^T + \hat{\mathbf{B}}_d^T\lambda_k^i\hat{\mathbf{x}}_k^{iT}\hat{\mathbf{C}}_y^T + \hat{\mathbf{D}}_y^T\mathbf{P}^T\Phi_1^T\lambda_k^i\hat{\mathbf{x}}_k^{iT}\Phi_2^T$$
$$+\Phi_1^T\lambda_k^i\hat{\mathbf{x}}_k^{iT}\Phi_2^T\mathbf{P}^T\hat{\mathbf{D}}_y^T + \Phi_1^T\lambda_k^i\ddot{x}_{gk}^{iT}[\hat{\mathbf{F}}_y^T + \mathbf{L}^T\hat{\mathbf{D}}_y^T]] = \mathbf{0} \tag{61}$$

$$\frac{\partial J'_{da}}{\partial \mathbf{L}} = \sum_{i=1}^{p}\sum_{k=0}^{N}[2\{\hat{\mathbf{D}}_z^T\mathbf{Q}\mathbf{z}_k^i + \hat{\mathbf{R}}[\mathbf{P}\Phi_2\hat{\mathbf{x}}_k^i + \mathbf{L}\ddot{x}_{gk}^i]\}\ddot{x}_{gk}^{iT} + [\hat{\mathbf{D}}_y^T\mathbf{P}\Phi_1^T + \hat{\mathbf{B}}_d^T]\lambda_k^i\ddot{x}_{gk}^{iT}] = \mathbf{0} \tag{62}$$

The structure of the multiple earthquake solution is analogous to the single earthquake case with the exception that, while one set of discrete state and discrete costate equations is recovered for each earthquake, only a single gradient equation is generated for each control matrix. This is the result of requiring a single set of control gains which are optimized over the set of p earthquakes.

In this class of problems related to seismic excitation, stability of the resulting control systems is assured by the fact that 1) the earthquake excitation is a very rich excitation, 2) we use an ensemble of earthquakes, and 3) the final time exceeds the duration of each of the earthquakes in the ensemble. In theory, it is conceivable that the algorithm could converge to an unstable feedback control law having the property that an unstable mode or modes vibrate exacly opposite to the earthquake. However, the complexity of the earthquake excitation makes such a cancellation very difficult. It becomes essentially impossible when we consider an ensemble of earthquakes. In addition, any instability is suppressed in the optimization process when it becomes dominant after the earthquake excitation ends. As a result, the control gains will produce an asymptotically stable system having all closed loop eigenvalues within the unit circle.

The proposed control algorithm has a natural extension for time-varying systems, which yields time-varying control gains.

ANALYSIS OF THE RESULTS

The proposed algorithm was used on the AMD controlled system from the benchmark problem. The physical structure is represented by a time invariant, 28 state space linear model, eqs. (1), whose system matrices are provided by Spencer et al [1]. A 10th order compensator, eqs. (12), was selected, where the output feedback measurements included the entire vector of responses that could be directly measured, $\mathrm{y} = [x_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}, \ddot{x}_g]^T$.

The merit of the controller has been based on criteria given in terms of both rms and peak response quantities. Ten nondimensional performance indices, J_i ($i = 1, 2, ..10$) were provided, the first five of which were in terms of rms responses while the remaining five were in terms of peak responses. Superior performance of the controller is indicated by smaller values of these performance indices. In addition, limitations on the actuator's acceleration, displacement and voltage were imposed in both the frequency and time domain (Table 1).

For the evaluation criteria in the frequency domain, the earthquake excitation $\ddot{x}_g$ has been represented as a stationary random process, with a spectral density function represented by the Kanai-Tajimi spectrum. The values of the frequency ω_g and damping ratio ξ_g corresponding to the worst-case condition were observed to be near those for the open-loop system; thus RMS evaluations were performed at the nominal ω_g = 37.3 rads/sec and ξ_g = 0.3 (duration of 300 secs.). For the evaluation criteria in the time domain, two recorded time histories of the ground acceleration (El Centro 1940 NS and Hachinohe 1968 NS) have been provided. Since only two known earthquake time histories were given as part of the benchmark study, it was decided to train the controller over a single 'known' earthquake (Hachinohe 1968 NS) and test over a single 'unknown' earthquake (El Centro 1940 NS). This allowed us to maintain the rigid distinction between 'known' earthquakes used to train the control systems and 'unknown' earthquakes used to simulate the future behavior of the controlled structure. However, the condition of training the controller just over a single known time history represents a limitation for the controller to capture the essential characteristics necessary to perform well during an arbitrary future earthquake.

Several controllers were designed by varying control and state weighting matrices in the proposed cost functional. For the numerical minimization process, a quasi–Newton optimization algorithm based on the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method was used. The results are presented in tables 2, 3, 4 and 5 where the performance indices and the rms and peak values of the actuator's acceleration, displacement and voltage are presented. For the performance indices in the time domain, the values corresponding to the two time histories (El Centro and Hachinohe) are reported so that we can keep the duality between "known" and "unknown" earthquakes, crucial for our approach. We include the results from the Hachinohe earthquake for completeness with respect to the benchmark study, and also to indicate that even for a 'known' earthquake it is impossible to cancel out the disturbance completely if there is a limitation on control actions. The effectiveness of these controllers is also compared with that of a reference controller proposed in the benchmark literature.

The first controller (table 2) is obtained by considering the weighting matrices Q and R such that $Q(1,1) = Q(2,2) = Q(3,3) = Q(12,12) = 1$ while $R = 12$. It is noteworthy to point out the advantage that, although the proposed approach sonsiders a reduced–order controller, weights are imposed on elements of the vector $\mathbf{z}$ which are quantities directly related to the full-order structural model. Fig. 1 presents the discrete transfer functions between the ground acceleration and the floor accelerations for the controlled structure. The results show the effectiveness of the proposed controller. Compared to the reference controller, the performance indices associated with the structural response (J_1, J_2, J_6, J_7) present an increment which varies from 1 to 18 percent while the indices associated with the control resources $(J_3, J_4, J_5, J_8, J_9, J_{10})$ drastically reduce up to 60 percent. Looking at the summation of all the performace indices, the proposed controller presents a reduction of the order of 10 percent. All the maximum values of the actuator's acceleration, displacement and voltage are well within the limits imposed by the study.

Similar results are obtaining for controller 2 (table 3) and controller 3 (table 4). In controller 2, the relative weights of control action and structural response are varied in an attempt to reduce the required control effort. A weighting matrix Q with $Q(1,1) = Q(2,2) = Q(3,3) = 1$, $Q(12,12) = 2$ is used while the weight associated with the control forces is $R = 25$. In this case, the results confirm the effectiveness of the proposed controller, with a more evident reduction in the control effort offset by an increase in the structural response.

In controller 3 (table 4), attention is given toward a further reduction in structural response. The weighting matrices Q and R have now the following values: $Q(1,1) = Q(2,2) = Q(3,3) = 1$, $Q(12,12) = 3$ and $R = 9$. In order to meet the constraints on the acceleration of the AMD, it is necessary to increase the associated weight in the cost functional ($Q(12,12)$). This, again, illustrates the advantage of including the entire vector of regulated responses in the definition of the cost functional and confirms the validity of the proposed approach and its flexibility in the trade–off of structural response and control force. Compared to those obtained for the reference controller and with the previous two controllers, the performance indices associated with the structural response show a substantial improvement of the order of 25 percent with respect to the previous controller. As expected, the required control force increases, as shown by the values of the performance indices J_3–J_5 and J_8–J_{10}.

Figs. 2 - 4 show the performances of these proposed controllers in reducing the structural vibrations for the case of an unknwon "future" earthquake (El Centro). These figure confirm the effectiveness of the proposed control algorithm.

A fourth controller (table 5) is designed in an attempt to further reduce the structural response so that the associated performance indeces are lower than those provided for the reference controller. This task is accomplished by increasing the values of the weighting matrix Q ($Q(1,1) = 1.2$, $Q(2,2) = 3.6$, $Q(3,3) = 1.2$, $Q(12,12) = 3.8$). As shown in table 5, the performance indices associated with the structural displacements and accelerations are slightly reduced with respect to those obtained for the reference model, with the exception of J_7 which is almost identical (0.7102 vs. 0.7180). On the contrary, some of the performance indices associated with the control force present values that are higher than those corresponding to the reference model. Looking at the global controller performance obtained by adding all the performance indices, such a proposed controller is performs slightly better than the reference one. In selecting all the worst performance indices for the proposed controller, values from both the unknown earthquake (Hachinohe) and the known earthquake (El Centro) have been used. All the restrictions imposed on the actuator's acceleration, displacement and voltage are satisfied.

The effectiveness of such a controller is confirmed by fig. 5, where the transfer functions between the ground acceleration and the acceleration of the three floors present a significant reduction of the peak values compared to the case where no active control force is applied. Figs. 6 and 7 show the controller performances in controlling the displacements and the accelerations of a structure subjected to an unknown ground excitation (El Centro earthquake).

The four example controllers explicitly show the potential trade-offs between structural response and control action. By changing the values of the weighting matrices Q and R, it is possible to obtain controllers which determine excellent structural behavior within the constraints associated with maximum control force and maximum actuator voltage and acceleration. All the proposed controllers are asymptotically stable, presenting all the closed loop discrete eigenvalues within the unit circle. Such controllers have the longest time constant ranging from 0.1186951 (controller 1) to 0.118669 (controller 4), corresponding to a range of maximum eigenvalues from 0.99988131 to 0.999881337. These represent the amount of decay in one sample time ($\Delta t = 0.001$ sec.). In performing the stability analysis, the augmented system, including the dynamics of the structure and of the controller, must be considered. These results also confirm the conclusions of the study presented in [2].

CONCLUSIONS

The results presented in this study confirm the effectiveness of the proposed control algorithm for the analysis of MDOF systems subjected to earthquake excitation. The attempt to synthesize the inclusion of the external excitation and open–loop control term with the integrated formalism for the control and modelling problems has proven successful. In this way, all the problems associated with the control of reduced-order models are eliminated. The control gains are obtained following an approach based on training the controller on an ensemble of known earthquake records. However, since only two earthquake records were provided in this study, an ensemble of one "known" earthquake time history has been used to train the controllers; this represents a serious limitation for the proposed approach. In addition, the effectiveness of the proposed methodology could have been compromised by the identification process used in the development of the evaluation model. Nevertheless, the proposed controllers show good structural performances with control forces well within the limits imposed by the benchmark problem. By changing the weighting factors on quantities directly related to the full–order model, it is possible to trade–offs better performances in terms of structural response and/or control forces.

REFERENCES

[1] B. F. Spencer Jr., S. J. Dyke, and H. S. Deoskar, 'Benchmark problems in structural control', to appear in a special issue of *Earthquake eng. and struct. dyn.* (1997).

[2] G. F. Panariello, R. Betti, and R. W. Longman, 'Optimal structural control via training on an ensemble of earthquakes', accepted for publication *J. eng. mech. div ASCE* (1997).

[3] G. W. Housner, T. T. Soong, and S.F. Masri, 'Second generation of active structural control in civil engineering', *First world conference on structural control*, **1**, Los Angeles, CA, USA (1994).

[4] T. T. Soong, *Active structural control : theory and practice* Longman Scientific and Technical,Essex, England, 1990.

[5] J. N. Yang, A. Akbarpout, and P. Ghaemmaghami, 'New control algoritms for structural control', *Journal of Engineering Mechanics*, **113** (9), 1369-86, 1987.

[6] T. Kobori, 'Future direction on research and development of seismic-response-controlled structure', *First world conference on structural control*, **1**, Los Angeles, CA, USA (1994).

[7] S. Bittanti, A. J. Laub, and J. C. Willems, (Eds.) *The Riccati Equation*, Springer-Verlag, Berlin, Germany, 1991.

[8] P. T. Kabamba and R. W. Longman, 'An integrated approach to reduced order control theory', *Third VPI & SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft*, Blacksburg, Virginia, USA, 1981.

[9] P. T. Kabamba and R. W. Longman, 'An integrated approach to reduced-order control theory', *Optimal Control Applications & Methods*, **4**, 405-415, 1983.

[10] L. Meirovitch, *Dynamics and Control of Structures*, John Wiley & Sons, New York, 1989.

[11] K. J. Astrom and B. Wittenmark, *Computer-controlled systems : theory and practice*, Prentice Hall Information and System Sciences Series, Prentice Hall, Englewood Cliffs, NJ, USA, 1990.

[12] D. A. Pierre, *Optimization theory with applications*, Dover Publications, New York, USA, 1986

[13] H. Kwakernaak and R. Sivan, *Linear optimal control systems*, Wiley, New YOrk, U.S.A. 1972.

[14] R. F. Stengel, *Optimal control and estimation*, Dover Publications, New York, U.S.A. 1994.

max. rms actuator voltage	**1.0 volts**
max. rms actuator displacement	**3.0 cm**
max. rms actuator acceleration	**2.0 g**
max. actuator voltage	**3.0 volts**
max. actuator displacement	**9.0 cm**
max. actuator acceleration	**6.0 g**

Table 1: Maximum allowable values for controller actions.

Controller #1	Reference	PBL
	ω_g = 37.3 rad/sec	ω_g = 37.3 rad/sec
	ξ_g = 0.3	ξ_g = 0.3
J_1	**0.2840**	**0.5043**
J_2	**0.4396**	**0.3909**
J_3	**0.5114**	**0.3968**
J_4	**0.5125**	**0.5055**
J_5	**0.6267**	**0.1161**
rms act. volt. (volts)	**0.1430**	**0.9048**
rms act. acc. (g's)	**1.1218**	**0.5121**
rms act. displ. (cm)	**0.6700**	**0.5043**

	Reference	PBL (El Centro)	PBL (Hachinohe)
J_6	**0.4556**	0.4267	**0.4598**
J_7	**0.7102**	0.6483	**0.8200**
J_8	**0.6680**	0.4762	**0.5069**
J_9	**0.7753**	0.4728	**0.5797**
J_{10}	**1.3360**	**1.0941**	0.9177
rms act. volt. (volts)	**0.5255**	**0.4513**	0.2328
rms act. acc. (g's)	**4.8275**	**5.5251**	2.3676
rms act. displ. (cm)	**2.0017**	**1.6049**	0.8415

summation worst J's	**6.3193**	**5.5841**

Table 2: Comparison between Reference and PBL Controller # 1 (in bold maximum values)

Controller #2	Reference	PBL
	ω_g = 37.3 rad/sec	ω_g = 37.3 rad/sec
	ξ_g = 0.3	ξ_g = 0.3
J_1	**0.2840**	**0.4183**
J_2	**0.4396**	**0.6572**
J_3	**0.5114**	**0.2275**
J_4	**0.5125**	**0.2325**
J_5	**0.6267**	**0.6212**
rms act. volt. (volts)	**0.1430**	**0.0739**
rms act. acc. (g's)	**1.1218**	**1.1120**
rms act. displ. (cm)	**0.6700**	**0.2980**

	Reference	PBL (El Centro)	PBL (Hachinohe)
J_6	**0.4556**	0.4733	**0.5008**
J_7	**0.7102**	0.7164	**0.9016**
J_8	**0.6680**	**0.3606**	0.2863
J_9	**0.7753**	**0.3650**	0.3538
J_{10}	**1.3360**	**1.0467**	0.8848
rms act. volt. (volts)	**0.5255**	**0.3293**	0.1413
rms act. acc. (g's)	**4.8275**	**5.2856**	2.2828
rms act. displ. (cm)	**2.0017**	**1.2154**	0.4752

summation worst J's	**6.3193**	**5.3314**

Table 3: Comparison between Reference and PBL Controller # 2 (in bold maximum values)

Controller #3	Reference	PBL
	ω_g = 37.3 rad/sec	ω_g = 37.3 rad/sec
	ξ_g = 0.3	ξ_g = 0.3
J_1	**0.2840**	**0.3209**
J_2	**0.4396**	**0.4961**
J_3	**0.5114**	**0.4299**
J_4	**0.5125**	**0.4346**
J_5	**0.6267**	**0.4742**
rms act. volt. (volts)	**0.1430**	**0.1394**
rms act. acc. (g's)	**1.1218**	**0.8489**
rms act. displ. (cm)	**0.6700**	**0.5632**

	Reference	PBL (El Centro)	PBL (Hachinohe)
J_6	**0.4556**	0.4222	**0.4513**
J_7	**0.7102**	0.6798	**0.7967**
J_8	**0.6680**	**0.5777**	0.5556
J_9	**0.7753**	0.5618	**0.6440**
J_{10}	**1.3360**	0.8657	**0.8894**
rms act. volt. (volts)	**0.5255**	**0.5067**	0.2656
rms act. acc. (g's)	**4.8275**	**4.3716**	2.2947
rms act. displ. (cm)	**2.0017**	**1.9467**	0.9223

summation worst J's	**6.3193**	**5.5148**

Table 4: Comparison between Reference and PBL Controller # 3 (in bold maximum values)

Controller #4	Reference	PBL
	ω_g = 37.3 rad/sec	ω_g = 37.3 rad/sec
	ξ_g = 0.3	ξ_g = 0.3
J_1	**0.2840**	**0.2636**
J_2	**0.4396**	**0.4042**
J_3	**0.5114**	**0.5742**
J_4	**0.5125**	**0.5783**
J_5	**0.6267**	**0.5804**
rms act. volt. (volts)	**0.1430**	**0.1807**
rms act. acc. (g’s)	**1.1218**	**1.0389**
rms act. displ. (cm)	**0.6700**	**0.7523**

	Reference	PBL (El Centro)	PBL (Hachinohe)
J_6	**0.4556**	0.3997	**0.4233**
J_7	**0.7102**	0.6282	**0.7180**
J_8	**0.6680**	**0.7651**	0.7595
J_9	**0.7753**	0.7463	**0.8705**
J_{10}	**1.3360**	**1.0806**	0.9964
rms act. volt. (volts)	**0.5255**	**0.6614**	0.3539
rms act. acc. (g’s)	**4.8275**	**5.4568**	2.5708
rms act. displ. (cm)	**2.0017**	**2.5785**	1.2608

summation worst J’s	**6.3193**	**6.2582**

Table 5: Comparison between Reference and PBL Controller # 4 (in bold maximum values)

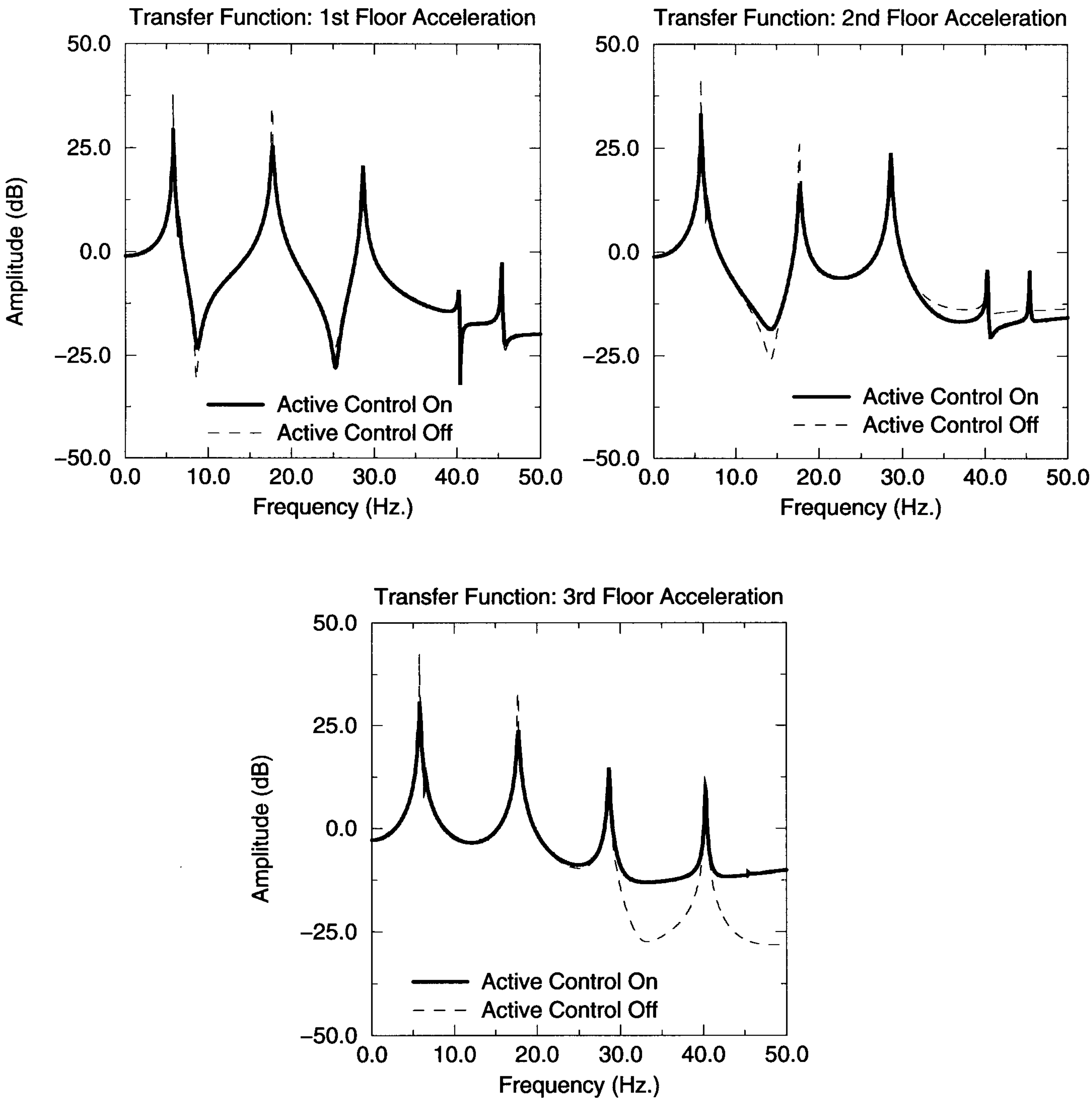

FIG. 1: Transfer Functions of Floor Accelerations: Controller 1

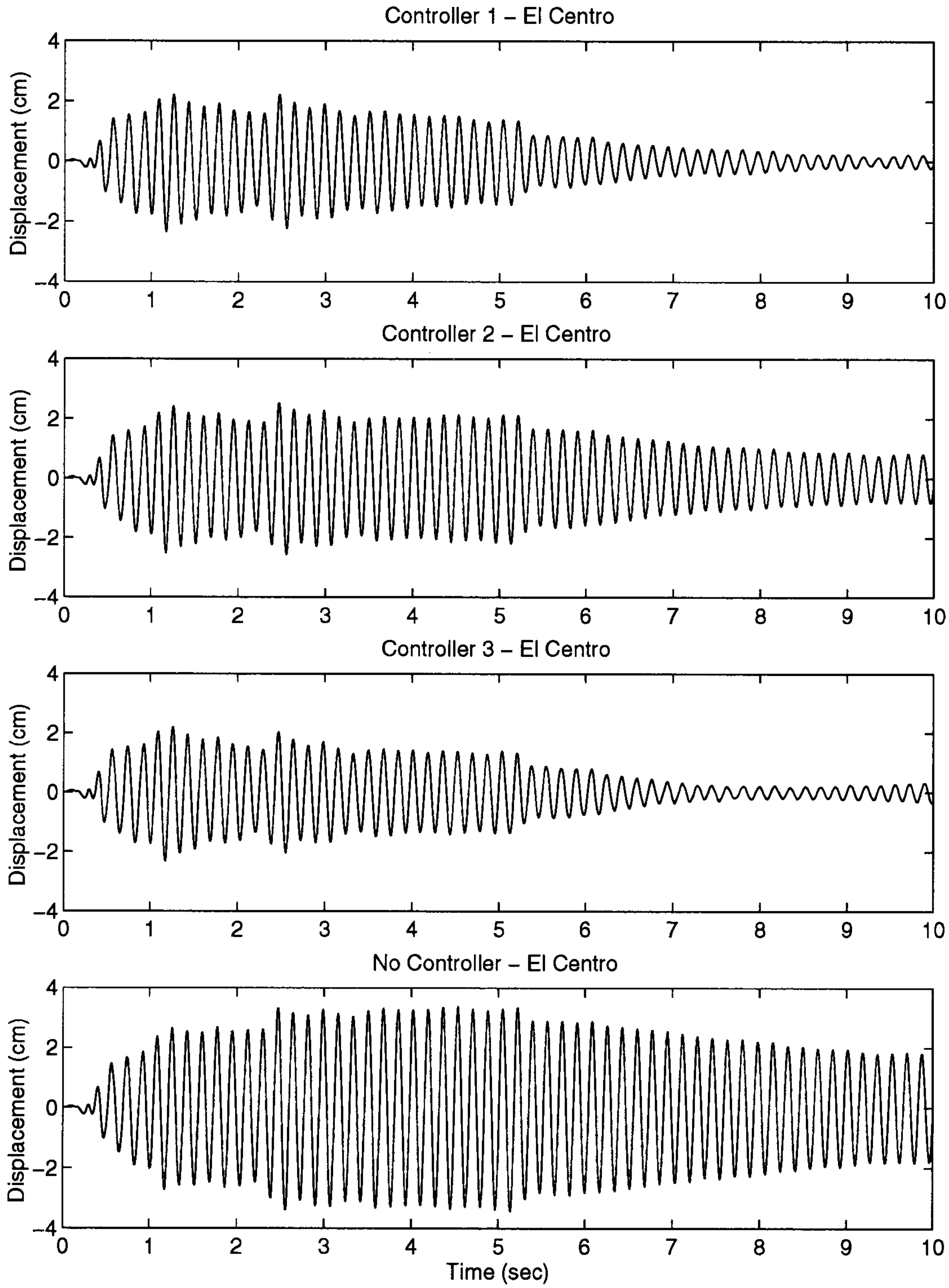

FIG. 2: El Centro 1940 NS 3rd Floor Displacements

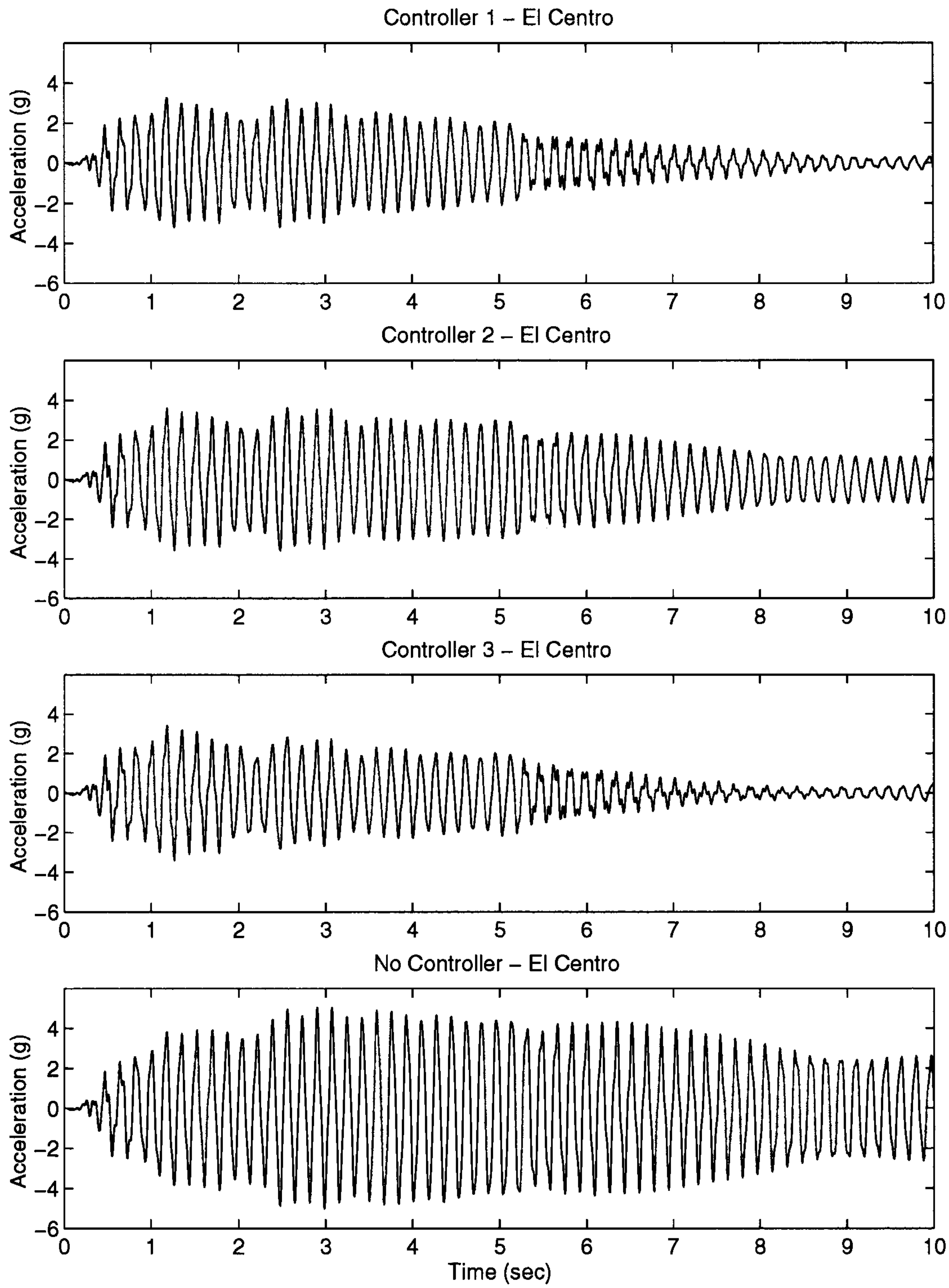

FIG. 3: El Centro 1940 NS 3rd Floor Accelerations

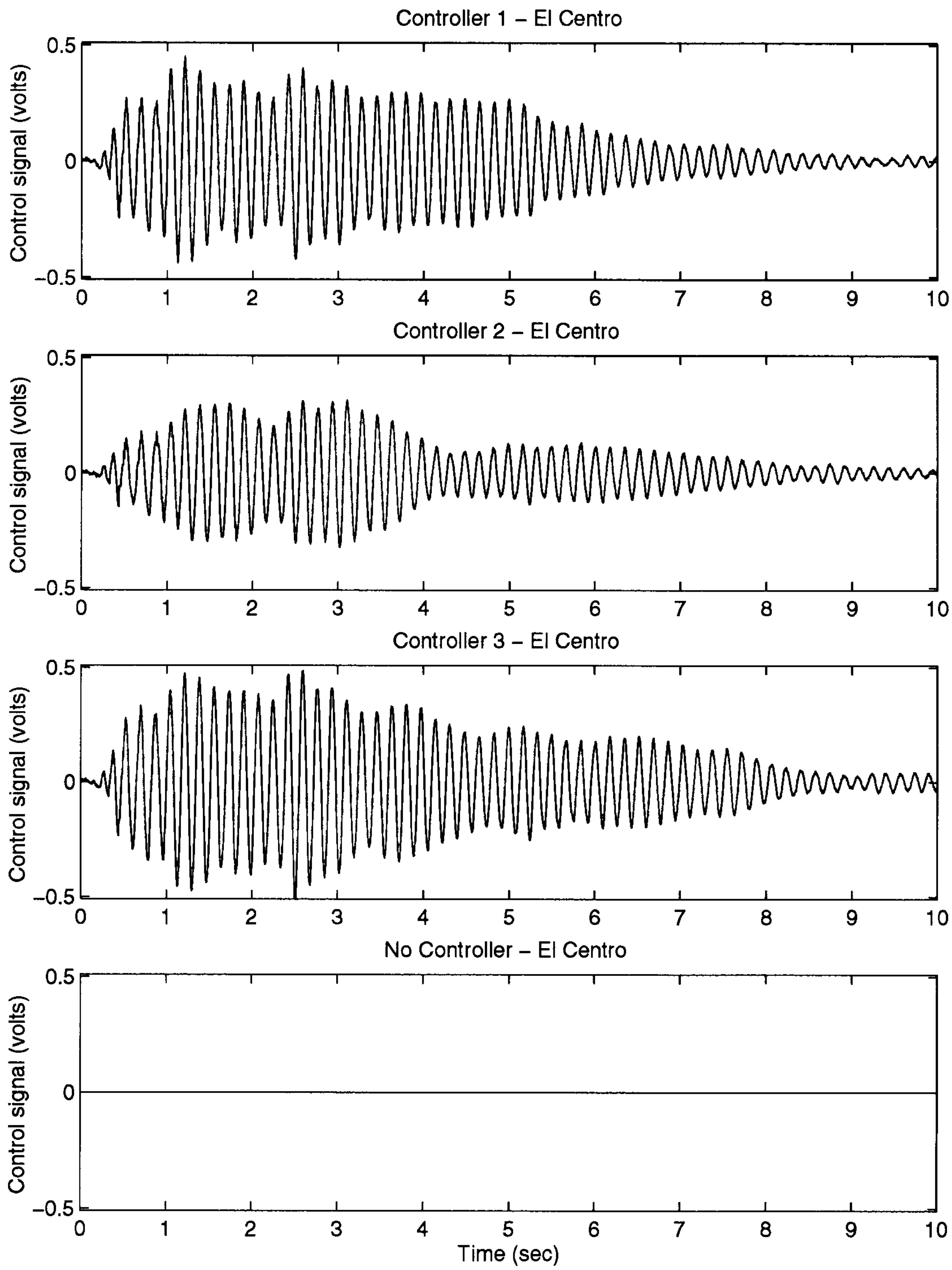

FIG. 4: El Centro 1940 NS Control Signals

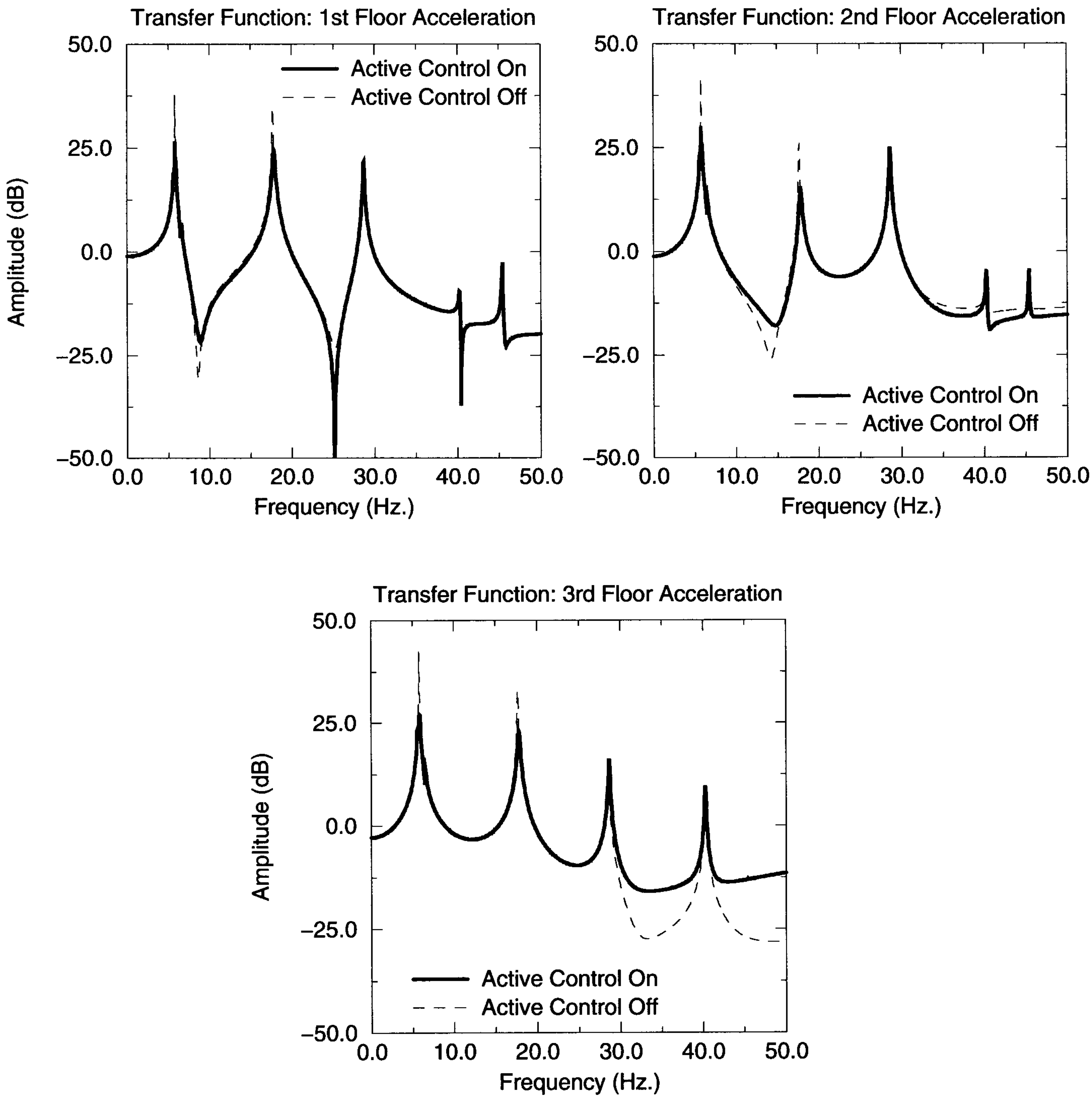

FIG. 5: Transfer Functions of Floor Accelerations: Controller 4

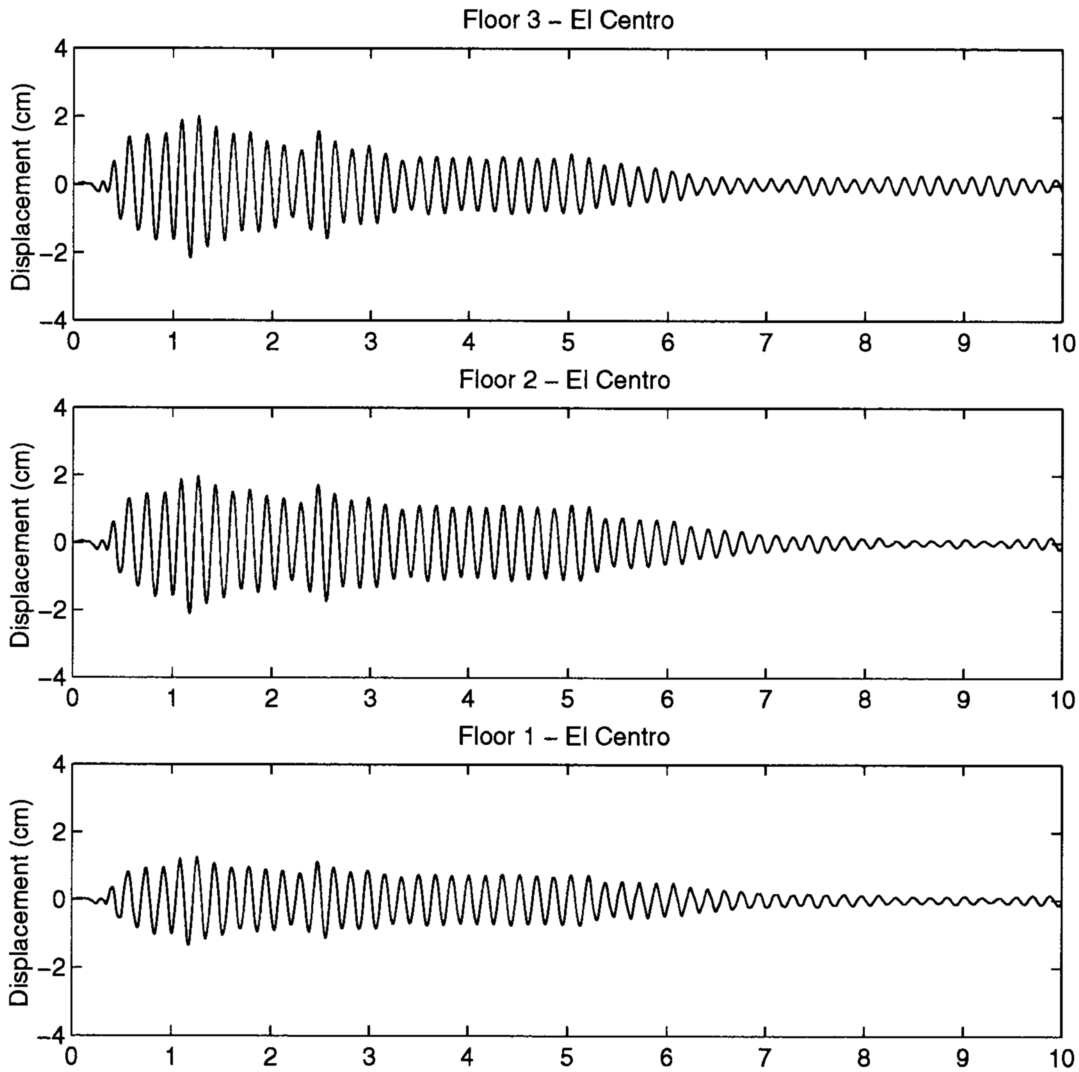

FIG. 6: El Centro 1940 NS Displacement Response: Controller 4

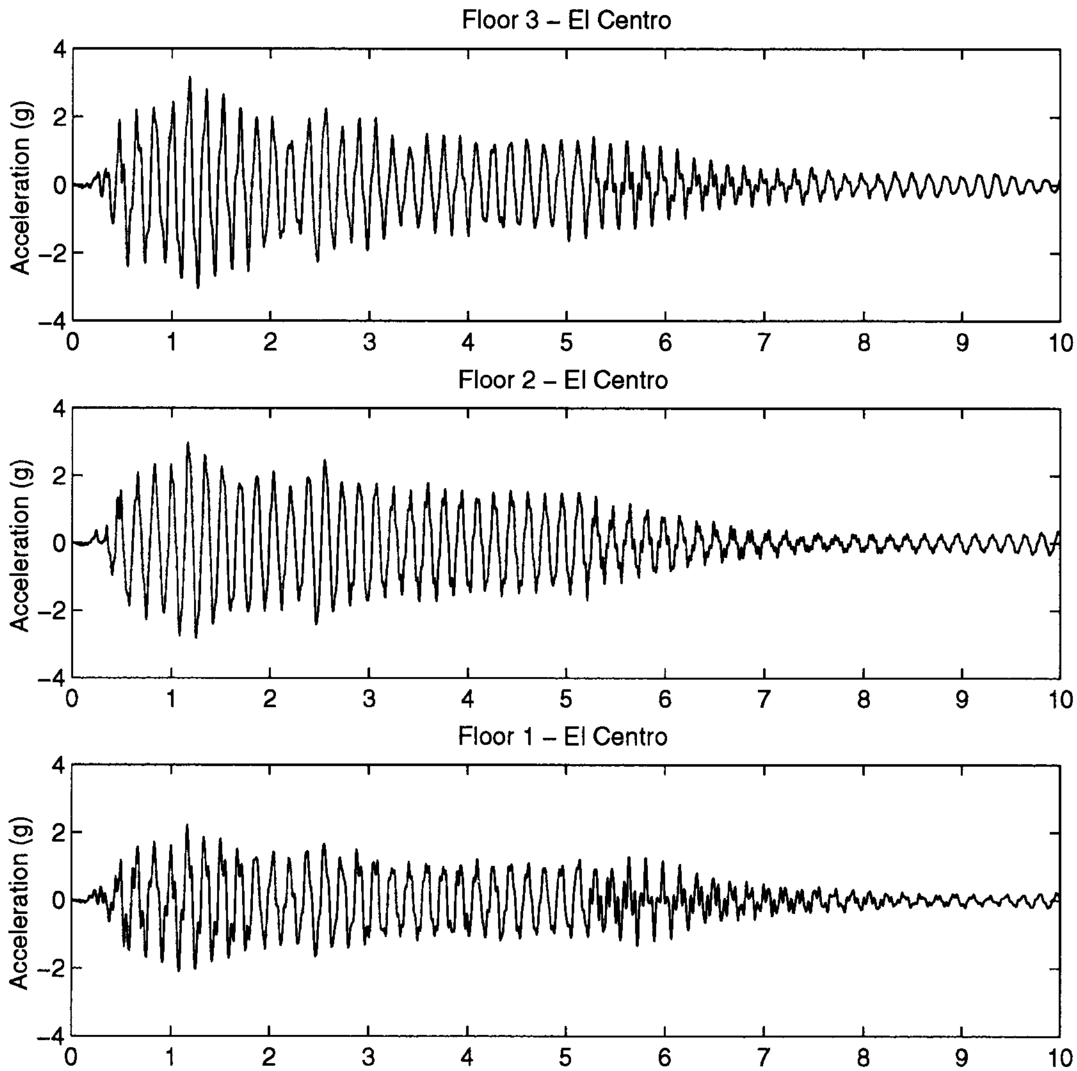

FIG. 7: El Centro 1940 NS Acceleration Response: Controller 4